Spin Structure Moments of the Proton and Deuteron
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Moments of the spin structure functions \( g_1 \) and \( g_2 \) of the proton and deuteron have been measured in the resonance region at intermediate four momentum transfer. We perform a Nachtmann moment analysis of this data, along with isovector and isoscalar combinations, in order to rigorously account for target mass effects. This analysis provides the first definitive evidence for dynamic higher twists.

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Moments of the nucleon spin structure functions \( g_1 \) and \( g_2 \) have proven to be powerful tools to test sum rule predictions and effective theories of QCD. In particular, the Operator Product Expansion (OPE) \( \Gamma_1(Q^2) \) relates these moments to reduced quark and gluon matrix elements. This perturbative approach to QCD is formulated in terms of the Cornwall-Norton (CN) moments \( \Gamma_{1,2}^{(n)}(Q^2) \):

\[
\Gamma_{1,2}^{(n)}(Q^2) = \int_0^1 dx x^{n-1} g_{1,2}(x, Q^2)
\]

(1)

where \( g_1 \) and \( g_2 \) are the spin structure functions of a spin-1/2 particle, \( x \) is the Bjorken scaling variable, and the four-momentum transfer squared of the scattering process is denoted by \( Q^2 \). Typically, the superscript is dropped for the simplest case of \( n = 1 \). In the OPE, the CN moments are expanded in powers of \( 1/Q \) or ‘twist’ \( \tau \), which is defined as the mass dimension minus the spin \( n \) of the QCD operator (\( n \geq \tau - 1 \) with \( n \) odd). The leading twist-2 term reproduces the successful predictions of the parton model, while higher twists arise from the non-perturbative multiparticle interactions. Phenomenological studies of higher twist effects have typically focused on two particular CN moments: \( \Gamma_1(Q^2) \) and \( I(Q^2) = \int_0^1 dx x^2 (2g_1 + 3g_2) \) in order to extract the twist-3 and twist-4 matrix elements. However, as recently stressed in \( [20, 21] \), this approach is only appropriate when terms of order \( M^2/Q^2 \) can be neglected. These terms, which are of purely kinematic origin, correspond to matrix elements of the same twist but increasing spin \( n \), and are given for each \( \tau \) by the \( n + 2 \) moments of the spin structure functions in the massless limit. Referred to as ‘target mass corrections’ (TMC) \( [22, 23, 24, 25, 26, 27, 28] \), they are formally related to twist-2 operators and must be cleanly separated.
At finite moments reduce to the more familiar CN moments: scaling variable Nachtmann moments \([25, 29]\). These moments utilize the target. They are defined as:

\[
M_n^1(Q^2) \equiv \frac{\bar{a}_n}{2} = \frac{1}{2} a_n E_n^1(Q^2, g) = \int_0^1 \frac{dx}{x} x^{n+1} \left( x - \frac{n^2}{(n+2)^2} \frac{M^2 x^2}{Q^2} \right) g_1 - \frac{n}{n+2} \frac{M^2 x^2}{Q^2} g_2
\]

\[
M_n^2(Q^2) \equiv \frac{\bar{a}_n}{2} = \frac{1}{2} d_n E_n^2(Q^2, g) = \int_0^1 \frac{dx}{x} x^{n+1} \left( \frac{x}{\xi} g_1 + \left\{ \frac{n}{n-1} \frac{x^2}{\xi^2} - \frac{n}{n+1} \frac{M^2 x^2}{Q^2} \right\} g_2 \right)
\]

Here \( n = 1, 3, \ldots \) for \( M_n^1 \), and \( n = 3, 5, \ldots \) for \( M_n^2 \), and \( g = 4\pi \alpha_s(Q^2) \) depends on the strong coupling constant \( \alpha_s(Q^2) \). The \( a_n \) (\( d_n \)) represent twist-2 (-3) matrix elements, while the \( E_n^1 \) are the corresponding Wilson coefficients, containing logarithmic QCD corrections. For convenience, these corrections are absorbed into the definition of the effective matrix elements \( \bar{a}_n \) and \( \bar{d}_n \).

In the limit \( M^2/Q^2 \to 0 \), (i.e. TMC vanish) the Nachtmann moments reduce to the more familiar CN moments:

\[
M_1^1(Q^2) \to \Gamma_1(Q^2) \quad 2M_2^3(Q^2) \to I(Q^2)
\]

At finite \( Q^2 \), the ratio of Nachtmann to CN moments gives a quantitative measure of the TMC:

\[
R_1(Q^2) = \frac{M_1^1(Q^2)}{\Gamma_1(Q^2)} \quad R_2(Q^2) = \frac{2M_2^3(Q^2)}{I(Q^2)}
\]

As emphasized in \([21]\), dynamical higher twists can be extracted from the measured \( g_1 \) and \( g_2 \) by using the Nachtmann moments \([23, 29]\). These moments utilize the scaling variable \( \xi = 2x/(1 + \sqrt{1 + (2M x^2)/Q^2}) \) which generalizes Bjorken \( x \) to account for the finite mass of the target. They are defined as:

\[
\Gamma_1(Q^2) \quad \text{and} \quad \Gamma_2(Q^2)
\]

FIG. 1: Spin structure function \( g_2(x, Q^2) \). Full circles : RSS data. Solid line : RSS Fit \([30]\). Shaded curve: \( g_2^{WW} \).

from the desired dynamical higher twists.

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\]

As pointed out in \([21, 29]\) the Nachtmann moments of order \( n \) relate the measured spin structure functions to the corresponding matrix element of a specific twist-\( \tau \) and \( n \) without admixture of higher \( n \) or \( \tau \), as occurs in the CN moments. As a consequence, the Nachtmann approach is accurate \([21]\) to \( O(M^2/Q^2)^4 \), whereas the CN approach ignores corrections of \( O(M^2/Q^2) \). The absence of precise \( g_2 \) data has until now limited the use of Nachtmann moments, but their use is critical in order to cleanly extract higher twist contributions.

Experiment E01-006 was conducted in Hall C of the Thomas Jefferson National Accelerator Facility by the Resonance Spin Structure (RSS) collaboration. We measured the parallel and perpendicular double spin asymmetries \( A_1 \) and \( A_\perp \) in the scattering of 100 nA polarized 5.755 GeV electrons on polarized protons and deuterons. We extracted the virtual photon asymmetries \( A_1 \) and \( A_\perp \) from \( A_1 \) and \( A_\perp \), using only the ratio of longitudinal to transverse cross sections \( R \) \([31, 52]\) as model dependent input. The spin structure functions \( g_1 \) and \( g_2 \) were then obtained utilizing a fit to the unpolarized structure function \( F_1 \) \([31, 52]\) based primarily on data measured previously in Hall C. Simultaneous determination of \( g_1 \) and \( g_2 \) enables us to evaluate the Nachtmann moments of Eqs. \([2]\) and \([3]\) without model input, unlike some spin-dependent experiments which only measure \( A_0 \). Scattered electrons were detected at an angle of 13.15° using the Hall C High Momentum Spectrometer. The kinematic coverage in invariant mass was 1.090 < \( W < 1.910 \) GeV, corresponding to a range \( x_{\text{min}} = 0.316 < x < x_{\text{max}} = 0.823 \) in Bjorken variable at an average four-momentum transfer.
The moment integrals can be decomposed into contributions from the elastic, resonance and $x \to 0$ regions. The elastic contribution (EL) at $x = 1$ can be easily evaluated using the form factor parametrizations of \[34, 35\]. We designate the unmeasured contribution for $0 < x_{\text{min}}$ by the shorthand (DIS), although we note that this contribution is not strictly deep inelastic scattering due to the low $Q^2$. The measured data fall in the resonance (RES) region $x_{\text{min}} < x < x_{\text{thr}}$, where the nucleon inelastic threshold is represented by $x_{\text{thr}}$. We note that the small difference between our experimental $x_{\text{max}}$ and $x_{\text{thr}}$ has negligible impact on the integrated results. Given the high quality deuteron and proton data we can evaluate both the neutron and nonsinglet moments as shown in Table I. Further details can be found in [30] and [33].

The unmeasured contribution to the $g_1$ integrals from the region $x < x_{\text{min}}$ is provided by a Regge-inspired fit to SLAC E143 [3] and E155 [4] $g_1^n$ and $g_1^d$ data within a band $Q^2 = 1.3 \pm 0.3 \text{ GeV}^2$. We find: $g_1 = ax^b(1 - x)^c(1 + c/Q^2)$ where $a = 0.392 \pm 0.254$, $b = 0.0676 \pm 0.084$, $c = 0.0636 \pm 0.0681$ for the proton, and $a = 1.778 \pm 0.0654$, $b = 0.739 \pm 0.0359$, and $c = -0.786 \pm 0.025$ for the deuteron. To estimate the contribution for the $g_2$ integrals for $x < x_{\text{min}}$, we assumed that the Wandzura-Wilczek [35] relation holds as $x \to 0$:

\[ g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy \quad (6) \]

In Eq. (6) we utilize the fit to our own resonance data [30] for $g_1$. We estimate a 15% uncertainty on this contribution by observing the spread of results that are obtained when using several different models for $g_1$ as input. Fig. 1 shows the behavior of our $g_2$ data near our low $x$ limit, which for the proton show that $g_2 = g_2^{WW}$ to be nearly constant and consistent with zero within errors [30], while for the deuteron they show a similar behavior, different from zero only at the one standard deviation (statistical) level. From this we conclude that use of the twist-2 $g_2^{WW}$ approximation is well justified in this region.

The unmeasured contribution (DIS) to $\bar{d}_2$ and $I(Q^2)$ is taken to be zero, reflecting the expected decreasing importance of higher twists at low $x$, and the further suppression of the low $x$ contributions by the $\xi^2$ or $x^2$ weights in the integrals. This expectation is supported by our low $x$ data as discussed above. The error in this assumption can be estimated from the value of $\delta g_2^d$, at $x_{\text{min}}$, as evaluated from our proton and deuteron fits. For $x < x_{\text{min}} = 0.316$, $\xi \approx x$ and the CN and Nachtman moments converge, so the contribution to both integrals is equal in this region. For the proton, we find an upper limit for $I^{\text{DIS}}$ of $0.0008$, by assuming a constant extrapolation of $\delta g_2^d$ as $x \to 0$. For the deuteron, we evaluated both a linear and constant extrapolation of $\delta g_2^d$ as $x \to 0$ to find $I^{\text{DIS}} = 0.0013$ and $0.0011$ respectively, both different from zero at one sigma. We averaged both assumptions and added quadratically one-half their difference to the error from the fit to obtain an upper limit $I^{\text{DIS}}$ of $0.0012$.

Table II provides numerical values for the lowest moments of $g_1$, which have also been partially discussed in a previous publication [30]. For the contribution of the resonances we evaluate the moments at constant $g_1(x, Q^2)$ $= 1.28 \text{ GeV}^2$ using a fit [30] to our measured data. The lowest moment of the isovector nucleon $g_1$ structure function $\Gamma_1^{V - A}$ is related to the nucleon axial charge $g_A$ via the Bjorken sum rule [39]. This relation is a direct consequence of QCD, and experimental tests [4, 6] of the sum rule at large $Q^2$ have played a critical role in establishing QCD as the correct theory of the strong interaction. Lower $Q^2$ measurements [40, 41] provide valuable information on the running of the strong coupling constant $\alpha_s$. Table II shows our results for the Bjorken sum rule, labeled $\Gamma_1^{\text{Non-Singlet}}$. Following the procedure outlined in [42, 43] we find $\alpha_s(Q^2 = 1.28 \text{ GeV}^2) = 0.331 \pm 0.042$.

The first moment of the $g_2$ structure function is predicted by the Burkhardt-Cottingham (BC) sum rule [44]:

\[ \Gamma_2 = \int_0^1 dx \, g_2(x, Q^2) = 0 \quad (7) \]

This sum rule arises from the unsubtracted dispersion relation for the spin-dependent virtual-virtual Compton scattering amplitude $S_2$, and is valid for any value of $Q^2$. Tests of the BC sum rule rely on precise measurements of the $g_2$ structure function, which has been historically neglected due to the technical difficulties of producing transversely polarized targets, and the lack of a simple interpretation of $g_2$ in the classic parton model. The E155 collaboration [4] found their data to be inconsistent with the proton BC sum rule at $Q^2 = 5 \text{ GeV}^2$, while the same group found the deuteron sum rule to hold within the large experimental uncertainty. At lower momentum transfer, the E94010 collaboration [13, 45] found that both the neutron and nuclear $^3\text{He}$ BC sum rules held for

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\Gamma_1^f$</th>
<th>$\bar{d}_2^f$</th>
<th>$\bar{d}_2^g$</th>
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<tbody>
<tr>
<td>RC</td>
<td>3.2</td>
<td>4.0</td>
<td>9.5</td>
</tr>
<tr>
<td>$F_1$</td>
<td>3.0</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$R$</td>
<td>0.9</td>
<td>1.8</td>
<td>1.2</td>
</tr>
<tr>
<td>$P_0P_1$</td>
<td>1.6</td>
<td>1.5</td>
<td>3.2</td>
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<tr>
<td>$Q^2$ dependence</td>
<td>0.33</td>
<td>0.1</td>
<td>4.3</td>
</tr>
<tr>
<td>Total</td>
<td>6.8</td>
<td>8.0</td>
<td>13.0</td>
</tr>
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</table>

TABLE I: Systematic uncertainties in % arising from the target dilution factor $f$, radiative corrections RC, model input for $F_1$ and $R$, fit evolution to constant $Q^2$, and beam and target polarimetry $P_0P_1$.
0.1 < \langle Q^2 \rangle < 0.9 \text{ GeV}^2$. Fig. 2 displays the first moment of $g_2$ for both the proton and neutron. The total integral exhibits a striking cancellation of the inelastic and elastic contributions, leading to satisfaction of the BC sum rule within uncertainties. Two upcoming JLab experiments will greatly expand our knowledge of $g_2$ with measurements planned in the ranges 0.02 < \langle Q^2 \rangle < 0.4 \text{ GeV}^2$ and 2.5 < \langle Q^2 \rangle < 6.5 \text{ GeV}^2.

We have calculated $d_2 = 2M_1^2$ and $I(Q^2)$ for the proton, deuteron, non-singlet and neutron structure functions in Table II. The difference of these two quantities (reflected in $R_2$) clearly demonstrates the importance of kinematic higher twists in the non-perturbative regime.

The values indicated in Table II represent only the inelastic part of $d_2$. The systematic and low $x$ extrapolation errors have been combined in quadrature. The non-singlet moment plus the twist-3 matrix element is entirely consistent with zero.

We can use the pQCD evolution [18, 49] of $\bar{d}_2^P$ and $\bar{d}_2^{NS}$ to compare our result $\bar{d}_2^P(5 \text{ GeV}^2) = 0.0021 \pm 0.0006$ with the result of SLAC's measurements [3] at $\langle Q^2 \rangle = 5 \text{ GeV}^2$ $\bar{d}_2^P = 0.0025 \pm 0.0015$, corrected for target mass effects according to [21]. SLAC's result does not include the elastic contribution (0.0003 at 5 GeV$^2$). Similarly, our $\bar{d}_2^P(5 \text{ GeV}^2) = 0.0053 \pm 0.0052$ and SLAC's inelastic, TMC corrected $\bar{d}_2^P = 0.0074 \pm 0.0045$ agree within errors. We have also calculated the NLO corrections to our singlet and non-singlet results [50, 51]. Applying the resulting Wilson coefficients at our kinematics $E_3^{28} = 1.436$ and $E_3^{2NS} = 1.26$ we get NLO $\bar{d}_2^P = 0.0026 \pm 0.0005 \pm 0.0006$, where the first error represents combined measured errors and the second one comes from the low $x$ extrapolation.

Higher twist contributions can be extracted by subtracting from the OPE expansion [18] of the measured first CN moment (asymptotic $\Gamma_1$) the twist-2 $\bar{a}_0 \equiv 2M_1^2$ and $\bar{a}_2 \equiv 2M_1^4$, and the twist-3 $\bar{d}_2$ matrix elements. We are left with the $\bar{f}_2$ twist-4 matrix element plus $O(M^4/Q^4)$ terms. Our results for the proton $\bar{f}_2 + O(M^4/Q^4) = -0.0013 \pm 0.0070$ and the deuteron $\bar{f}_2 + O(M^4/Q^4) = -0.0008 \pm 0.0100$ only indicate that there may be a significant cancellation among the higher twists, but the errors (which only include the contributions of $\Gamma_1$ and $\bar{d}_2$) are too large to confirm this.

The target mass corrections to $\Gamma_1$ given by the ratio $R_1$ are small at our kinematics (5% for $g_2^p$, 2% for $g_2^n$), due to a combination of dominant contributions from the $x$ region below our lowest measured $x_{\text{min}} = 0.316$, and the nearly identical values of $x$ and $\xi$ in this region. For the neutron and Bjorken sum, the TMC cancel almost exactly. On the other hand, Ref. [21] concludes that experimental determinations of $I(Q^2)$ significantly overestimate contributions to the twist-3 matrix element for $Q^2 < 10 \text{ GeV}^2$, which is supported by our measured values of $R_2$.

In summary, we have measured the spin structure function moments of the proton, deuteron and neutron in the resonance region at $Q^2 = 1.28 \text{ GeV}^2$. We find good agreement with world data for $\Gamma_1$, and satisfaction of the BC

<table>
<thead>
<tr>
<th>I(Q^2)</th>
<th>Deuteron</th>
<th>Neutron</th>
<th>P-N</th>
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<tr>
<td>28 GeV</td>
<td>$\bar{d}_2^P$</td>
<td>$\bar{d}_2^{NS}$</td>
<td>$\bar{d}_2^P$</td>
</tr>
<tr>
<td>28 GeV</td>
<td>0.0021</td>
<td>0.0006</td>
<td>0.0026</td>
</tr>
<tr>
<td>5 GeV</td>
<td>0.0025</td>
<td>0.0015</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

TABLE II: CN and Nachtmann Moments at $Q^2 = 1.28 \text{ GeV}^2$. Deuteron results exclude contributions below the pion production threshold. Non-Singlet $\Gamma^{NS} = \Gamma^p - \Gamma^n$. Neutron $\Gamma^n = \Gamma^d/\Gamma^D - \Gamma^n$. Note: The elastic contribution is included only in the $\Gamma_2$ TOT value.
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