HW #5

Problem 7.12

a. To Find:

Does the single crystal yield under the given applied stress? If not, at what stress will it yield?

b. Given:

φ = 43.1°  λ = 47.9°  τ_{CRSS} = 20.7 MPa  Applied stress \( \sigma = 45 \) MPa

c. Assumptions:

(i) The applied force is uniaxial

(ii) Single crystal

(iii) The metal is pure (alloying agents/impurities can change the value of \( \tau_{CRSS} \)).

(iv) The testing is done at room temperature – the value of \( \tau_{CRSS} \) can change with temperature.

(v) The values of \( \phi, \lambda, \tau_{CRSS} \) and applied stress \( \sigma \) are accurate.

d. Solution:

Method 1: Calculate applied \( \tau_R \) and compare with \( \tau_{CRSS} \).

\[
\tau_R = \sigma \cos \phi \cos \lambda = (45 \text{ MPa})(\cos 43.1 \degree)(\cos 47.9 \degree) = 22.0 \text{ MPa} \quad (3181 \text{ psi})
\]

\[\Rightarrow \tau_R > \tau_{CRSS}\]

The resolved shear stress is greater than the critical resolved shear stress. Hence, the single crystal will yield.

Method 2: Calculate yield strength and compare with applied stress.

\[
\sigma_Y = \frac{\tau_{CRSS}}{\cos \phi \cos \lambda} = \frac{(20.7 \text{ MPa})}{(\cos 43.1 \degree)(\cos 47.9 \degree)} = 42.29 \text{ MPa}
\]

\[\Rightarrow \sigma > \sigma_Y\]

The applied stress is greater than the yield strength. Hence, the single crystal will yield.
Problem 7.13

a. To Find:
   (a) Most favored slip direction
   (b) $\tau_{\text{CRSS}}$

b. Given:

   $\phi = 28.1^\circ$  $\lambda = \text{one of } 62.4^\circ, 72.0^\circ, 81.1^\circ$  
   Yield strength $\sigma_y = 45 \text{ MPa}$

c. Assumptions:

   (i) The applied force is uniaxial
   (ii) Single crystal
   (iii) The metal is pure alloying agents/impurities can change the value of $\tau_{\text{CRSS}}$.
   (iv) The testing is done at room temperature – the value of $\tau_{\text{CRSS}}$ can change with temperature.

d. Solution:

   (a) Slip will occur along the direction for which $(\cos \phi \cos \lambda)$ is a maximum, which, in this case, 
   corresponds to the largest value of $\cos \lambda$. Cosines for the possible $\lambda$ values are as follows:

   \[
   \begin{align*}
   \cos(62.4^\circ) &= 0.46 \\
   \cos(72.0^\circ) &= 0.31 \\
   \cos(81.1^\circ) &= 0.15 \\
   \end{align*}
   \]

   Hence, the slip direction is at an angle of $62.4^\circ$ with the tensile axis.

   (b) $\tau_{\text{crss}} = \sigma_y (\cos \phi \cos \lambda)_{\text{max}}$

   \[
   = (1.95 \text{ MPa} \left[\cos(28.1^\circ) \cos(62.4^\circ)\right]) = 0.80 \text{ MPa}
   \]

(a) $62.4^\circ$

(b) 0.80 MPa
Problem 7.29

a. To Find:

Compare the hardness of two specimens after cold-work

b. Given:

<table>
<thead>
<tr>
<th></th>
<th>Circular (diameter, mm)</th>
<th>Rectangular (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original dimensions</td>
<td>15.2</td>
<td>125 × 175</td>
</tr>
<tr>
<td>Deformed dimensions</td>
<td>11.4</td>
<td>75 × 200</td>
</tr>
</tbody>
</table>

c. Assumptions:

(i) Given data is accurate.

(ii) The applied force is uniaxial and in a direction perpendicular to the cross-section.

(iii) The deformation is carried out at the same temperature (say room temperature) since we do not wish to consider the possibility of recovery - recovery could cause a decrease in dislocation density.

d. Solution:

Greater the degree of cold work, greater is the hardness of the specimen.

\[
\% \text{CW} = \left( \frac{A_0 - A_d}{A_0} \right) \times 100
\]

For the specimen with circular cross-section:

\[
\% \text{CW} = \left[ \frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \right] \times 100 = \left[ \frac{\pi \left( \frac{15.2 \ mm}{2} \right)^2 - \pi \left( \frac{11.4 \ mm}{2} \right)^2}{\pi \left( \frac{15.2 \ mm}{2} \right)^2} \right] \times 100 = 43.8\%
\]

For the specimen with rectangular cross-section:

\[
\% \text{CW} = \left( \frac{(125 \ mm)(175 \ mm) - (75 \ mm)(200 \ mm)}{(125 \ mm)(175 \ mm)} \right) \times 100 = 31.4\%
\]

Thus, the specimen with circular cross-section will be harder after plastic deformation.
Problem 7.41

a. To Find:

Yield strength of brass after heat-treatment (σ₂)

b. Given:

Yield strength σ₁ = 160 MPa when grain diameter, d₁ = 0.008 mm
k_y = 12 MPa.mm\(^{1/2}\)
Heat-treatment : 600 °C for 1000s

c. Assumptions:

(i) Given data is accurate.

(ii) The composition of brass used in this experiment is the same as that in Fig.7.25.

d. Solution:

Step 1: To evaluate σ₀ from given values of σ₁, k_y and d₁.

σ_y = σ₀ + k_yd\(^{-1/2}\)

σ_y = σ₁ = 160 MPa when d= d₁ = 0.008 mm

⇒ σ₀ = 160MPa −(12.0MPa.mm\(^{1/2}\))(0.008mm\(^{-1/2}\)) = 25.8MPa

To evaluate the grain size after heat treatment using Fig. 7.25.

Step 2: The last paragraph on Page 135 explains how logarithmically scaled axes are to be interpreted.

Based on this, the grain diameter after a 1000s/60 s/min = 16.67 minute heat treatment at 600 °C (d₂) is 0.04 mm. [Depending on how they students the graph, the new grain diameter values could vary between 0.03 and 0.06 microns.]

Step 3: To calculate the new yield strength (σ_y = σ₂) corresponding to the new grain diameter (d=d₂).

σ_y = σ₀ + k_yd\(^{-1/2}\)

σ_y = σ₂ = 25.8 MPa+(12.0 MPa-mm\(^{1/2}\))(0.04mm\(^{-1/2}\)) = 85.8MPa

Based on the value of new grain diameter (d₂) used, this value could vary between 74.8 MPa and 95.08 MPa.

85.8 MPa
**Problem 7.D4**

**a. To Find:**

Determine which materials among copper, brass and 1040 steel are candidates for the given application.

**b. Given:**

The given application requires the material to have a yield strength > 345 MPa AND a ductility of >20% elongation.

The materials may be cold-worked.

**c. Assumptions:**

(i) The yield strength and ductility requirements for the application have been estimated accurately.

(ii) If the material at 0% cold work has a low yield strength, it is cold-worked to increase its yield strength.

(iii) The data in Fig. 7.19 a and 7.19 c is accurate.

**d. Solution:**

Although cold work leads to an increase in yield strength, it leads to a drop in ductility.

   
   **Cu:** Yield strength > 345 MPa when cold work > 60%
   
   **Brass:** Yield strength > 345 MPa when cold work > 20 %
   
   **1040 Steel:** Yield strength > 345 MPa when cold work > 0 %

2. Considering ONLY the ductility criterion – Ref. 7.19 (c).
   
   **Cu:** Ductility > 20% elongation when cold work < 15%
   
   **Brass:** Ductility > 20% elongation when cold work < 22.5%
   
   **1040 Steel:** Ductility > 20% elongation when cold work < 5%

3. **Cu:** There exists no % cold work which allows this material to have a yield strength > 345 MPa AND a ductility of >20% elongation.
   
   **Brass:** Between 20 and 22.5% cold-work, this material has a yield strength > 345 MPa AND a ductility of >20% elongation. *[Since the range is very narrow, some may quote a slightly different range or a single value like 22.5% cold work – it’s ok]*
   
   **1040 Steel:** Between 0 and 5% cold-work, this material has a yield strength > 345 MPa AND a ductility of >20% elongation.

**Brass, 1040 Steel**