Lecture 3 Gauss’s Law Ch. 23

• Cartoon - Electric field is analogous to gravitational field
• Physlet  /webphysics.davidson.edu/physletprob
• Topics
  • Flux
  • Electric Flux and Example
  • Gauss’ Law
  • Coulombs Law from Gauss’ Law
  • Isolated conductor and Electric field outside conductor
  • Application of Gauss’ Law
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• Demo
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Electric Flux

Flux is a measure of the number of field lines passing through an area.

Electric flux is the number of Electric field lines penetrating a surface or an area.

In general, \( \text{Flux} = \Phi = \text{normal component of the field} \times \text{area} \)

Electric Flux = \( \Phi = (E \cos \theta)A = \vec{E} \cdot \vec{A} \) where \( \vec{A} = |\vec{A}| \cdot \hat{n} \)

\[ a \quad \vec{E} \parallel \vec{A} \quad \Rightarrow \theta = 0 \]
So, \( \Phi = EA \cos 0 = EA \)

\[ b \quad \vec{E} \perp \vec{A} \quad \Rightarrow \theta \neq 0 \]
Let \( \theta = 45^\circ \) Then,
\( \Phi = EA \cos 45^\circ = 0.707EA \)
Gauss’s Law

• Gauss’s law makes it possible to find the electric field easily in highly symmetric situations.

• Drawing electric field lines around charges leads us to Gauss’ Law

• The idea is to draw a closed surface like a cylinder, sphere or other shape around any charge distribution, then some field line will exit through the surface and some will enter or reenter. If we count those that leave as positive and those that enter as negative, then the net number leaving will give a measure of the net positive charge inside.

\[
\Phi_{\text{net}} = \frac{q_{\text{enc}}}{\varepsilon_0}
\]

If there is no charge inside, then the flux is 0

\[
\Phi_{\text{net}} = 0
\]
Simple Example

Find the electric flux through a cylindrical surface in a uniform electric field $E$

- Uniform electric field
- Cylindrical surface choose because of symmetry
- No charge inside
- Flux should be 0
- We want to calculate $\Phi = \oint \vec{E} \cdot d\vec{A}$
Remember

- The flux $\Phi$ through a plane surface of area $A$ due to a uniform field $E$ is a simple product:
  $\Phi = EA$ where $E$ is normal to the area $A$.

- $\Phi = E_n A = 0 \times A = 0$ because the normal component of $E$ is 0

$\Phi = E_n A = E \cos \theta \cdot A$
Find the electric flux through a cylindrical surface in a uniform electric field $E$.

\[ \Phi = \oint E \cdot d\vec{A} = \oint E \cos \theta dA \]

a. $\Phi = \int E \cos 180 dA = -\int E dA = -E\pi R^2$

b. $\Phi = \int E \cos 90 dA = 0$

c. $\Phi = \int E \cos 0 dA = \int E dA = E\pi R^2$

Flux from a. + b. + c. = 0

What would be the flux if the cylinder were vertical?

Suppose it were any shape?
Derivation of Gauss’ Law from a point charge and using Coulomb's law: Summary of steps

- Start with an isolated point charge.
- Choose a sphere around the charge.
- Calculate \( \Phi_{\text{net}} = \int \bar{E} \cdot d\bar{A} \)
- Show that \( \Phi_{\text{net}} = \frac{q_{\text{enc}}}{\varepsilon_0} \)
Start with a point charge.

Note that the electric lines of flux are radial for a point charge.

Choose a spherical surface to simplify the calculation of the flux.
• Calculate the net flux through the closed surface.

\[
\text{Net Flux} = \Phi_{\text{net}} = \oint \vec{E} \cdot d\vec{A} = \oint E \cos \theta dA = \oint EdA
\]

\[
E \parallel n \quad \cos 0^\circ = 1
\]

For a Point charge \( E = \frac{kq}{r^2} \)

\[
\Phi = \oint EdA = \oint \frac{kq}{r^2} dA
\]

\[
\Phi = \frac{kq}{r^2} \oint dA = \frac{kq}{r^2} (4\pi r^2)
\]

\[
\Phi = 4\pi kq
\]

\[
4\pi k = \frac{1}{\varepsilon_0} \quad \text{where} \quad \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}
\]

\[
\Phi_{\text{net}} = \frac{q_{\text{enc}}}{\varepsilon_0} \quad \text{Gauss’ Law}
\]
Gauss’ Law

\[ \Phi_{net} = \frac{q_{enc}}{\varepsilon_0} \]

This result can be extended to any shape surface with any number of point charges inside and outside the surface as long as we evaluate the net flux through it.
**Approximate Flux**

\[ \Phi = \sum \vec{E} \cdot \Delta \vec{A} \]

**Exact Flux**

\[ \Phi = \oiint \vec{E} \cdot d\vec{A} \]

\[ d\vec{A} = \hat{n}dA \]

Circle means you integrate over a closed surface.
You should know how to prove these things

1. The electric field inside a conductor is 0.
2. The total net charge inside a conductor is 0. It resides on the surface.
3. Find electric field just outside the surface of a conductor.
4. Find electric field around two parallel flat conducting planes.
5. Find electric field of a large non-conducting sheet of charge.
6. Find electric field of an infinitely long uniformly line of charge.

7. Find E inside and outside of a long non-conducting solid cylinder of uniform charge density.
8. Find E for a thin cylindrical shell of surface charge density $\sigma$.
9. Find E inside and outside a solid non-conducting sphere of uniform charge density $\rho$. 
1. Electric field inside a conductor is 0. Why?

- Inside a conductor in electrostatic equilibrium the electric field is always zero. (averaged over many atomic volumes)

- The electrons in a conductor move around so that they cancel out any electric field inside the conductor resulting from free charges anywhere including outside the conductor. This results in a net force of $0$ on any particular charge inside the conductor.

\[ \vec{F} = q \vec{E} \]
2. The net charge inside a conductor is 0.

- Any net electric charge resides on the surface of the conductor within a few angstroms ($10^{-10}$ m).
- Draw a Gaussian surface just inside the conductor. We know everywhere on this surface $E=0$.
- Hence, the net flux is zero. From Gauss’s Law the net charge inside is zero.
- Show Faraday ice pail demo.
3. Find electric field just outside the surface of a conductor

- The electric field just outside a conductor has magnitude \( \frac{\sigma}{\varepsilon_0} \) and is directed perpendicular to the surface.

  - Draw a small pill box that extends into the conductor. Since there is no field inside, all the flux comes out through the top.

\[
EA = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}
\]

\[
E = \frac{\sigma}{\varepsilon_0}
\]
4. Find electric field around two parallel flat conducting planes.

\[ E = \frac{\sigma_1}{\varepsilon_0} \]

\[ E = \frac{\sigma_1}{\varepsilon_0} \]

\[ E = \frac{2\sigma_1}{\varepsilon_0} \]
6. Find the electric field for an infinite long wire

Charge per unit length $\frac{Q}{L} = \lambda$

$E_n = \vec{E} \cdot \hat{n}$

$E \parallel \hat{n} \Rightarrow \theta = 0^\circ$

$\oint E_n \, dA = E \oint dA = E \cdot 2\pi rh$

$\phi = \oint E_n \, dA = \frac{q}{\varepsilon_0}$

$\phi = \text{endcaps} + \text{side}$

$= 0 + E \cdot 2\pi rh = \frac{\lambda h}{\varepsilon_0}$

$\vec{E} \perp \hat{n}$

$\theta = 90^\circ$

$\cos 90^\circ = 0$

$E = \frac{\lambda}{2\pi \varepsilon_0 r}$
9. Electric field inside and outside a non-conducting solid uniformly charged sphere

- Often used as a model of the nucleus.
- Electron scattering experiments have shown that the charge density is constant for some radius and then suddenly drops off at about

$$2 - 3 \times 10^{-14} \text{ m}.$$ 

For the nucleus, 

$$\rho = 10^{-26} \frac{C}{m^3}$$ 

$\rho = \text{Charge density per unit volume}$ 

$R \times 10^{-14} \text{ m}$
Electric Field inside and outside a uniformly charged sphere

\[ \rho = \frac{Q}{\frac{4}{3} \pi R^3}, \quad r \leq R \]

\[ Q = \text{Total charge} = Z \times 1.6 \times 10^{-19} \text{C} \]

**Inside the sphere:**
To find the charge at a distance \( r < R \)
Draw a gaussian surface of radius \( r \)
By symmetry \( E \) is radial and parallel to normal at the surface. By Gauss’s Law:

\[ \oint E_n \, dA = \frac{q}{\varepsilon_0} \]

\[ E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{\rho}{3} \frac{4\pi R^3}{\varepsilon_0} \]

\[ E = \frac{\rho r}{3\varepsilon_0} \]
Electric Field inside and outside a uniformly charged sphere

\[ \frac{Q}{\frac{4}{3}\pi R^3}, \quad r \leq R \]

Q = Total charge
\[ = Z \times 1.6 \times 10^{-19} C \]

Outside the sphere:

\[ E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_0} \]

\[ E = \frac{\rho R^3}{3\varepsilon_0 r^2} \]

Same as a point charge q
Electric field vs. radius for a conducting sphere

\[ E_r \propto r \]

\[ E_r \propto \frac{1}{r^2} \]
Comment on Sample Problem 23 -4
The figure below shows a cross section through a very large nonconducting slab of thickness $d=9.40$ mm and uniform volume charge density $p=5.80$ fC/m$^3$. The origin of an $x$ axis is at the slab’s centre. What is the magnitude of the slab’s electric field at an $x$ coordinate of (a) 0, (b) 2.00 mm, (c) 4.70 mm, (d) 26 mm?
(Elmo) In the figure below, a solid sphere, of radius $a = 2.10 \text{ cm}$ is concentric with a spherical conducting shell of inner radius $b = 2.00a$ and outer radius $c = 2.40a$. The sphere has a net charge $q_1 = +4.00 \text{ fC}$ which is distributed uniformly through the sphere; the shell has a net charge of $q_2 = -q_1$. 

Chapter 23 Problem 49
Questions

- (a) What is the magnitude of the electric field at radial distance $r = 0$?
- (b) What is the magnitude of the electric field at radial distance $r = a/4$?
- (c) What is the magnitude of the electric field at radial distance $r = a$?
- (d) What is the magnitude of the electric field at radial distance $r = 1.30a$?
- (e) What is the magnitude of the electric field at radial distance $r = 2.10a$?
- (f) What is the magnitude of the electric field at radial distance $r = 3.40a$?
- (g) What is the net charge on the inner surface of the shell?
- (h) What is the net charge on the outer surface of the shell?
• 2) Repeat question 1 for the case of a non-uniform field.

• 3) Suppose you put some charge on an initially-neutral, solid, perfectly-conducting sphere (where the sphere is not in a pre-existing electric field). Describe the electric field inside the conductor, at the surface of the conductor, and outside the conductor as a result of the unbalanced charge. Describe the distribution of the charge in and on the conductor.

• 4) Repeat questions 1-3 for the case of a hollow perfectly-conducting spherical shell (with the interior being vacuum).

• 5) How would your answers to questions 1-4 change if the conductor had some shape other than spherical?
External Electric field penetrates conductor

Free charges inside conductor move around onto the surface in such a way as to cancel out the field inside.
E=0 inside sphere
Orientation of electric field is perpendicular to surface

If the electric field were not perpendicular, then there would be a component of electric field along the surface causing charges to accelerate on the surface and it would not be in electrostatic equilibrium contrary to experience.
Chapter 23 Problem 2

An electric field given by $E = 7.0 \, i - 3(y^2 + 7.0) \, j$ pierces the Gaussian cube of Figure 23-05. (E is in newtons per coulomb and $x$ is in meters.) What is the electric flux through the following surfaces?

(a) the top face
(b) the bottom face
(c) the left face
(d) the back face
(e) the net electric flux through the cube
A long straight wire has fixed negative charge with a linear charge density of magnitude 3.1 nC/m. The wire is to be enclosed by a coaxial, thin-walled, nonconducting cylindrical shell of radius 1.8 cm. The shell is to have positive charge on its outside surface with a surface charge density $\sigma$ that makes the net external electric field zero. Calculate $\sigma$. 
Two charged concentric spherical shells have radii of 10.0 cm and 14.0 cm. The charge on the inner shell is $3.90 \times 10^{-8}$ C and that on the outer shell is $1.70 \times 10^{-8}$ C. Find the magnitude of the electric field at the following points.
(a) at $r = 12.5$ cm
(b) at $r = 20.5$ cm