Let \( n_{ijkt} \) be the number of reported health professionals of type \( i \) in county \( k \) of state \( j \) at time \( t \). Assume \( t = 2000, 2003; k = 1, 2, ..., K_j \). Define

\[
n_{ijt} = \sum_{k=1}^{K_j} n_{ijkt}
\]

to be the reported number of health professionals of type \( i \) in state \( j \) at time \( t \).

We have reason to believe that variation in \( n_{ijt} / n_{imt} \) over time for two states \( j, m \) is not reliable because of variation across states in the way data was defined and/or collected in 2003. Thus, we should adjust the 2003 data to reflect this.

In particular, a better estimate for 2003 is

\[
\frac{n_{ijk2003}^*}{n_{ijk2003}} = \frac{n_{ij2000}}{n_{ij2003}} n_{ij2003}
\]

This adjustment imposes the condition that

\[
\sum_{k=1}^{K_j} n_{ijk2003}^* = \sum_{k=1}^{K_j} \frac{n_{ij2000}}{n_{ij2003}} n_{ij2003}
\]

\[
= \frac{n_{ij2000}}{n_{ij2003}} \sum_{k=1}^{K_j} n_{ij2003}
\]

\[
= \frac{n_{ij2000}}{n_{ij2003}} n_{ij2003}
\]

\[
= n_{ij2000}.
\]

If we also have reliable information on growth rates in health professionals of type \( i \), we can use them to improve our estimates a little. Define

\[
n_{it} = \sum_{j} n_{ijt},
\]

and assume we have a reliable estimate of \( n_{i2003} / n_{i2000} \). Then we can improve our estimate for each county as

\[
\frac{n_{ijk2003}^{**}}{n_{ij2000}} = \frac{n_{i2003}}{n_{i2000}} \frac{\sum_j n_{ij2003}^*}{n_{ij2000}}
\]

This adjustment imposes the condition that

\[
\sum_{j} \sum_{k=1}^{K_j} n_{ijk2003}^{**} = \sum_{j} \sum_{k=1}^{K_j} \frac{n_{i2003}}{n_{i2000}} n_{ij2003}^*
\]

\[
= \frac{n_{i2003}}{n_{i2000}} \sum_{j} \sum_{k=1}^{K_j} n_{ij2003}
\]

\[
= \frac{n_{i2003}}{n_{i2000}} \sum_{j} n_{ij2003}
\]

\[
= \frac{n_{i2003}}{n_{i2000}} n_{i2000}
\]

\[
= n_{i2003}.
\]