The Great Recession: A Self-Fulfilling Global Panic\textsuperscript{1}

Philippe Bacchetta  
University of Lausanne  
Swiss Finance Institute  
CEPR  

Eric van Wincoop  
University of Virginia  
NBER  

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Abstract

While the 2008-2009 financial crisis originated in the United States, we witnessed steep declines in output, consumption and investment of similar magnitudes around the globe. This raises two questions. First, given the partial integration of both goods and financial markets, what can account for the remarkable global business cycle synchronicity during this period? Second, what can explain the difference relative to previous recessions, where we witnessed far weaker co-movement? To address these questions, we develop a two-country model that allows for self-fulfilling business cycle panics. We show that a business cycle panic will necessarily be synchronized across countries as long as there is a minimum level of economic integration. Moreover, we show that several factors generated particular vulnerability to such a global panic in 2008: tight credit, the zero lower bound, unresponsive fiscal policy and increased economic integration.
1 Introduction

The 2008-2009 Great Recession clearly had its origins in the United States, where an historic drop in house prices had a deep impact on financial institutions and markets. It is remarkable then, as illustrated in Figure 1, that the steep decline in output, consumption, investment and corporate profits during the second half of 2008 and beginning of 2009 was about the same in the rest of the world as in the United States.\(^1\) Figure 2 shows that the decline in expectations of future GDP growth, as well as the increase in uncertainty about future growth, was also of a similar magnitude in the rest of the world as in the United States.\(^2\) This co-movement of business cycles and of expectations is surprising both in the context of existing theory and historical experience. Figure 3 shows that during the Great Depression the decline in output in the rest of the world was much smaller than in the United States. There is extensive evidence that output correlations in the 2008-2009 recession were unprecedented (e.g., see Imbs, 2010, Perri and Quadrini, 2013, or International Monetary Fund, 2013), while there has been no indication of an increase in co-movements before the crisis.\(^3\)

The strong co-movement also poses a theoretical challenge. Apart from exogenous global shocks, co-movement of business cycles in existing models is a result of a transmission of shocks across countries. Transmission can be either negative, such as in models with technology shocks, or positive, such as in models with financial shocks. But unless goods and financial markets are perfectly integrated, even positive transmission is partial at best.\(^4\) The assumption of perfectly inte-

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\(^1\)The numbers for corporate profits in the last panel of Figure 1 have been derived by aggregating profits from firms listed in the Worldscope database. We selected continuing firms over the interval and windsorized the top and bottom tails at 1 percent. The resulting profit series are divided by the GDP deflator. Only G7 countries are included due to data limitations.

\(^2\)The data for Figure 2 come from Consensus Economics, who survey about 250 "prominent financial and economic" forecasters. Each January, forecasters are asked to give probabilities for GDP growth rate intervals for the current year. We compute the average and the variance for each country, as explained in more detail in Appendix A. For the non-US data line, we use the average across the 17 other countries in the sample.

\(^3\)On the contrary, Hirata et al. (2013) find that over the past 25 years the global component of business cycles has actually declined relative to local components (region and country-specific).

\(^4\)van Wincoop (2013) considers a model matching the observed financial home bias and shows that the transmission of credit shocks across countries is limited. Credit shocks are perfectly transmitted across countries only when both goods and financial markets are perfectly integrated,
grated markets is sharply contradicted by extensive evidence of large trade costs in goods markets, rejection of perfect financial integration (Backus-Smith puzzle) and strong home bias in both goods and financial markets. Moreover, recent empirical evidence suggests that the unprecedented synchronization of business cycles and asset prices during the Great Recession cannot be explained through standard trade and financial linkages.\textsuperscript{5} International Monetary Fund (2013) suggests that the unusual co-movement is a result of an undetermined common shock, for example in the form of a global panic.

This then leads to two questions that we aim to address in this paper. First, given the limited extent of goods and financial integration, how can we explain that the sharp decline in business cycles and expectations was similar in the rest of the world as in the United States during the Great Recession? Second, what can explain the difference relative to previous recessions?

To answer these questions we develop a two-country model that explains the recession as a demand collapse resulting from a self-fulfilling shock to expectations (or panic) as opposed to an exogenous shock to fundamentals. To answer the first question, we show that expectation shocks are endogenously coordinated across countries as long as integration passes a minimum threshold. It is not possible for two countries to have very different beliefs about the future when they are sufficiently integrated. In order to answer the second question, we develop a New Keynesian model that generates vulnerability to self-fulfilling expectation shocks only under certain conditions, which are precisely ones that were present at the onset of the Great Recession. These include tight credit and constraints on monetary and fiscal policy.

The view that the Great Recession could result from a self-fulfilling expectation shock has already gained significant traction in the literature in closed economy models.\textsuperscript{6} When defining the Great Recession as the sharp decline in output over the three quarters from Q3, 2008 to Q1, 2009, this view of an expectation shock

\textsuperscript{5}See Rose and Spiegel (2010), Kamin and Pounder (2012), Kalemli-Ozcan et al. (2013) and International Monetary Fund (2013).

or panic is quite natural. It is also consistent with evidence of a decline in expectations, shown in Figure 2, which was synchronized across countries.

The main contribution of the paper is to show that, in presence of self-fulfilling expectation shocks, business cycles can be synchronized even under limited economic integration. This is the result of complementarity between domestic and foreign economies that generates an endogenous coordination of equilibria. The intuition is related to the typical feature found in the literature that the very existence of multiple equilibria depends on the value of fundamentals. There is a unique "good" equilibrium when fundamentals are strong, a unique "bad" equilibrium when fundamentals are weak, and multiple equilibria for intermediate values of fundamentals. While such fundamentals are usually exogenous, here the endogenous state of the foreign economy is a key fundamental that affects the existence of equilibria in the domestic economy. If the foreign economy is strong, the domestic economy may not be vulnerable to a self-fulfilling bad equilibrium. Similarly, if the foreign economy is really weak, only a bad equilibrium may be feasible in the domestic country. Their interconnectedness makes it impossible for one country to have self-fulfilling favorable beliefs about the future while the other country has very negative beliefs. A self-fulfilling business cycle panic, if it happens, is necessarily global. We show that the required minimum threshold level of economic integration does not need to be high.

A couple of other papers have considered self-fulfilling shocks to business cycles and asset prices in open economy contexts. None of these highlights the endogenous coordination of beliefs under limited integration. Bacchetta and van Wincoop (2013) develop a two-country model with self-fulfilling risk-panics. But even when financial markets are perfectly integrated, such panics are generally not synchronized across countries. Perri and Quadrini (2013) consider a two-country model with self-fulfilling credit shocks. While output and consumption are perfectly synchronized across countries, this is the result of perfect financial and goods market integration and also arises with exogenous credit shocks. Martin and Rey (2006) develop a two-country model of a developed and emerging market. They focus on self-fulfilling shocks to the demand for goods and assets of the emerging market, which can arise under partial goods market and financial integration. But they do not consider global panics.

The other contribution of the paper is to explain why we were particularly vulnerable to a global panic in 2008. While most elements of the model are totally
standard, two features generate the possibility of self-fulfilling beliefs that can give rise to a panic. First, we adopt a New Keynesian framework, where firms set prices at the start of each period. Second, we assume that firm operations are constrained by internal funds as a result of borrowing constraints. When profits are very low and credit is tight, some firms will then have to cut their production. These assumptions can generate the following self-fulfilling circularity. If consumers expect lower future income, they will decrease their consumption. As a result of nominal rigidities, this drop in demand reduces output and current profits. If this decline in profits is strong enough, and firms face binding borrowing constraints, they will cut future production. This reduces future income, so that expectations are self-fulfilling.\(^7\) Lower profits also imply a greater sensitivity of firms to future shocks and therefore an increase in uncertainty about future output. The presence of investment amplifies these mechanisms.

This mechanism of self-fulfilling beliefs connects to the Great Recession episode in various ways. First, it is consistent with the sharp decline in profits seen in the data, of similar magnitude in the U.S. and the rest of the world (Figure 1). Second, it is consistent with the tight credit conditions resulting from large balance sheet losses in the financial sector. We take these credit conditions as exogenous. Third, the demand shock is consistent with micro evidence that firms were more affected by a sudden sharp drop in demand than sudden reduced access to credit (e.g. Kahle and Stulz (2013), Ngyyen and Qiuang (2013)).\(^8\) Finally, we show that the self-fulfilling mechanism relies critically on constraints in monetary and fiscal policy. Monetary policy was constrained by the zero lower bound, while fiscal policy was constrained by historically high debt levels.

The model implies particular vulnerability to a global panic at the end of 2008. First, we show that when credit conditions are easier, self-fulfilling panics are not feasible in equilibrium. Tight credit makes firms more vulnerable when hit by a drop in demand that lowers profits. Second, we show that self-fulfilling panics are not possible when the central bank can conduct significant countercyclical

\(^7\)This relates to the classic Paradox of Thrift, where higher saving implies lower demand, which reduces output and may actually end up lowering saving. We will discuss the Paradox of Thrift in the context of our model in Section 5. For recent contributions, see Eggertsson and Krugman (2012), Eggertsson (2010) and Christiano (2004).

\(^8\)It is important to distinguish tighter credit from a sudden decline in access to credit, which does not occur in our model.
policy when we are further from the zero lower bound. Third, we show that with strong countercyclical fiscal policy, self-fulfilling panics would again be ruled out. Finally, consistent with the discussion about synchronization above, the significant increase in both trade and financial integration over the past two decades generated particular vulnerability to a panic that is global in nature.

In Section 2 we start with a simple reduced-form multiple equilibrium model to illustrate why equilibria are coordinated across countries when they are sufficiently integrated. While in Section 3 we discuss a fully specified model that has more direct relevance to the Great Recession, the example of Section 2 suggests that the coordination of equilibria is a more general result that should not be particularly sensitive to various assumptions. The full model in Section 3 is a benchmark model that is analytically tractable. Various extensions, requiring a numerical approach, are considered towards the end of the paper.

Section 4 analyzes the equilibria and determines when business cycle panics are global. Our main result, stated in Proposition 2, is that partial integration is sufficient to guarantee that business cycles are perfectly synchronized during a panic. We show numerically that the extent of integration required is relatively small. Section 5 shows that countries are more vulnerable to global panics with tight credit, low interest rates or rigid fiscal policies. Section 6 considers various extensions and Section 7 concludes.

2 Global Panics with Partial Integration: A Generic Example

In order to illustrate the basic point that a panic is necessarily coordinated across countries with limited integration, we start with a very simple generic multiple equilibria model. This model could be seen as a reduced form of a fully specified model, such as the one presented later in this paper. In autarky, the model would just have two equations:

\[ c = f(y) \]  
(1)
\[ c = y \]  
(2)

where \( f'(y) > 0 \). (1) represents consumption as a positive function of output or income. (2) represents goods market equilibrium, equating output and consump-
tion. Figure 4 illustrates two particular cases, with respectively two and three equilibria. Dependent on the curvature of the function \( f(y) \), it is clear that there may be any number of equilibria.

The simplest two-country version of this model looks as follows. Assume a Home and a Foreign country, with consumption and output respectively \( c \) and \( y \) in Home and \( c^* \) and \( y^* \) in Foreign. Let the Home country consume \( \psi c \) Home goods and \( (1 - \psi)c \) Foreign goods. Analogously, the Foreign country consumes \( \psi c^* \) Foreign goods and \( (1 - \psi)c^* \) Home goods. We can think of \( \psi = 1 \) as autarky, \( \psi = 0.5 \) as perfect integration and any value of \( \psi \) between 0.5 and 1 as partial integration. With the consumption functions as before, \( c = f(y) \) for the Home country and \( c^* = f(y^*) \) for the Foreign country, the Home and Foreign goods market equilibrium conditions are

\[
\begin{align*}
  y &= \psi f(y) + (1 - \psi)f(y^*) \\
  y^* &= (1 - \psi)f(y) + \psi f(y^*)
\end{align*}
\]

Because of symmetry, these two schedules are the mirror image of each other in the 45 degree line.\(^9\)

In the case of autarky (\( \psi = 1 \)), the equilibrium in the Home country is independent of the equilibrium in the Foreign country. For example, when \( f(y) \) corresponds to the left panel of Figure 4, \( y \) and \( y^* \) can independently take on the values \( y^A \) and \( y^B \). There are then four equilibria, including two symmetric and two asymmetric equilibria. An example of the latter is an equilibrium where \( y = y^A \) and \( y^* = y^B \). When \( f(y) \) corresponds to the right panel of Figure 4, there are nine possible equilibria, including three symmetric and six asymmetric ones.

In Appendix B we analyze all equilibria for \( y \) and \( y^* \) when the two countries are partially integrated. We find that when \( f(y) \) has the shape of either the left or the right panel in Figure 4, only symmetric equilibria exist when

\[
0.5 \leq \psi < \bar{\psi} = 0.5 + 0.5 \frac{1}{f'(y^B)}
\]

Note that \( f'(y^B) > 1 \), so that \( \bar{\psi} \) lies between 0.5 and 1. Therefore a sufficient degree of integration (\( \psi < \bar{\psi} \)) is enough to guarantee that only symmetric equilibria exist.

\(^9\)Notice that equations (3) and (4) allow for complementarity between Home and Foreign output, in the sense that \( dy/dy^* > 0 \) and \( dy^*/dy > 0 \) at least for some \( y \) and \( y^* \). Appendix B shows that this complementarity becomes stronger for lower \( \psi \) and that a stronger complementarity eliminates asymmetric equilibria.
Since $\tilde{\psi} > 0.5$, integration does not need to be perfect in order for equilibria to be fully coordinated across countries. Appendix B shows that this result holds both for stable and unstable equilibria.\textsuperscript{10}

As a result of a sunspot there may be a jump from one equilibrium to another. We refer to a downward jump (e.g., from C to A) as a panic. The result above implies that such a panic will be perfectly coordinated across countries under sufficient integration. When integration is insufficient ($\psi > \tilde{\psi}$), asymmetric equilibria exist as well and a panic may not be synchronized.

Figure 5 provides some intuition for these results. It is well known that in models with multiple equilibria, the nature of the equilibria depends on the value of the fundamentals. This is the case here as well. Figure 5 illustrates what happens to equilibria in the Home country when the state of the Foreign economy is either strong ($y^* = y^C$; left panel) or weak ($y^* = y^A$; right panel). The solid line is the same as in Figure 4 and represents autarky. Figure 5 shows how this line shifts when the countries become partially integrated ($\psi < 1$).

When the Foreign economy is strong with $y^* = y^C$, the schedule becomes a weighted average of $f(y)$ and $f(y^C)$, with weights of respectively $\psi$ and $1 - \psi$. The strong Foreign economy shifts upward the Home schedule for all values of $y$ below $y^C$. For sufficient integration we see that the Home demand schedule shifts up enough that there is only one equilibrium, which is $y = y^C$. The strong Foreign economy is a favorable fundamental for the Home country that for sufficient integration precludes the possibility of a bad equilibrium in Home. The two countries coordinate on the good equilibrium.

Similarly, the right panel of Figure 5 shows that when the Foreign economy is weak and $y^* = y^A$, the Home demand schedule is shifted down for all values of $y$ above $y^A$. For sufficient integration, the Home schedule drops enough that there is only one equilibrium, which is $y = y^A$. The weak Foreign economy is a bad fundamental for the Home country that for sufficient integration precludes the possibility of a good equilibrium. The two countries then coordinate on the bad equilibrium.

\textsuperscript{10}In the extreme case where $f(y) = y$, equations (3) and (4) become $y = y^*$. There is then a continuum of equilibria that are all symmetric. In that case the threshold is $\tilde{\psi} = 1$, so that equilibria are always symmetric once countries move out of autarky ($\psi < 1$). We thank Fabrizio Perri for pointing out this case. This is a limiting case as the threshold $\tilde{\psi}$ approaches 1 in (5) when the slope of $f(y)$ is close to 1, so that $f'(y^B)$ is close to 1.
Ruling out the unstable equilibrium B, under sufficient integration a panic leads to a synchronized drop in output in both countries from $y^C$ to $y^A$. One can call this “contagion of fear”. Fear travels. Fear in the Foreign country (Foreign panic) leads to fear in the Home country (Home panic). But it is critical that fear travels in both directions. Fear in the Foreign country would not be rational when the Home country’s economy is strong. With sufficient integration the fate of the economies becomes intertwined. It becomes impossible for one country to coordinate on a bad equilibrium and the other on a good equilibrium.

3 Full Model

We now turn to the full model. In this section we consider a benchmark model that is analytically tractable. There are two countries, Home and Foreign, and two periods, 1 and 2. The basic two-period New Keynesian structure is similar to closed economy models found in the literature, starting with Krugman (1998).\[11\] Prices are pre-set, while wages are flexible. There is partial integration of goods markets through trade. Countries are in financial autarky, with financial assets (claims on firms, a bond, and money) only held domestically. Goods are only used for consumption, abstracting from investment. There are households, firms, a government and a central bank. There is no uncertainty about the future (period 2). The only potential shock in the model is a sunspot shock in period 1 that can generate self-fulfilling shifts in expectations. In Section 6 we examine several extensions, including investment, uncertainty and financial integration.

3.1 Households

Households make consumption and leisure decisions in both periods. Households in the Home country maximize

$$\frac{1}{1 - \gamma} c_1^{1-\gamma} + \lambda l_1 + \beta \left( \frac{1}{1 - \gamma} c_2^{1-\gamma} + \lambda l_2 \right)$$

\[6\]

\[11\] See Mankiw and Weinzierl (2011) or Fernandez-Villaverde et al. (2012) for recent contributions. Aghion et al. (2000) analyze a small open economy.
where \( l_t \) is the fraction of time devoted to leisure in period \( t \) and \( c_t \) is the period-\( t \) consumption index of Home and Foreign goods:

\[
c_t = \left( \frac{c_{H,t}}{\psi} \right)^\psi \left( \frac{c_{F,t}}{1 - \psi} \right)^{1-\psi} \tag{7}
\]

where

\[
c_{H,t} = \left( \int_0^{n_{H,t}} c_{H,t}(j)^{\frac{\mu-1}{\mu}} \, dj \right)^{\frac{\mu}{\mu-1}} \tag{8}
\]

\[
c_{F,t} = \left( \int_0^{n_{F,t}} c_{F,t}(j)^{\frac{\mu-1}{\mu}} \, dj \right)^{\frac{\mu}{\mu-1}} \tag{9}
\]

Here \( c_{H,t} \) is the consumption index of Home goods and \( c_{F,t} \) the consumption index of Foreign goods. Consumption of respectively the Home and Foreign good \( j \) is \( c_{H,t}(j) \) and \( c_{F,t}(j) \). The number of Home and Foreign goods in period \( t \) is \( n_{H,t} \) and \( n_{F,t} \), which are equal to the number of Home and Foreign firms. The elasticity of substitution among goods of the same country is \( \mu > 1 \), while the elasticity of substitution between Home and Foreign goods is 1 (we examine non-unitary elasticities in Section 6). There is a preference home bias towards domestic goods as we assume \( \psi > 0.5 \). The specification is symmetric for the Foreign country, with the overall consumption index denoted as \( c_t^* \) and \( c_{H,t}^*(j) \), \( c_{F,t}^*(j) \) denoting the consumption of individual Home and Foreign goods consumption by Foreign households.

The parameter \( \psi \) captures the degree of goods market integration. A value of \( \psi > 0.5 \) implies a positive preference for domestic goods, which is well-known to be indistinguishable from introducing positive trade costs without a preference home bias.\(^{12}\) The limit \( \psi = 0.5 \) implies perfect goods market integration. As we will see, \( \psi = 0.5 \) also implies that in equilibrium \( c_t = c_t^* \), so that financial markets are effectively complete even though there is no asset trade.\(^{13}\) This is a feature that results specifically from the Cobb-Douglas specification and is familiar from Cole and Obstfeld (1991). We can then think of \( \psi = 0.5 \) as perfect economic integration across the two countries.

In period 1 Home households earn labor income \( W_1(1 - l_1) \), where \( W_1 \) is the nominal wage rate. They also earn a dividend \( \Pi_1^C \) and receive a transfer of \( M_1 \)

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\(^{12}\)See for example Anderson and van Wincoop (2003).

\(^{13}\)Financial markets are complete when the ratio of marginal utilities of consumption across the two countries is equal to the real exchange rate, which is 1 when \( \psi = 0.5 \).
in money balances from the central bank. They use these resources to consume, pay a tax $T_1$ to the government, buy Home nominal bonds with interest rate $i$ and hold money balances:

\[
W_1(1 - l_1) + \Pi_C^1 + \bar{M}_1 = \int_0^{n_{H,1}} P_{H,1}(j) c_{H,1}(j) dj + \int_0^{n_{F,1}} S_1 P_{F,1}(j) c_{F,1}(j) dj + T_1 + B + M_1
\]

where $P_{H,t}(j)$ and $P_{F,t}(j)$ are the price of respectively Home and Foreign good $j$ in the Home and Foreign currency. $S_t$ is the nominal exchange rate in period $t$ (Home currency per unit of Foreign currency).

In period 2 Home households earn labor income $W_2(1 - l_2)$, earn a dividend $C_2$, receive $(1 + i)B$ from bond holdings, carry over $M_1$ in money balances from period 1, and receive an additional money transfer of $\bar{M}_2 - \bar{M}_1$ from the central bank. These resources are then used to consume, pay a tax $T_2$ to the government and hold money balances $M_2$:\footnote{As usual in finite-time models, there is an implicit assumption on the final use of money, e.g., agents need to return the money stock to the central bank.}

\[
W_2(1 - l_2) + \Pi_C^2 + (1 + i)B + M_1 + (\bar{M}_2 - \bar{M}_1) = \int_0^{n_{H,2}} P_{H,2}(j) c_{H,2}(j) dj + \int_0^{n_{F,2}} S_2 P_{F,2}(j) c_{F,2}(j) dj + T_2 + M_2
\]

We assume a cash-in-advance constraint, with the buyer’s currency being used for payment:

\[
\int_0^{n_{H,t}} P_{H,t}(j) c_{H,t}(j) dj + \int_0^{n_{F,t}} S_t P_{F,t}(j) c_{F,t}(j) dj \leq M_t
\]

The constraint will always bind in period 2. It will bind in period 1 when the nominal interest rate $i$ is positive. When $i = 0$, the constraint will generally not bind in period 1.

Households choose consumption and leisure to maximize (6). The first-order
conditions are

\[ c_t^{-\gamma} = \beta(1 + i) \frac{P_1}{P_2} c_2^{-\gamma} \]  
\[ c_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\mu} c_{H,t} \]  
\[ c_{F,t}(j) = \left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\mu} c_{F,t} \]  
\[ c_{H,t} = \psi \frac{P_t}{P_{H,t}} c_t \]  
\[ c_{F,t} = (1 - \psi) \frac{P_t}{S_t P_{F,t}} c_t \]  
\[ \frac{W_t}{P_t} = \lambda c_t^{-\gamma} \]

where

\[ P_{H,t} = \left( \int_0^{n_{H,t}} P_{H,t}(j)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \]
\[ P_{F,t} = \left( \int_0^{n_{F,t}} P_{F,t}(j)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} \]
\[ P_t = P_{H,t}^{\psi \left[ S_t P_{F,t} \right]^{-\psi}} \]

\( P_{H,t} \) and \( P_{F,t} \) are price indices of Home and Foreign goods that are denominated in respectively Home and Foreign currencies. \( P_t \) is the overall price index, denominated in the Home currency.

Equation (13) is a standard intertemporal consumption Euler equation. (14)-(15) represent the optimal consumption allocation across goods within each country. (16)-(17) represent the optimal consumption allocation across the two countries. (18) represents the consumption-leisure trade-off. As usual, the inverse of \( \gamma \) measures the intertemporal rate of substitution. However, in equation (18) \( \gamma \) also measures the wage elasticity to consumption.

There is an analogous set of first-order conditions for Foreign households. Other than for Home and Foreign prices and price indices, we only need to add * superscripts to the variables and exchange \( \psi \) and \( 1 - \psi \). The Foreign price index is

\[ P_t^* = (P_{H,t}/S_t)^{1-\psi} P_{F,t}^{\psi}. \]
3.2 The Government and the Central Bank

The government and central bank policies are analogous in the two countries. We therefore again only describe the Home country. The Home government only buys Home goods. The total government consumption index is analogous to the CES index for private Home consumption:

\[ g_t = \left( \int_0^{r_{H,t}} g_t(j) \frac{r_{j-1}}{r} \, dj \right)^{\frac{\mu}{\mu - 1}} \]  

(19)

In the benchmark case we will simply set \( g_t = 0 \). But we will also consider a positive constant level of government spending, where \( g_t = \bar{g} \). Moreover, in Section 5 we consider the role of countercyclical fiscal policy, where \( g_t = \bar{g} - \Theta(c_1 - \bar{c}) \), with \( \bar{c} \) consumption in the non-panic equilibrium of the model and \( \Theta \geq 0 \).

Optimal allocation of government spending across the different goods implies

\[ g_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\mu} g_t \]  

(20)

We have \( \int_0^{r_{H,t}} P_{H,t}(j)g_t(j)\,dj = P_{H,t}g_t \). Since the timing of taxation across the two periods does not matter due to Ricardian equivalence, we simply impose the balanced budget condition

\[ T_t = P_{H,t}g_t \]  

(21)

The central bank’s behavior is modeled as in other two-period models (e.g., Krugman, 1998, or Mankiw and Weinzierl, 2001). The central bank credibly sets second-period money supply to stabilize second-period prices. We assume that the central bank has a zero inflation target from period 1 to period 2, so that \( P_2 = P_1 \). Since the cash-in-advance constraint is binding in period 2, we have \( M_2 = P_2c_2 \), and the second-period price level can be controlled through the second period money supply.

In the first period the central bank sets the nominal interest rate \( i \). For now we will assume that the central bank sets the interest rate such that \( (1 + i)\beta = 1 \). This corresponds to the interest rate in the flexible price equilibrium of the model. The non-panic equilibrium of the model will then correspond to the flexible price equilibrium. In Section 5 we consider what happens when during a panic the central bank lowers the interest rate to stimulate demand. Such a policy will not avert a panic when we are close to the zero lower bound. The central bank then has limited ability to counter a business cycle decline and the equilibrium will be similar to that without any countercyclical central bank action.
3.3 Firms

The number of firms operating in period 1 is based on prior decisions and is therefore taken as given. We normalize it at 1 for both countries, so $n_{H,1} = n_{F,1} = 1$. At the end of period 1, firms decide whether to continue to operate in period 2. We denote the number of period-2 firms by $n_{H,2} = n$ and $n_{F,2} = n^*$. We do not allow new firms to enter.\footnote{We could allow for entry under a fixed cost. If the fixed cost is large enough we revert to our current setup. Lower fixed costs that leads to limited entry, only partially replacing exiting firms, will only affect results quantitatively, not qualitatively.} We focus our description mainly on Home firms. Results are analogous for Foreign firms. Output of Home firm $j$ in period $t$ is

$$y_t(j) = (AL_t(j))^\alpha$$

(22)

where $L_t(j)$ is labor input, $A$ a constant labor productivity parameter and $\alpha$ is between 0 and 1.

Firms set prices at the start of each period. This Keynesian assumption only bites for period 1 as no unexpected shocks happen after firms set prices at the start of period 2. As we will see in Section 4, in period 1 there may be multiple equilibria, with consumption lower when a panic equilibrium occurs. This occurs with some exogenous probability as it is driven by a sunspot whose arrival is unknown in advance. Firms need to set prices at the start of period 1 before knowing whether the panic will occur. Production will then adjust to demand. Lower consumption during a panic lowers demand for goods and therefore production. Labor demand is then adjusted to satisfy the demand for goods. This Keynesian aspect is critical to the self-fulfilling business cycle panic in the model.

Since prices in period 1 are preset, and their level does not matter for what follows, we simply assume that all Home firms set the same price of $P_{H1}$, so that $P_{H1}(j) = P_{H1}$. Similarly, for the Foreign firms $P_{F1}(j) = P_{H1}$. In period 2 Home firm $j$ sets its price $P_{H2}(j)$ to maximize profits

$$\Pi_2(j) = P_{H2}(j)y_2(j) - \frac{W_2}{A}y_2(j)^{1/\alpha}$$

(23)

subject to

$$y_2(j) = c_{H,2}(j) + g_2(j) + c^*_{H,2}(j) = \left(\frac{P_{H2}(j)}{P_{H2}}\right)^{-\mu} \left[\psi \frac{P_2}{P_{H2}} c_2 + g_2 + (1-\psi) \frac{S_2 P^*_2}{P_{H2}} c^*_2\right]$$

(24)
The optimal price is a markup $\frac{\mu}{\mu - 1}$ over the marginal cost:

$$P_{H,2}(j) = \frac{\mu}{\mu - 1} \frac{W_2}{\alpha A} y_2(j)^{1 - \alpha}$$

(25)

Second-period profits are then

$$\Pi_2(j) = \kappa \frac{1}{A} W_2 y_2(j)^{1/\alpha}$$

(26)

where $\kappa = \frac{\mu(1 - \alpha) + \alpha}{(\mu - 1)\alpha}$. Since all firms face the same demand and the same wage, they set the same price. From the definition of the Home price index we have $P_{H,2} = P_{H,2}(j)^{1/(1-\mu)}$.

We now turn to period 1. Self-fulfilling expectations can happen in the model because of a circularity between periods 1 and 2. Expectations of low period-2 income reduce period-1 consumption, which reduces period-1 demand, output and profits, which negatively impacts period-2 production and makes the decline in expected income self-fulfilling. Two key assumptions are needed to make this circularity work. First, as already discussed, we assume nominal rigidities, so that the decline in period-1 consumption reduces period-1 output and profits. A second assumption is needed to draw a link from period-1 profits to period-2 production. We assume that firms are constrained by internal funds through borrowing constraints. When period-1 losses are large, and firms are unable to fill the gap through borrowing, they cease operations in period 2. Note that during the Great Recession there was indeed a sharp decline in profits (Figure 1) as well as tight credit.

Bankruptcy should be seen as a metaphor for a much broader range of ways that period 1 firm losses in combination with borrowing constraints can impact future economic activity. One could alternatively assume that firms continue to operate at a smaller scale for example by reducing branches or departments, reducing investment, or reducing worker training. All of these generate additional funds that may avoid bankruptcy, but nonetheless have the same effect of reducing period-2 production.\textsuperscript{16}

We assume some heterogeneity of firms in order to avoid the extreme that either all or none of the firms go bankrupt at the end of period 1. We separate firms into

\textsuperscript{16}Even if we take the bankruptcy in the model literally, we show in an extension in Section 6.4 that self-fulfilling panics can happen in the model without any firms actually going bankrupt. Just the belief that Great Depression style widescale bankruptcies might occur is sufficient, even if it does not actually materialize.
two groups, with a vulnerable group of $1 - \bar{n}$ firms that is in weaker financial shape than the other group. We model this by introducing a fixed cost that reduces the real value of profits by $z$ for the vulnerable group. This cost captures business costs other than wages. The other firms do not face this additional cost. As discussed further at the end of Section 4.4, the binomial distribution of the cost across firms is assumed only for analytical convenience and is not critical to the results.

Total profits of Home firm $j$ in period 1, $\Pi_1(j)$, are equal to

$$\Pi_1(j) = \Pi_1 - P_1z(j) = P_{H,1}y_1 - W_1L_1 - P_1z(j)$$

(27)

where $z(j) = 0$ for a fraction $\bar{n}$ of firms and $z(j) = z$ for a fraction $1 - \bar{n}$ of firms. It is also useful to define $\Pi_1$ as period 1 profits before paying this cost. When firm $j$ is unable to fully pay the fixed cost, it is declared bankrupt and cannot produce in period 2. We assume that $z(j)$ does not affect aggregate resources and is paid to an agency. In case of bankruptcy, the agency seizes $\Pi_1$. The agency operates at no cost and transfers its income to households.

Since $\Pi_1 > 0$, the $\bar{n}$ firms for which $z(j)$ is zero always have positive profits in period 1 and therefore do not need to borrow to continue their operation into period 2. The other $1 - \bar{n}$ firms may need to borrow when their first-period profits are negative. But they face a maximum limit on their borrowing capacity. Let $D(j)$ be borrowing by firm $j$ at the end of period 1. The firm then owes $(1+i)D(j)$ in period 2. It is assumed that this can be no larger than a fraction $\phi$ of second period profits:

$$(1 + i)D(j) \leq \phi \Pi_2(j)$$

(28)

This standard borrowing constraint reflects that lenders can seize at most a fraction $\phi$ of second period profits in case of non-payment. Second-period profits are positive and known at the end of period 1.

The vulnerable group of $1 - \bar{n}$ firms facing the cost $z$ are fragile in that they will go bankrupt if their debt limit is insufficient to cover negative profits in period 1. This is the case when

$$\Pi_1 + \phi \frac{\Pi_2}{1 + i} < P_1z$$

(29)

\[1^{17}\text{We choose to do so through an additive term in profits only because it simplifies the algebra. Results would not change fundamentally if instead we introduced differences in firm productivity, which interacts multiplicatively with } W_1L_1.\]
Another way to look at the bankruptcy condition is to define the real quantity of funds $\pi$ available to pay for the fixed cost:

$$\pi \equiv \pi_1 + \phi \frac{\pi_2}{1+i}$$

(30)

where $\pi_1 = \Pi_1/P_1$ and $\pi_2 = \Pi_2/P_2$. From (29), the $1 - \bar{n}$ fragile firms will go bankrupt when

$$\pi < z$$

(31)

Therefore the number of firms in period 2 is either 1 or $\pi$, depending on whether $\pi \geq z$ or $\pi < z$.18

Let $D$ denote aggregate borrowing by firms. The total dividends received by households include dividends from firms and from the service agency. Dividends received in periods 1 and 2 are

$$\Pi_1^C = \Pi_1 + D$$

(32)

$$\Pi_2^C = n\Pi_2 - (1+i)D$$

(33)

### 3.4 Market Clearing

For the Home country the market clearing conditions are

$$y_t(j) = c_{H,t}(j) + g_t(j) + c^*_{H,t}(j) \quad t = 1, 2$$

(34)

$$n_{H,t}L_t = 1 - l_t \quad t = 1, 2$$

(35)

$$M_t = \overline{M}_t \quad t = 1, 2$$

(36)

$$B = D$$

(37)

These represent respectively the goods markets clearing conditions, the labor market clearing condition, the money market clearing condition and the bond market

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18While we have discussed production in period 2, a brief comment is in order about production in period 1. As we will see, there may be multiple equilibria, with either no defaults ($\pi > z$) or positive defaults ($\pi < z$). The latter depends on the arrival of a sunspot. But firms need to commit to produce, for example by incurring a capital expense, before the arrival of this sunspot is known. Even vulnerable firms will choose to produce in period 1 as long as $(1 - p)E[u_c(\pi_1 + \pi_2/(1+i) - z)|\text{no sunspot}]$ is larger than the capital expense, where $p$ is the probability of the sunspot arrival and $u_c = c^{1-\gamma}$ is the marginal utility from period 1 consumption. Note that conditional on default the owners of the vulnerable firms have earnings of zero as there is limited liability.
clearing condition. There is an analogous set of market clearing conditions for the Foreign country.

If we substitute into the household budget constraints (10)-(11) the bond, money and labor market clearing conditions, along with the dividend expressions (32)-(33), we get

\[ P_{H,t}c_{H,t} + P_{H,t}g_t + S_tP_{F,t}c_{F,t} = \int_0^{n_{H,t}} P_{H,t}(j)y_t(j) dj \]  

(38)

This says that national consumption is equal to GDP. The trade balance is therefore zero. Indeed, multiplying the goods market clearing condition (34) by \( P_{H,t}(j) \) and aggregating and substituting into the right hand side of (38), gives the balanced trade condition

\[ S_tP_{F,t}c_{F,t} = P_{H,t}c_{H,t}^* \]  

(39)

Using the expressions for \( c_{F,t} \) and \( c_{H,t}^* \), this can also be written as

\[ P_t c_t = S_t P_t^* c_t^* \]  

(40)

The nominal value of consumption is equal across the two countries. This does not imply that real consumption is equal as the real exchange rate \( S_tP_t^*/P_t \) is not necessarily equal to 1 when \( \psi > 0.5 \). Only when markets are perfectly integrated (\( \psi = 0.5 \)) is the real exchange rate equal to 1 and \( c_t = c_t^* \).

Together with the definitions of the price indices, (40) also gives an expression for relative prices that we will use below:

\[ \frac{P_{H,t}}{P_t} = \left( \frac{c_t^*}{c_t} \right)^{\frac{1-\psi}{2\psi-1}} \]  

(41)

The Foreign relative prices are the reciprocal: \( P_{F,t}/P_t^* = P_t/P_{H,t} \).

### 3.5 Equilibrium

Appendix C provides a description of the main equilibrium conditions. Assuming \((1+i)\beta = 1\) and \(g_t = 0\), the equilibrium can be reduced to a set of 6 equations in
\[ c_1, c^*_1, \pi, \pi^*, n \text{ and } n^*:\]

\[ c_1 = \frac{1}{\theta} n^{(1-\delta)\zeta}(n^*)^{\delta\zeta} \quad (42) \]

\[ c^*_1 = \frac{1}{\theta} n^{\kappa\zeta}(n^*)^{(1-\delta)\zeta} \quad (43) \]

\[ \pi = c_1 - \frac{\lambda}{A} c_1^{\gamma+1/\alpha} \left( \frac{P_1}{P_{H,1}} \right)^{1/\alpha} + \frac{\phi \beta \kappa \lambda}{A} (c_1)^{\gamma+1/\alpha} \left( \frac{P_1}{P_{H,1}} \right)^{1/\alpha} n^{-\frac{\mu}{(\alpha-1)\alpha}} \quad (44) \]

\[ \pi^* = c_1^* - \frac{\lambda}{A} (c_1^*)^{\gamma+1/\alpha} \left( \frac{P_1^*}{P_{F,1}} \right)^{1/\alpha} + \frac{\phi \beta \kappa \lambda}{A} (c_1^*)^{\gamma+1/\alpha} \left( \frac{P_1^*}{P_{F,1}} \right)^{1/\alpha} (n^*)^{-\frac{\mu}{(\alpha-1)\alpha}} \quad (45) \]

\[ n = \begin{cases} \bar{n} & \text{if } \pi < z \\ 1 & \text{if } \pi \geq z \end{cases} \quad (46) \]

\[ n^* = \begin{cases} \bar{n} & \text{if } \pi^* < z \\ 1 & \text{if } \pi^* \geq z \end{cases} \quad (47) \]

where

\[ \theta = \left( \frac{\lambda \mu}{(\mu-1)\alpha A} \right)^{\alpha/(1-\alpha+\alpha \gamma)} \]

\[ \zeta = \frac{\alpha + \mu(1-\alpha)}{(\mu-1)(1-\alpha+\alpha \gamma)} \]

\[ \delta = (1-\psi)/[(1-\alpha+\alpha \gamma)(2\psi-1) + 2(1-\psi)] \]

and the relative prices depend on \( c_1/c^*_1 \) as in (41).

Appendix C provides algebraic details behind these equations. Equations (42)-(43) are derived by combining the Home and Foreign counterpart of the optimal second period price setting equation (25), the labor supply schedule \( W_2/P_2 = \lambda c_2^\gamma, P_{H,2}(j)/P_{H,2} = n^{1/(\mu-1)} \), the consumption Euler equations, and the assumed monetary policy. Note that the relationship between the number of firms and consumption depends on the parameter \( \zeta \). If it is zero, the number of firms has no impact on income and consumption. This would be the case when \( \alpha = 1 \) and \( \mu = \infty \). In that case production is linear and goods are perfect substitutes. The number of firms is then irrelevant. A smaller number of firms reduces real income and consumption when either the production function is concave \( \alpha < 1 \) or goods are imperfect substitutes \( \mu < \infty \). In the former case it essentially reduces aggregate productivity, while in the latter case it raises the cost of living.

Equation (44) is the expression for available funds \( \pi = \pi_1 + \phi \pi_2/(1+i) \), using \( W_t/P_t = \lambda c_t^\gamma \), (40) and the fact \( c_2 = c_1 \) from the consumption Euler equations.
Equation (45) is the Foreign counterpart for available funds. After substituting the expression (41) for the relative price, available funds depend on $c_1$, $c_1^*$, $n$ and $n^*$. Finally, (46)-(47) follow from the description of default in Section 3.3.

Before turning to the solution of the model, some brief comments are in order about the flexible price equilibrium, where first-period prices are perfectly flexible. We show in Appendix C that the equilibrium is then unique. This results from the absence of a Keynesian demand effect. Independent of parameters, first-period consumption is $c_1 = c_1^* = 1/\theta$, while first-period profits are $\pi_1 = \pi_1^* = [\mu(1 - \alpha) + \alpha]/(\mu\theta)$. We will assume that in the flexible price equilibrium first-period profits of all firms are positive:

**Assumption 1** $z < [\mu(1 - \alpha) + \alpha]/(\mu\theta)$

The right hand side of the expression in Assumption 1 is equal to $\pi_1 = \pi_1^*$ in the flexible price equilibrium. We then also have $z < \pi$ since $\pi_2 > 0$, so that no firms go bankrupt ($n = n^* = 1$). Finally, we find that the equilibrium interest rates are given by $(1 + i)\beta = (1 + i^*)\beta = 1$. As mentioned above, this corresponds to the policy we assume in our benchmark model. The global non-panic equilibrium in the benchmark Keynesian model will then correspond exactly to the flexible price equilibrium.

### 4 Multiple Equilibria and Global Panics

The model can generate multiple equilibria with either $n = 1$ (no bankruptcies) or $n = \bar{n}$ (with bankruptcies). When both equilibria exist, we call the equilibrium with bankruptcies the *panic* equilibrium as it is simply generated by low expectations. There are potentially four equilibria, characterized by the values of $n$ and $n^*$. We refer to equilibria where $n = n^*$ as symmetric equilibria. The case where $n = n^* = 1$ is a global non-panic equilibrium. If in addition there is an equilibrium where $n = n^* = \bar{n}$ we refer to it as a *global panic*. But there may also be asymmetric equilibria, where only one country panics and the other does not. There are potentially two asymmetric equilibria, with either $n = \bar{n}$ and $n^* = 1$ or $n = 1$ and $n^* = \bar{n}$.

In this section we first focus on symmetric equilibria in which $n = n^*$. In that case first-period consumption, output and profits are also equal across the two
countries. Then we look at equilibria when countries are in autarky, where $\psi = 1$. Finally, we consider all equilibria for any value of $\psi$ between 0.5 and 1. We will show that when economies are in autarky ($\psi = 1$), asymmetric equilibria always exist. However, consistent with the results from the reduced-form model of Section 2, when countries are sufficiently integrated ($\psi$ below a threshold) there are only symmetric equilibria and a panic is necessarily global.

4.1 Symmetric Equilibria

Considering symmetric equilibria allows us to clearly illustrate the mechanism behind a global panic. Moreover, considering global panics first is natural as in the absence of a global panic equilibrium the model does not feature any type of panic equilibrium, including asymmetric panics.

It is immediate from (42)-(47) that $n = n^*$ implies $c_1 = c_1^*$ and $\pi = \pi^*$. The equilibria are then characterized by $(c_1, \pi, n)$ that satisfy

\begin{align}
\frac{c_1}{\theta} &= \frac{n^\zeta}{\theta} \quad (48) \\
\pi &= c_1 - \frac{\lambda}{A} c_1^{\gamma+1/\alpha} + \phi \beta \frac{\mu(1-\alpha) + \alpha}{\mu \theta} n^{\zeta-1} \quad (49)
\end{align}

\begin{align}
n &= \pi \quad \text{if } \pi < z \\
1 &= \pi \quad \text{if } \pi \geq z \quad (50)
\end{align}

A higher number of period 2 firms $n$ raises period 2 production and income for reasons discussed above, which raises period 1 consumption. Substituting (48) into (49) we can write available funds $\pi$ as a function of only $n$. Let $\pi(1)$ and $\pi(\bar{n})$ represent available funds without and with bankruptcies in the symmetric equilibrium. We will assume that parameters are such that available funds are higher without bankruptcies:

**Assumption 2** $\pi(1) > \pi(\bar{n})$

This can be written in terms of a condition on the various parameters in the model.\footnote{The condition is $\pi(1) > \pi(\bar{n})$, which implies $\alpha \gamma (\mu - 1) \leq 1$.}

\[19\text{The condition is } (\bar{n}^{-\zeta} - 1) + \frac{1}{\bar{n}} (\bar{n}^{\zeta} - 1) + \phi \beta (\bar{n}^{-\zeta} - \frac{1}{\bar{n}^\zeta}) > 0. \text{ The condition is not satisfied for a high } \gamma \text{ as real wages then decline significantly during a panic, which raises profits. We will return to this issue in Section 4.5.} \]
Together with Assumption 1, which implies that \( z < \pi(1) \), the equilibria follow directly from (48)-(50) and are summarized in the following proposition.

**Proposition 1** When Assumptions 1 and 2 hold, there are one or two symmetric equilibria. They are characterized by:

1. \((n, c_1) = (1, 1/\theta)\) if \( \pi(\overline{\pi}) \geq z \)

2. \((n, c_1) = (1, 1/\theta)\) or \((n, c_1) = (\bar{n}, \overline{\pi}c/\theta)\) if \( \pi(\overline{\pi}) < z < \pi(1) \)

For the case where \( \phi = 0 \), so that \( \pi = \pi_1 \), Figure 6 illustrates the multiple equilibria in Proposition 1. The hump-shaped curve represents the first-period profits function (49). The vertical lines represent (48) for the two levels of \( n \) and the cut-off point is determined by the level of \( z \). When \( \zeta > 1 \), both vertical lines cross the profit schedule when it is upward sloping. When \( z \) is in the intermediate range \((\pi(\overline{\pi}) < z < \pi(1))\), there are two equilibria, A and B. Equilibrium A is a good one, which we refer to as the non-panic equilibrium. First-period consumption and profits are high and no firms go bankrupt \((n = 1)\). Equilibrium B is the bad one, which we refer to as the panic equilibrium. First-period consumption and profits are low and \( 1 - \overline{\pi} \) firms go bankrupt.

The presence of two equilibria is a result of the possibility of self-fulfilling business cycle panics. This occurs due to reinforcing linkages between periods 1 and 2. The link from period 2 to period 1 is standard as low expected period-2 income leads to low period-1 consumption. The link from period 1 to period 2 operates through profits and borrowing constraints. Low period-1 consumption leads to low (negative) period-1 firm profits due to nominal rigidities. When credit is sufficiently tight, this leads to bankruptcies and therefore a low number of firms in period 2. This implies low period-2 output, making the belief of low period-2 income self-fulfilling.

It is useful to emphasize that this is by no means the only possible way to model the link from the present to the future. One can think of many alternatives that would deliver similar results. Low current demand may affect future output through inventory buildup, lower current investment or production chains. In addition, lower output today may reduce future output when a reduction in productive capacity is combined with sunk costs. Together with the standard link from the future to the present through expected income, these alternative mechanisms for linking the present to the future will also generate self-fulfilling beliefs.
The mechanism we emphasize here operates through low internal funds (profits) and borrowing constraints as these conditions were particularly relevant to the Great Recession. As discussed previously in Section 3.3, low profits in combination with borrowing constraints may lower future output through cost-cutting measures rather than through bankruptcy, e.g., through reduced R&D, less training of labor, closing some departments or branches or less investment.

4.2 Autarky

When $\psi = 1$ the two economies are in autarky. They only consume their own goods, so that the relative prices $P_t / P_{H,t}$ and $P^*_{t} / P_{F,t}$ are equal to 1 in both periods. It then follows from (42)-(47) that for each country the equilibria correspond exactly to the symmetric equilibria described above. But in autarky the equilibrium in one country has no impact on the equilibrium of another country. When $\pi(\overline{\pi}) < z < \pi(1)$ there are then four possible outcomes. Either country may be in the panic equilibrium B or the non-panic equilibrium A, independent of the other country. Therefore it is possible for both countries to experience a panic together, but it is also possible for just one of the two countries to experience a panic (asymmetric equilibria).

There is no a priori reason why the two countries would panic simultaneously. There may be arguments outside of the model why a panic would be global. For example, if the trigger that sets off the panic is particularly frightening, the two countries may react together. But if this trigger event takes place in the Home country$^{20}$, it would seem odd that the Foreign country would react to it in the absence of any integration between the two countries.

4.3 When Are Panics Global?

In this section we examine all equilibria for values of $\psi$ between 0.5 and 1. We have already described the symmetric equilibria, where $(n, n^*) = (1, 1)$ or $(n, n^*) = (\bar{n}, \bar{n})$. We now need to consider asymmetric equilibria as well, where either $(n, n^*) = (\bar{n}, 1)$ or $(n, n^*) = (1, \bar{n})$. We are particularly interested in circumstances where only the two symmetric equilibria exist. When a panic occurs, it

$^{20}$An example is the bankruptcy of Lehman Brothers or more generally events surrounding U.S. financial markets in the Fall of 2008.
will then necessarily be global.

We will assume that symmetric multiple equilibria exist, i.e., \( \pi(n) < z < \pi(1) \) from Proposition 1. As discussed in Section 4.2, this implies that multiple equilibria also exist in individual countries in autarky. This means that asymmetric equilibria exist when \( \psi = 1 \). However, as we move away from autarky, i.e., as we lower \( \psi \), the asymmetric equilibria will no longer exist, so that panics can only be global. This is stated in the following proposition.

**Proposition 2** Assume \( \pi(n) < z < \pi(1) \), so that there are multiple equilibria. There is a threshold \( \psi(z) > 0.5 \) such that only the symmetric equilibria exist when \( \psi < \psi(z) \).

**Proof.** See Appendix D. ■

Using (42), Figure 7 illustrates Proposition 2 by plotting all equilibrium Home consumption levels as a function of \( \psi \). Symmetric equilibria give perfectly horizontal schedules as consumption is \( c_1 = n^\xi/\theta \), which is unaffected by the level of integration. This is not the case in the asymmetric equilibria. For example, a Foreign panic affects Home consumption more the greater the extent of integration (the lower \( \psi \)).

When \( \psi \) is below the threshold \( \psi(z) \), only the two symmetric equilibria exist. In that case panics are necessarily global. In other words, when the level of trade is sufficiently high, or home bias sufficiently low, a panic will be perfectly coordinated across the two countries. However, the two countries do not need to be perfectly integrated. A panic will be necessarily global for all values of \( \psi \) larger than 0.5 and less than \( \psi(z) \). A sufficient degree of integration, not perfect integration, is needed to guarantee that panics will be global. As we show in Section 4.5, the cutoff for \( \psi \) will generally be far above 0.5, so that we do not need to be anywhere close to full integration to assure that panics will be perfectly coordinated across countries.

Before we turn to the intuition behind this key result, it is useful to first draw out some of the implications. First, Proposition 2 implies that when the two countries are sufficiently integrated (\( \psi > \psi(z) \)) a panic leads to a drop in consumption that is common across countries. Consumption in both countries drops from \( 1/\theta \) to \( n^\xi/\theta \). Second, output drops equally in both countries and the same as consumption.\(^{21} \) Third, profits decline in a synchronized way. Fourth,

\(^{21}\)The real value of Home output in period 1 is \( P_1c_1/P_{H,1} \) from (38), while \( P_1/P_{H,1} \) depends
future output is expected to drop in both countries by the same amount as well. All these pieces of evidence are consistent with the business cycle and survey data reported in Figures 1 and 2.

4.4 Intuition Behind Global Panics

Unless countries are perfectly integrated, business cycle shocks are only partially transmitted across countries in standard models. This is the case in our model as well in the sense that an asymmetric panic in one country is only partially transmitted to the other country. But the key to perfect business cycle co-movement here is that limited transmission impacts the range of feasible equilibria. In particular, under sufficient integration we can rule out asymmetric equilibria, so that a panic is necessarily global. The intuition for this result is analogous to that discussed in the context of the simple reduced form model of Section 2. The key point is again that the state of the other economy is now an endogenous fundamental that affects the existence of equilibria in the domestic country under sufficient integration. This leads to a coordination of equilibria.

To see this, it is useful to start from (42)-(43), which are repeated here for convenience:

\[ c_1 = \frac{1}{\theta} n^{(1-\delta)\zeta} (n^*)^\delta \zeta \]  
\[ c_* = \frac{1}{\theta} n^\delta (n^*)^{(1-\delta)\zeta} \]  

Since \( \delta \) is between 0 and 0.5 when \( \psi \) is between 0.5 and 1, one implication immediately follows. Assume that there is a panic in just one country, say Foreign \((n = 1 \text{ and } n^* = \bar{n})\). The panic in the Foreign economy has a negative impact on Home consumption as it drops from \(1/\theta\) (without a Foreign panic) to \(\bar{n}^\delta \zeta / \theta\). But this transmission is only partial when economic integration is partial \((0.5 < \psi < 1)\). Foreign consumption is \(\bar{n}^{(1-\delta)\zeta}\), which is lower than Home consumption \(\bar{n}^\delta \zeta / \theta\) as \(\delta < 0.5\). This partial transmission is standard in open economy models with partial integration. However, in our framework limited transmission plays the additional role of impacting the very existence of equilibria.

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\( c_1/c_* \) from (41) and therefore stays equal to 1. The drop in Home real GDP in period 1 is therefore the same as the drop in Home consumption. The same is the case for the Foreign country.
Consider equilibria in the Home country. As before, there are two schedules. Conditional on the state $n^*$ (1 or $\bar{n}$) in the Foreign economy, these are given by (42) and

$$\pi = c_1 - \frac{\lambda}{A} c_1^{\gamma+1/\alpha} \left( \frac{\theta c_1}{(n^*)^\gamma} \right)^{\frac{\delta}{1-\delta}}$$  \hspace{1cm} (53)

$$n = \begin{cases} \bar{n} & \text{if } \pi < z \\ 1 & \text{if } \pi \geq z \end{cases}$$  \hspace{1cm} (54)

For a given state of the Foreign economy, it gives two possible values of $c_1$ corresponding to the two possible values of $n$. This is analogous to the two vertical lines in Figure 6. (53) is the humped shaped profit schedule, where we have assumed for simplicity that $\phi = 0$.\footnote{It corresponds to (44), substituting the expression for the relative price as a function of relative consumption. Using (51)-(52) the latter can be written as $c_1/c_1^* = (\theta c_1/(n^*)^\gamma)^{(1-2\delta)/(1-\delta)}$.} When $\psi = 1$, so that $\delta = 0$ (the case of autarky), these schedules are the same as in the case of symmetric equilibria.

Figure 8 shows how Home equilibria are affected when the countries are partially integrated. First consider the left panel, where it is assumed that there is a panic in the Foreign country ($n^* = \bar{n}$). The left vertical schedule (for $n = \bar{n}$) remains unaffected by integration as $c_1 = \bar{n}^\gamma/\theta$ when both countries are in a panic. But the right vertical schedule shifts to the left as a panic in the Foreign country has a negative effect on Home consumption when there is no panic in the Home country. This is a regular transmission effect. Similarly, the profit schedule shifts down for values of $c_1$ above the global panic level ($c_1 > \bar{n}^\gamma/\theta$). Home profits are dragged down by the Foreign panic.

There are now two possibilities. When the two schedules do not move a lot (economic integration is limited), there remain two equilibria in the Home country. But when economic integration is sufficient, the good equilibrium may no longer exist, as illustrated in the left panel of Figure 8. In that case the additional bad fundamental (Foreign panic) pushes the Home economy into a singular bad equilibrium (panic). This precludes asymmetric equilibria as it is not possible to have a panic in only one country.

Next consider the right panel of Figure 8, which is conditional on no Foreign panic ($n^* = 1$). The right vertical schedule (for $n = 1$) remains unaffected by economic integration as $c_1 = 1/\theta$ when neither country panics. But the left vertical schedule shifts to the right as the strong Foreign economy has a positive effect on
Home consumption when there is a Home panic. The profit schedule now shifts up for values of $c_1$ below $1/\theta$, the global non-panic level. Home profits are raised by the strong Foreign economy. Here again, when economic integration is sufficient asymmetric equilibria are not possible. The additional good fundamental (strong Foreign economy) implies that the Home economy is no longer vulnerable to a panic. Only the non-panic equilibrium is feasible.

In order to rule out the existence of asymmetric equilibria it is sufficient that either in the left or right panel of Figure 8 the schedules shift enough such that one of the equilibria goes away. For that, it must be the case that either $\pi(n = 1|n^* = \bar{n}) < z$ or $\pi(n = \bar{n}|n^* = 1) > z$. Proposition 2 implies that one of these conditions will be satisfied for $\psi$ less than a cutoff $\psi(z) > 0.5$. From Figure 8 we see that very little integration is needed when $z$ is close to either the panic or the no-panic profit levels. But more generally, Appendix D shows that independent of $z$ there is a threshold $\bar{\psi} > 0.5$ such that at least one of these conditions must be satisfied when $\psi < \bar{\psi}$. The cutoff $\psi(z)$ in Proposition 2 lies somewhere between $\bar{\psi}$ and 1.

We should finally point out that Proposition 2 is not an artifact of the specific way that we have modeled the cost $z$ that defaulting firms face. Instead of the binary assumption that some firms face the cost $z$ and some do not, in a previous draft of the paper we assumed that there is a cross-sectional distribution of the cost across firms that is uniform over an interval $[a, b]$. The fraction of firms that goes bankrupt then becomes endogenous. That version of the model is more complicated and requires a numerical solution. The numerical results are nonetheless consistent with Proposition 2. There are in general again two symmetric equilibria, while the asymmetric equilibria disappear when $\psi$ drops below a cutoff that is above 0.5. As follows from the simple reduced form example of Section 2, the logic behind this result is more general than the specifics of this particular model.

### 4.5 Numerical Illustration

While the model is obviously highly stylized, it is still useful to provide a numerical illustration for reasonable levels of parameters to see what level of integration is sufficient to guarantee that a panic is synchronized across countries. We will

---

set the elasticity $\mu$ equal to $3$.\footnote{Broda and Weinstein (2006) estimate this elasticity using 8-digit, 5-digit and 3-digit industry levels. In all cases they find that the median elasticity across industries is just below 3.} We set $\alpha = 0.75$. This delivers a labor share of $\alpha(\mu - 1)/\mu = 0.5$, which is consistent with 2010 data for the U.S., Japan and the Euro zone on the ratio of employee compensation to GDP. We normalize private consumption in the non-panic state to be 1 by setting $\lambda/A$ such that $\theta = 1$. We re-introduce government spending, which was only suppressed in the previous subsections for analytic tractability. We set $g_t = \bar{g} = 0.3$ in both periods, implying that government consumption as a fraction of GDP is $0.3/1.3=0.23$. This is consistent with recent data from industrialized countries for government spending (consumption plus investment) relative to GDP. For now we set $\phi = 0$, so that the borrowing constraint is very tight: firms cannot borrow at all. We will investigate the role of borrowing constraints further in the next section.

The only parameter left is $\gamma$. It is hard to calibrate as it plays three roles in the model: rate of risk aversion, inverse of intertemporal elasticity of substitution and real wage cyclicality. The real wage is $\lambda c^{\gamma}$. Based on estimates of risk-aversion and the intertemporal elasticity of substitution $\gamma$ should be larger than 1. But this is inconsistent with the evidence that the average real wage rate is not very cyclical. Moreover, given realistic choices for the other parameters the model implies counterintuitively that $\pi(1) < \pi(\bar{n})$ when $\gamma$ is set at 1 or larger. The reason is that in the panic state the real wage is much lower, which raises firm profits. In order to avoid this strong cyclicity of the wage rate, we consider results both for the case where $\gamma$ is well below 1 and the extension where nominal or real wages are rigid (preset at the start of each period). This extension is straightforward and described in Appendix E.

When we set $\gamma = 0.2$, so that the real wage rate is not very cyclical, we find $\tilde{\psi} = 0.9$, independent of the level of $\bar{n}$. The actual cutoff $\psi(z)$ then lies somewhere between 0.9 and 1, dependent on the value of $z$. Only limited trade is then sufficient to guarantee a global panic. When 10% of private consumption goods are imported, a panic is necessarily global and therefore business cycles will be perfectly synchronized during the panic. $\tilde{\psi}$ will be only slightly lower, at 0.88, when we set $\gamma$ infinitesimally close to 0, so that the real wage rate is not cyclical at all.

As discussed further in Appendix E, under both nominal and real wage rigidity wages are set at the start of each period under the assumption that there will be
no panic.\textsuperscript{25} Results will be very similar when setting the probability of a panic at a small positive number. This does not affect period 2 as there are no further unexpected shocks during period 2. When the real wage is negotiated at the start of period 1, it will then be set at its equilibrium non-panic level. When instead the nominal wage rate is agreed to in advance, the real wage will be equal to the non-panic real wage rate times $P_1/P_1$, where $P_1$ is the price index without a panic.

We now set \( \gamma \) at 3, which is a standard value when measuring risk aversion or the inverse of the intertemporal elasticity of substitution.

Under real wage rigidity we find that \( \bar{\psi} \) is 0.89. Note that this is not exactly the same model as under flexible real wages with \( \gamma \) very small since the second period equilibrium does depend on \( \gamma \). Nonetheless the result is virtually identical and it again does not depend on \( \bar{n} \). Under nominal wage rigidity \( \bar{\psi} \) is a bit lower at 0.77, so that \( \psi(z) \) is in the range of 0.77 to 1. But it is still the case that limited trade is needed to guarantee perfect synchronization of panics across countries. It is sufficient that 23% of private consumption goods are imported. This number may be even less depending on the precise value of \( z \).

We can also numerically evaluate the extent of traditional business cycle transmission associated with asymmetric shocks. Since there are no exogenous asymmetric shocks in the model, we consider transmission associated with an asymmetric panic. Take the example of real wage rigidity where \( \bar{\psi} = 0.89 \). Assume that \( \psi(z) = \bar{\psi} \) and that \( \psi = 0.9 > \bar{\psi} \). We are then in the region where asymmetric panics are possible. Using the parameter values discussed above, the drop in log Foreign consumption is then only a fraction 0.05 of the drop in log Home consumption. Transmission is positive but small. But only slightly more trade integration (\( \psi \) equal to 0.89 or less) guarantees that panics are global, allowing us to explain the perfect business cycle synchronization while retaining limited integration.

5 Vulnerabilities

We now consider factors that make countries vulnerable to self-fulfilling panics. We focus on symmetric equilibria. If symmetric panics do not exist, no type of panic, including asymmetric ones, exist in the model. The question is therefore under

\textsuperscript{25}Even though firms preset their prices, there is a difference between nominal and real wage rigidity due to the exchange rate impact on the price level.
what conditions of the fundamentals of the model are there multiple equilibria, as opposed to only the non-panic equilibrium. We consider a version of the model that is general enough to focus on the role of credit, monetary policy and fiscal policy. These are captured by respectively $\phi$, $i$ and $g_t$. At the same time we will simplify by setting $\zeta = \alpha = 1$. This leads to a cleaner set of equilibrium equations, but is not critical to the results. As shown in Appendix C, the schedules that determine the symmetric equilibrium are then

\begin{equation}
    c_1 = \left[\beta(1+i)\right]^{-1/\gamma}n
\end{equation}

\begin{equation}
    \pi = c_1 + g_1 - \frac{\lambda}{A}c_1^\gamma(c_1 + g_1) + \frac{\phi}{\mu} \left(1 + \frac{g_2}{n}\right)
\end{equation}

\begin{equation}
    n = \begin{cases} 
        \pi & \text{if } \pi < z \\
        1 & \text{if } \pi \geq z
    \end{cases}
\end{equation}

We consider different versions of this set of equilibrium equations, dependent on the vulnerability of interest. We can think of $\phi = 0$, $g_t = 0$ and $(1+i)\beta = 1$ as a benchmark that we deviate from one parameter at a time.

### 5.1 Credit

In order to consider the role of credit we focus on the impact of the parameter $\phi$, while setting $\beta(1+i) = 1$ and $g_t = 0$. Equilibrium is then characterized by two schedules:

\begin{equation}
    c_1 = \frac{n}{\theta} \quad \text{with } n = \bar{n} \text{ if } \pi < z \text{ and } n = 1 \text{ if } \pi \geq z
\end{equation}

\begin{equation}
    \pi = c_1 - \frac{\lambda}{A}c_1^{1+\gamma} + \frac{\phi\beta}{\mu} \frac{1}{\theta}
\end{equation}

These schedules are shown in Figure 9 for two values of $\phi$. The vertical lines represent the consumption schedule while the humped shaped line reflects the available fund schedule. A higher $\phi$ raises the available fund schedule. Figure 9 shows that when $\phi$ is low, so that credit is tight, there may be two equilibria, so that self-fulfilling panics are possible. But when credit is loose, so that $\phi$ is high, only the non-panic equilibrium exists. The more firms are able to borrow, the less fragile they are. They are then better able to withstand a drop in demand that lowers first-period profits. This in turn can make a self-fulfilling panic impossible. While it remains the case that conditions in period 2 affect consumption in period 1, the
linkage in the other direction is broken under loose credit conditions. Even with low consumption in period 1, leading to low profits, firms can avoid bankruptcy by borrowing.

5.2 Monetary Policy

So far we have assumed that monetary policy is a zero inflation policy and \((1+i)\beta = 1\), so that the non-panic equilibrium corresponds to the flexible price equilibrium. But it is sensible for the central bank to lower the interest rate when faced with a panic that reduces output and consumption. However, the central bank may be constrained by the zero lower bound. We will now assume that \(\phi = 0\) and \(q_t = 0\), but we no longer restrict monetary policy to be \((1+i)\beta = 1\). The symmetric equilibrium is then determined by

\[
\begin{align*}
    c_1 &= \left[\beta(1+i)\right]^{-1/\gamma} \frac{n}{\theta} \quad \text{with } n = \bar{n} \text{ if } \pi < z \text{ and } n = 1 \text{ if } \pi \geq z \\
    \pi &= c_1 - \frac{\lambda}{A} c_1^{1+\gamma}
\end{align*}
\]

The interest rate only enters the consumption schedule. Lowering the interest rate raises consumption and therefore shifts the consumption schedule to the right.

Now consider the following policy. In the absence of a panic the central bank keeps \((1+i)\beta = 1\), so that we achieve the flexible price equilibrium. But when a panic occurs the central bank lowers the interest rate. The chart on the left-hand side of Figure 10 considers the case where the central bank lowers the interest rate all the way to zero during a panic. When \(\beta\) is only slightly below 1, so that the non-panic interest rate \(i = \bar{i} = 1/\beta - 1\) is already close to zero, this involves only a small rightward shift of the left vertical line of the consumption schedule. We see that in that case the central bank cannot avoid a panic due to the zero lower bound. There is a panic equilibrium at \(B'\) that is quite close to the panic equilibrium \(B\) under the passive policy \((1+i)\beta = 1\). The reason for this is that the central bank does not have much room to maneuver when the interest rate is already close to 0.

When instead \(\beta\) is well below 1, so that we are far from the zero lower bound without a panic, the interest rate can be lowered much more during a panic. This leads to a larger rightward shift of the left vertical line of the consumption schedule. When the central bank follows this policy, it is clear from Figure 10 that a panic
can be avoided altogether. A large drop in the interest rate leads to a significant rise in first period consumption, which dampens the decline in firm profits and thus avoids defaults. Only the non-panic equilibrium A exists.

The chart on the right hand side of Figure 10 illustrates this point as well. We can think of (60) as a downward sloping IS curve in the space of \((i, c_1)\). A panic lowers \(n\), which shifts the IS curve to the left. When policy is passive, so that \(i = 1/\beta - 1\), the panic leads to a significant drop in first-period consumption. We shift from point A to point B, corresponding to the same points in the chart on the left. If instead the central bank lowers the interest rate to zero during the panic, we move to point \(B'\). The chart is drawn for the case where \(\beta\) is only slightly below 1, so that the interest is already close to zero without a panic. Lowering the interest further, all the way to zero, will then not raise consumption very much. Profits will then remain very weak and we are unable to escape bankruptcies and therefore the panic.

There is another policy option that theoretically exists and allows the central bank to avoid a panic even when close to the zero lower bound. Instead of a zero inflation policy it could adopt a high inflation policy during a panic. The consumption schedule is then

\[
c_1 = [\beta(1 + i)\frac{P_1}{P_2}]^{-1/\gamma}\frac{n}{\theta} \quad \text{with } n = \bar{n} \text{ if } \pi < z \text{ and } n = 1 \text{ if } \pi \geq z
\]  

(62)

High inflation expectations can significantly lower the real interest rate, which can lead to a large rightward shift of the left vertical line of the consumption schedule in the left chart of Figure 10 even when we are at the zero lower bound. The panic equilibrium would then no longer exist. This policy has been widely discussed but suffers from a credibility problem as ex-post the central bank has little incentive to generate the promised inflation.

\[26\text{Such credibility issues cannot be properly analyzed in our model as we have not modeled the cost of inflation.}\]
5.3 Fiscal Policy

Figure 11 illustrates the role of fiscal policy. In this case we set $\phi = 0$ and $(1+i)\beta = 1$, so that the two schedules become

$$c_1 = \frac{n}{\theta} \quad \text{with } n = \bar{n} \text{ if } \pi < z \text{ and } n = 1 \text{ if } \pi \geq z$$

(63)

$$\pi = c_1 + g_1 = \frac{\lambda}{A} c_1^\gamma (c_1 + g_1)$$

(64)

First consider the case where fiscal policy takes the form $g_1 = \bar{g}$, which is illustrated in the left chart of Figure 11. A higher level of $\bar{g}$ then shifts upward the available funds schedule.\(^{27}\) The chart illustrates that when government consumption is sufficiently high, the panic equilibrium is ruled out. Only the non-panic equilibrium without bankruptcies exists. With a very high level of government consumption, it is impossible to have a self-fulfilling business cycle panic because government spending is not affected by expectations. Even if private consumption were to decline substantially, period 1 profits would remain relatively strong because of the stable government spending. This precludes the fragile firms from going bankrupt, thus avoiding a self-fulfilling panic.

The chart on the right hand side considers the role of countercyclical fiscal policy. The broken humped shaped schedule assumes that fiscal policy takes the form $g_1 = g - \Theta(c_1 - 1/\theta)$. In that case government consumption is the same as under the $g_1 = \bar{g}$ policy in the absence of a panic. But when a panic occurs, which lowers first period consumption, government spending is higher. When fiscal policy is sufficiently countercyclical, as measured by the parameter $\Theta$, the chart shows that the panic equilibrium no longer exists. When the drop in private consumption during a panic is sufficiently offset by an increase in government consumption, firm profits remain relatively strong and bankruptcies are avoided.\(^{28}\)

\(^{27}\)The derivative of $\pi$ with respect to $\bar{g}$ is $1 - (\lambda/A)c_1^\gamma$. When $c_1 = 1/\theta$, as in the non-panic equilibrium, this derivative is $1/\mu$, which is positive. Only for first-period consumption values well above that can the derivative be negative, but those are not of interest to us as first-period consumption can be no larger than $1/\theta$.

\(^{28}\)Another type of countercyclical fiscal policy that will avoid the panic equilibrium is to recapitalize firms during a panic, as with the GM bailout. More specifically, the government would need to transfer or lend $z - \pi$ to vulnerable firms conditional on a panic. This would preclude the panic equilibrium altogether.
5.4 Vulnerabilities during the 2008 Crisis

There are three ways in which the world economy was particularly vulnerable to a self-fulfilling panic in 2008. First, credit was known to be tight due to large losses experienced by banks and other financial institutions since early 2007, leading to deleveraging in the financial system. Second, interest rates around the world were close to zero even prior to the Fall of 2008, leaving central banks little room to maneuver. Third, the Great Recession took place against the backdrop of high levels of government debt, which limited the ability of fiscal authorities to respond with strong countercyclical policies. Moreover, several countries had adopted fiscal rules, also limiting the flexibility of fiscal policy. These three factors were combined with increased global economic integration in recent decades, which made the world particularly vulnerable to a global panic.

6 Extensions

In this section we consider five extensions to the benchmark model. While these extensions make the model more realistic, they do not alter the main results derived in the benchmark case. The first extension introduces international risk sharing, which leads to further integration across the two countries. The second extension allows for a non-unitary elasticity of substitution between Home and Foreign goods. The third extension adds investment and is able to explain a synchronized drop in investment as observed during the Great Recession. The fourth extension adds uncertainty about $z$. A panic then also leads to an increase in uncertainty about future output that is common across countries, consistent with what we saw during the Great Recession as documented in Figure 2. Finally, the last extension examines asymmetric countries.

6.1 Financial Integration

In the model so far the two countries trade goods but are in financial autarky. We have seen that a limited degree of goods market integration is sufficient to

\footnote{Even before fiscal debt around the globe rose significantly as a result of the recession itself, gross public debt as a percent of GDP stood close to 80\% among advanced economies (see International Monetary Fund, 2012). With the exception of the end of World War II, this is the highest level in over a century.}
guarantee that a business cycle panic is global. We now add to this financial integration. We only consider the extreme case of full risk sharing.\footnote{Intermediate cases with partial financial integration can be accomplished in many ways and this is not necessarily captured well through one parameter in a way that is analogous to $\psi$ for goods market integration.}

There is room for risk sharing as business cycle panics are a result of sunspot shocks that may affect only one country. Under complete markets the ratio of marginal utilities of consumption is equal to the real exchange rate:

$$\frac{c_t^{-\gamma}}{(c_t^*)^{-\gamma}} = \frac{P_t}{S_tP_t^*}$$

(65)

This replaces the condition $P_t c_t = S_t P_t^* c_t^*$ under financial autarky. As long as $\gamma$ is different from 1 these two conditions will differ.\footnote{We assume that only households share risk. Firms do not have access to risk-sharing because of standard principal agents problems that also lead to borrowing constraints.} The expression (41) for relative prices no longer holds and is replaced by (65). This is the only change to the model. The equations (74)-(81) in Appendix C that summarize the equilibrium all remain the same, but the relative prices $P_{H,t}/P_t$ and $P_{F,t}/P_t$ that enter these equations are now based on (65).\footnote{We have $P_t/P_{H,t} = (c_t/c_t^*)^{(1-\psi)/(2\psi-1)}$ and $P^*_t/P_{F,t} = P_{H,t}/P_t$.}

We find numerically that risk sharing tends to further increase the cutoff level of $\psi$ below which a panic is necessarily global. With financial integration, less trade integration is then needed to assure a global panic. For example, in the numerical exercise in Section 4.5 we found that $\psi$ was 0.89 under real wage rigidities and 0.77 under nominal wage rigidities.\footnote{As explained, without wage rigidities we needed to set $\gamma$ close to zero to avoid excessive wage cyclical, which is particularly unrealistic in the present context of risk-sharing where $\gamma$ plays a role as the rate of relative risk-aversion. With wage rigidities we set $\gamma = 3$.} With risk sharing these numbers increase to respectively 0.95 and 0.84.

To understand the role of risk sharing, consider for example the left panel of Figure 8. We saw that trade integration makes it less likely that there is an equilibrium where there is no panic in the Home country if there is a panic in the Foreign country. The Foreign panic reduces both consumption and profits in the Home country. With sufficient integration the good, non-panic, equilibrium then no longer exists in the Home country. This is reinforced with financial integration. If there is a panic only in the Foreign country, there will be a transfer of resources from Home to Foreign. This lowers Home consumption (shifting the right vertical
line further to the left). The transfer also lowers relative demand for Home goods, which lowers the relative price of Home goods, which further lowers Home profits. The non-panic equilibrium is therefore even less likely to exist in the Home country. A similar argument can be made for the right panel of Figure 8.

6.2 Elasticity of Substitution

Throughout the paper so far we have assumed a unitary elasticity of substitution between Home and Foreign goods. We now relax this assumption by adopting a CES specification with an elasticity of substitution of $\nu$ between Home and Foreign goods:

$$c_t = \left[ \psi^{1/\nu} \frac{c_t}{c_{H,t}} + (1 - \psi)^{1/\nu} \frac{c_t}{c_{F,t}} \right]^{\frac{\nu}{\nu - 1}} \tag{66}$$

The specification for $c^*_t$ is analogous, with the weights $\psi$ and $1 - \psi$ switched.

As was the case for risk sharing, equations (74)-(81) in Appendix C that describe the equilibrium of the model remain unchanged. The only change again applies to the expression for relative prices that enter these equations. Relative prices are derived from the balanced trade condition $S_t P_{F,t} c_{F,t} = P_{H,t} c^*_{H,t}$. With a unitary elasticity this implies $P_t c_t = S_t P_t c^* t$. With an elasticity $\nu$ this generalizes to

$$\left( \frac{S_t P_{F,t}}{P_{H,t}} \right)^{\nu - 1} \left( \frac{S_t P^*_t}{P_t} \right)^{\nu} = \frac{c_t}{c^*_t} \tag{67}$$

The left hand side is a function of the relative price $S_t P_{F,t}/P_{H,t}$, so this gives an implicit solution of the relative price as a function of $c_t/c^*_t$.\(^{34}\)

We find numerically that the cutoff $\psi(z)$ rises when we lower $\nu$ below 1 and falls when we raise $\nu$ above 1. There is evidence that $\nu$ is in fact lower than 1. For example, Hooper, Johnson and Marquez (2000) estimate import price elasticities to be well below 1 for the G-7 countries. Using the parameter assumptions from Section 4.5 we find that lowering $\nu$ from 1 to 0.7 raises $\psi$ from 0.91 to 0.95 for the flexible wage case, from 0.9 to 0.95 for the rigid real wage case and from 0.77 to 0.89 for the case of rigid nominal wages. These results imply that with trade elasticities less than 1 even less trade is needed to guarantee that panics will be global in nature.

\(^{34}\)It is well known that for sufficiently low elasticities of substitution (in our case below 0.5), this balanced trade condition has more than one solution for the relative price. This is an entirely separate form of multiplicity, discussed for example by Bodenstein (2010).
The elasticity of substitution plays a role by affecting transmission, which we saw has in turn an effect on the very existence of equilibria. Consider an asymmetric equilibrium with a Foreign panic but no Home panic. While we have seen that a Foreign panic has a negative effect on the Home profit schedule (see (53)), there is one factor that weakens this negative transmission by operating in the opposite direction. The lower relative supply of Foreign goods raises the relative price of Foreign goods. This in turn leads to a substitution towards Home goods, raising demand for Home goods and Home profits. This factor is weakened for a lower elasticity of substitution. The negative transmission of a Foreign panic to Home profits is then stronger and makes it less likely that the no-panic equilibrium in the Home country exists (Figure 8).

6.3 Investment

As shown in Figure 1, investment also declined sharply during the Great Recession. And the decline was again of similar magnitude in the rest of the world as in the United States. To capture this, we now consider a simple extension that allows for investment. We assume that firms that do not go bankrupt need to invest in period 1 in order to operate in period 2. To simplify, we assume a given level of required investment per firm of $\bar{k}$. This investment is measured as the same index of Home and Foreign goods as for consumption. Investment demand for individual goods therefore takes the same form as for consumption, with $c_1$ replaced by $I_1$ and $c^*_1$ by $I^*_1$. Aggregate investment is $I_1 = n\bar{k}$ and $I^*_1 = n^*\bar{k}$.

The equilibrium conditions (74)-(81) listed in Appendix C remain the same with two exceptions that affect the available funds schedule. First, investment $\bar{k}$ needs to be subtracted from first period profits. Second, $c_1$ and $c^*_1$ need to be replaced by $c_1 + I_1$ and $c^*_1 + I^*_1$ (with the exception of wages, which only depend on consumption as in (18)). The only other change is to the expression for the relative price in period 1. It is derived from the balanced trade condition. Without investment we showed that it can be written as $P_1 c_1 = S_1 P_1^* c^*_1$. With investment it becomes $P_1 (c_1 + I_1) = S_1 P_1^* (c^*_1 + I^*_1)$. Correspondingly, in the expression (41) for the period-1 relative price we again need to replace $c_1$ and $c^*_1$ with $c_1 + I_1$ and $c^*_1 + I^*_1$.

The change in the expression for the relative price makes it more difficult to derive analytical results, but the numerical results are consistent with Propositions
If we set $\bar{k}$ such that the ratio of investment to GDP is 0.15 without a panic (the average for the U.S. since 1990), and set the other parameters the same as in Section 4.5, the values of $\psi$ remain virtually the same. Therefore it is again the case that limited integration is sufficient to assure that a panic is global. The only difference is that now during a global panic there is also a synchronized drop in investment in both countries.

Another interesting point relates to the Paradox of Thrift. All agents in the economy attempt to save more because of an anticipated drop in future income. But in the end equilibrium saving will be lower around the world. This occurs because the increase in desired saving leads to a drop in demand in period 1, which lowers output and income in period 1. For intertemporal smoothing reasons this reduces period-1 saving. In the model without investment, equilibrium saving remains at zero during a global panic. But since we now have an endogenous decline in investment during the panic, global saving must have declined as well. This is consistent with the data, which show a decline in global saving and investment during the 2008-2009 crisis.

### 6.4 Uncertainty

A simple way to introduce uncertainty is to assume that the level of the fixed cost $z$ is not known in advance. Let us assume that $z$ can take the values $z_L$ or $z_H$, with $z_H > z_L > 0$. The probability of either value is 0.5 and the draw is uncorrelated across countries. As we will see, this generates business cycle uncertainty only when there is a panic, consistent with the evidence of a significant spike in GDP uncertainty during the Great Recession, documented in Figure 2.

Of the equilibrium conditions (74)-(81) listed in Appendix C, only the consumption Euler equations will change. Previously period 2 consumption was known in period 1, while now it may be uncertain. Assuming $\phi = 0$, the Home fragile firms default when $\pi_1 < z$. This depends on the level of $z$. We assume that the fixed cost is paid at the end of period 1 and is unknown when consumption decisions are made.

Let $p_D$ be the probability of default. We have $p_D = 0$ when $\pi_1 \geq z_H$, $p_D = 1$ when $\pi_1 < z_L$ and $p_D = 0.5$ when $z_L \leq \pi_1 < z_H$. In the latter case, default will depend on whether the draw of $z$ is $z_L$ or $z_H$. The probability of default $p^*_D$ in the Foreign country depends similarly on $\pi^*_1$. 

37
Let \( c_2(n, n^*) \) and \( c_2^*(n, n^*) \) be second-period consumption in both countries as a function of the number of firms. This takes the form \( c_2 = \frac{1}{\delta} n^{(1-\delta)\zeta} (n^*)^{\delta\zeta} \) when \( g_2 = 0 \), but more generally is derived from (74)-(75) in Appendix C. There are now 4 possible outcomes, dependent on whether or not there is default in Home and Foreign. This leads to the following consumption Euler equation for Home (assuming \((1 + i)\beta = 1\)):

\[
c_1^{-\gamma} = p_Dp_D^*c_2(n, \bar{n})^{-\gamma} + (1 - p_D)(1 - p_D)c_2(1, 1)^{-\gamma} + \]
\[
p_D(1 - p_D^*)c_2(n, 1)^{-\gamma} + (1 - p_D)p_D^*c_2(1, \bar{n})^{-\gamma}
\]

(68)

The Foreign consumption Euler equation is analogous.

We can numerically verify the equilibria by considering all 9 possible values of the pair \((p_D, p_D^*)\). Given a set of values for these default probabilities, we can compute first-period consumption from the consumption Euler equations. This gives us expressions for first-period profits in both countries, which maps into values of \( p_D \) and \( p_D^* \) as described above. When the latter are consistent with their assumed values, there is an equilibrium.

To provide an illustration of the type of equilibria that this can generate, consider again the parameter values in Section 4.5. Let \( z_L = 0.5 \) and \( z_H = 0.58 \). In the case of rigid real wages we find that for \( \psi < 0.92 \) there are two equilibria. In one equilibrium there is no panic in either country. Consumption and profits are high and the probability of default is zero. In the second equilibrium there is a panic. Consumption and profits are weak. The probability of default is 0.5 as there will not be default when \( z = z_L \). The panic is synchronized across the two countries. When \( \psi > 0.92 \) these same two equilibria still exist. In addition there are now mixed equilibria where only one country panics, with a 0.5 probability of default, and the other does not.

The main difference relative to the previous equilibria is that in a panic equilibrium there is now a positive probability of default rather than certain default. The key result of the paper, that a limited extent of trade integration is sufficient to guarantee that panics are global, still holds. The same equilibria also apply to nominal wage rigidities, with the cutoff for \( \psi \) being 0.83, as well as flexible wages.\(^{35}\)

Business cycle uncertainty is now endogenous and only spikes during a panic. Without a panic, consumption and profits are strong. No firms default, whether

\(^{35}\)In the case of flexible wages we need to set different values for \( z_L \) and \( z_H \). For example, if we set them at 0.4 and 0.54 the same types of equilibria occur, with the cutoff for \( \psi \) being 0.92.
$z = z_L$ or $z = z_H$, assuming that Assumption 1 holds for $z = z_H$. The exogenous uncertainty about $z$ therefore does not generate output uncertainty. In a panic, however, consumption and profits are weak. In that case the value of $z$ does matter. When $z = z_L$ defaults can still be avoided even though profits are weak. But when $z = z_H$ the fragile firms will default. Therefore the uncertainty about $z$ translates into output and consumption uncertainty only during a panic.

The endogenous uncertainty also contributes to the self-fulfilling mechanism itself. Without uncertainty we saw that the self-fulfilling beliefs operate through the expected level of second period income. Lower expected income leads to lower consumption, which causes lower profits that generates bankruptcies that are consistent with the belief of lower expected future income. With uncertainty the second moment plays a role as well.\(^{36}\) Higher income uncertainty leads to lower consumption as a result of precautionary saving. This in turn lowers profits, which makes the fragile firms more sensitive to fixed cost shocks. This generates uncertainty about defaults, making the belief of income uncertainty self-fulfilling.

It is also useful to note that panics do not necessarily imply bankruptcies in this extension. When $z_L \leq \pi_1 < z_H$ in a panic, bankruptcies only occur when $z = z_H$. It is the increased expectation of bankruptcies and uncertainty about bankruptcies that drives the panic. But dependent on $z$, these bankruptcies may not necessarily materialize. Moreover, since $z$ and $z^*$ are uncorrelated, bankruptcies may occur in only one country, even when the panic is global. In other words, perfect co-movement may only occur in a global panic and not in subsequent periods.

### 6.5 Asymmetric Countries

So far we have assumed that the parameters, including the policy parameters, are all exactly the same across the two countries. This means that the exogenous fundamentals faced by both countries are the same. It is natural to ask what happens when these parameters differ. For example, what would happen if one

\(^{36}\) A large literature following Bloom (2009) has focused on the impact of exogenous uncertainty shocks on the business cycle. In contrast to this literature, here the uncertainty is entirely endogenous. Even though there is exogenous uncertainty about $z$, business cycle uncertainty is endogenous and only arises during a panic. Moreover, and again in constrast to the existing literature, the uncertainty is self-fulfilling. Basu and Bundick (2012) use a sticky price model to show that the increase in uncertainty had a large role in worsening the Great Recession. Ravn and Sterk (2012) focus on the impact of job uncertainty on the Great Recession.
country is further from the zero lower bound than the other or one country is less constrained to conduct countercyclical fiscal policy than the other?

In order to answer this, we consider two types of asymmetries. The first is a difference across countries in credit constraints, measured by different values of the parameter $\phi$. Results will be analogous if instead we consider differences in the ability to conduct countercyclical fiscal or monetary policy. The other asymmetry is a difference in country size.

First consider credit constraints. Let $\phi$ be equal to $\phi_H$ in the Home country and $\phi_F$ in the Foreign country. Without loss of generality, assume that credit is less constrained in the Foreign country, so that $\phi_F > \phi_H$. Under Assumption 1, the symmetric non-panic equilibrium always exists. The first question is under what conditions a symmetric panic equilibrium exists. Symmetric equilibria are still given by the schedules (48)-(50). The only difference is that $\pi$ and $\pi^*$ are no longer equal. The weaker credit constraint in the Foreign country shifts up its available fund schedule. In order for the global panic equilibrium to exist we therefore must make sure that the panic equilibrium exists in the Foreign country when $n = n^* = \bar{n}$. This is the case when $\pi^*(\bar{n}) < z$. In terms of Figure 9, it means that the available funds schedule crosses the vertical line below $z$.

More generally, if the exogenous fundamentals are “too strong” in the Foreign country, a panic equilibrium in the Foreign country does not exist, independent of conditions in the Home country. In that case a global panic is not possible. For a global panic equilibrium to exist it must be the case that a panic equilibrium exists for both countries if they were in autarky. While conditions do not need to be equally bad, fundamentals in both countries must be sufficiently weak that both are vulnerable to a panic in autarky.

Assuming that a global panic exists, in general it remains the case that for sufficient integration either both countries panic or neither panics (Proposition 2 still holds). As shown in Appendix D, a sufficient condition for this to be the case is that

$$\kappa\beta(\phi_F - \phi_H) < (1 - \bar{n}^*) + \left(\frac{1}{\bar{n}} - 1\right)\kappa\beta\phi_H$$

This will hold as long as either the difference in fundamentals is not too big or the magnitude of the panic is sufficiently large. If this condition is not satisfied, it may be possible (dependent on the value of $z$) for the Foreign country not to have a panic while the Home country does experience a panic, independent of the
degree of economic integration.

To summarize, even when fundamentals differ across countries, a panic equilibrium that is necessarily global in nature may still exist. This requires that countries are sufficiently integrated and that in both countries fundamentals overall are sufficiently weak. This may for example be the case even if credit is less tight in one of the countries, but it still faces significant constraints on monetary and fiscal policy.

We finally briefly discuss asymmetry through a difference in country size. Consider the extreme where the Foreign country is infinitesimal in size relative to the Home country. In that case the Home country is essentially in autarky as the other country has an infinitesimal effect on its economy. When fundamentals are sufficiently weak in the Home country, both a panic and non-panic equilibrium are possible. With sufficient integration, the conditions in the Foreign country will be almost entirely determined by those in the Home country because of its large relative size. This implies that when the Home country panics, the Foreign country panics as well. Similarly, when the Home country does not panic, neither does the Foreign country. A panic will again necessarily be global for sufficient integration. In this case even less integration is needed to guarantee that a Home panic leads to a Foreign panic as now the Foreign country is even more dependent on the Home country than in the benchmark model where both countries are of equal size.

7 Conclusion

The paper is motivated by the close business cycle co-movement during the Great Recession in a world where both goods and financial markets are far from perfectly integrated. Even though the housing and financial shock originated in the United States, business cycles in the rest of the world were impacted to a similar extent. Given limited trade and financial integration across countries this is surprising as standard models with exogenous shocks and limited integration generate only partial transmission. It is also surprising given the much lower co-movement of business cycles during prior recessions.

To explain this we have developed a two-country model with self-fulfilling business cycle panics. We have shown that the model is consistent with high international co-movement observed during the Great Recession. We find that limited
economic integration is sufficient to assure that a panic, when it occurs, is necessarily perfectly synchronized across countries. In a panic there is an equal drop of consumption, investment, output, expected output and profits across countries. Moreover, perceived uncertainty increases equally across countries.

At the same time, we shed light on the fact that such strong business cycle co-movement as seen during the Great Recession is historically unusual. We have argued that several factors made the 2008 episode particularly vulnerable to such a global panic: tight credit, very low interest rates, rigid fiscal policy, combined with increased economic integration across countries. And of course there was an unusually strong trigger event for a panic in the form of U.S. financial market turmoil. The combination of these conditions separates the 2008 episode from previous recessions.
Appendix

A. GDP Forecast Expectation and Variance

This Appendix describes in some more detail how the numbers in Figure 2 are computed. The data has been purchased from Consensus Economics. In their January newsletter of “Consensus Forecast” and “Asia Pacific Consensus Forecasts” they publish one-year-ahead GDP forecast probabilities since 1999 for the countries listed in the Figure. More specifically, for every country and year there are seven intervals of growth forecasts (e.g. 1-2%, 2-3%). The precise intervals may change from year to year. The data reports probabilities of each interval as the percentage of forecasts that lie in that interval. We compute the expectation and variance of the forecasts by using the midpoint of each interval, together with the probabilities of the intervals.

One issue is that the intervals at both ends of the range are not bounded (e.g., an interval can be “<-1%”). In that case we adopt two scenarios to choose a midpoint for the interval. In the first scenario, we choose a midpoint by assuming that the interval width is the same as that for the other intervals. In the second scenario we choose a midpoint by assuming that the interval width is twice that for the other intervals. This leads to almost identical results. Figure 2 shows the results for the first scenario.

B. A Generic Example

In this Appendix we derive the result in Section 2 that limited integration leads to a coordination of equilibria. We start with the case of the left panel of Figure 4, where there are two equilibria in autarky. For the two-country version of the model one can draw the Home and Foreign goods market equilibria in \((y, y^*)\) space in order to evaluate all possible equilibria. We will focus here on the Foreign market equilibrium as the Home equilibrium schedule is simply symmetric in the 45 degree line. We can rewrite the Foreign goods market equilibrium (4) as

\[
f(y) = \frac{1}{1 - \psi} (y^* - \psi f(y^*))
\]  

(70)

Since both \(f' > 0\) and \(f'' > 0\) in the left panel of Figure 4, it follows that the Foreign market equilibrium schedule takes the form of a humped shaped solution of \(y\) as a function of \(y^*\).
First consider $\psi < 1/f'(y^B) < 1$. This is only possible if $f'(y^B) < 2$ as $\psi$ is between 0.5 and 1. In that case it is immediate that the Foreign schedule is positively sloped at the symmetric equilibrium B where output is $y^B$ in both countries. By symmetry the Home schedule is then positively sloped as well. This case is represented by Panel 1 of Figure A1. Clearly, in this case only symmetric equilibria exist.

Next assume $\psi > 1/f'(y^B)$. This is automatically the case when $f'(y^B) > 2$ and may also be the case when $f'(y^B) \leq 2$. In that case the humped shaped Foreign schedule is negatively sloped at the symmetric equilibrium B. There are now two cases to consider, corresponding to Panels 2 and 3 in Figure A1. In Panel 2 the Foreign schedule crosses point B with a negative slope that is less than 1 in absolute value. By symmetry the Home schedule then has a negative slope that is larger than 1 in absolute value. The Home schedule therefore has a more negative slope. It is immediate that in this case there are again only symmetric equilibria. Asymmetric equilibria occur only when the Foreign schedule crosses point B with a negative slope that is larger than 1 in absolute value and the Home schedule with a negative slope that is smaller than 1 in absolute value. This is represented by Panel 3.

Equilibria are therefore necessarily symmetric when the Foreign schedule has a slope at point B that is larger than -1 (less negative than -1). This is the case when

$$\frac{1 - \psi f'(y^B)}{(1 - \psi)f'(y^B)} > -1$$  \hspace{1cm} (71)$$

Since $\psi > 1/f'(y^B)$, the numerator is negative. Dividing both sides by -1 and multiplying by the denominator, gives

$$\psi f'(y^B) - 1 < (1 - \psi)f'(y^B)$$  \hspace{1cm} (72)$$

This implies

$$\psi < \bar{\psi} = 0.5 + 0.5 \frac{1}{f'(y^B)}$$  \hspace{1cm} (73)$$

$\bar{\psi}$ is between 0.5 and 1 as $f'(y^B)$ is larger than 1.

We can summarize these results by saying that for $\psi < \bar{\psi}$ there are only symmetric equilibria and for $\psi > \bar{\psi}$ there are both symmetric and asymmetric equilibria. To see this, we know that only symmetric equilibria exist as long as $\psi$ is larger than $1/f'(y^B)$ and less than $\bar{\psi}$. When $f'(y^B) > 2$ this includes all values
of $\psi$ between 0.5 and $\bar{\psi}$. When $f'(y^B) < 2$, it includes all values of $\psi$ on the interval from $1/f'(y^B) > 0.5$ to $\bar{\psi}$ (note that $\bar{\psi} > 1/f'(y^B)$ as $f'(y^B) > 1$). But we know that in addition there are only symmetric equilibria for $\psi$ less than $1/f'(y^B)$. Therefore the conclusion is again that all values of $\psi$ between 0.5 and $\bar{\psi}$ lead to symmetric equilibria. When $\psi > \bar{\psi}$ asymmetric equilibria exist as well. In this case we are in Panel 3 of Figure A1.

These results carry over to the case of 3 equilibria in autarky, as illustrated by the right panel of Figure 4. For the stable symmetric equilibria A and C the $c = f(y)$ schedule crosses the 45-degree line from above, so with a slope of less than 1. Therefore $1 - \psi f'(y)$ is positive at these points. The Foreign schedule then has a positive slope at A and C and by symmetry so does the Home schedule.

There are again different cases to consider, dependent on the slope of the schedules at point B. These different cases are illustrated in Figure A2. Panel 1 assumes that $\psi < 1/f'(y^B)$, which as before leads to a positive slope of both schedules at point B. It is then immediate that only symmetric equilibria exist. Also analogous to the previous results, only symmetric equilibria exist when at point B the Foreign schedule has a negative slope that is less than 1 in absolute value. This is illustrated in Panel 2. Taking Panels 1 and 2 together, the previous result that there are only symmetric equilibria for $\psi$ between 0.5 and $\bar{\psi}$ continues to hold.

When $\psi > \bar{\psi}$, so that the Foreign schedule has a negative slope at point B that is larger in absolute value than 1, there will again be asymmetric equilibria, as illustrated in Panels 3, 4 and 5 of Figure A1. However, as we slightly raise $\psi$ above $\bar{\psi}$ the first type of asymmetric equilibria that show up are unstable. This is illustrated in Panel 3. For example, with $y^*$ as in point E, the level of $y$ at point E is the middle of the 3 equilibria for the Home country, which is the unstable one. As we raise $\psi$ further, eventually we get 6 mixed equilibria as illustrated in Panel 4. The only stable ones are H and K. If we only count stable equilibria, it follows that the cutoff for $\psi$ above which asymmetric equilibria start to appear is even larger than $\bar{\psi}$. This cutoff is denoted $\bar{\psi}$ in Figure A2. When $\psi = 1$, in the last panel, the equilibria in one country do not depend on output in the other country. This leads to three horizontal and vertical lines, with H and K again being the stable asymmetric equilibria.

C. Model Equilibrium
In this Appendix we show how the model can be described a set of eight equations. Throughout the paper we use these equations to look at various special cases. These equations are:

\[
\frac{\mu}{\mu - 1} \frac{\lambda}{\alpha A} c_2 \frac{P_2}{P_{H,2}} \left( \frac{P_2}{P_{H,2}} c_2 + g_2 \right)^{\frac{1-\alpha}{\alpha}} = n^\kappa
\]

(74)

\[
\frac{\mu}{\mu - 1} \frac{\lambda}{\alpha A} (c_2)^\gamma \frac{P_2^*}{P_{F,2}} \left( \frac{P_2^*}{P_{F,2}} c_2^* + g_2^* \right)^{\frac{1-\alpha}{\alpha}} = (n^*)^\kappa
\]

(75)

\[
c_1^\gamma = \beta (1 + i) c_2^\gamma
\]

(76)

\[
[c_1^*]^{-\gamma} = \beta (1 + i^*) [c_2^*]^{-\gamma}
\]

(77)

\[
\pi = c_1 + \frac{P_{H,1}}{P_1} g_1 - \frac{\lambda}{A} c_1 \left( \frac{P_1}{P_{H,1}} c_1 + g_1 \right)^{1/\alpha}
\]

\[
+ \frac{\phi \kappa \lambda}{1 + i} c_2 n^{-\frac{\mu}{\alpha}} \left[ \frac{P_2}{P_{H,2}} c_2 + g_2 \right]^{1/\alpha}
\]

(78)

\[
\pi^* = c_1^* + \frac{P_{F,1}}{P_1} g_1^* - \frac{\lambda}{A} (c_1^*)^\gamma \left( \frac{P_1^*}{P_{F,1}} c_1^* + g_1^* \right)^{1/\alpha}
\]

\[
+ \frac{\phi \kappa \lambda}{1 + i^*} (c_2^*)^\gamma (n^*)^{-\frac{\mu}{\alpha}} \left[ \frac{P_2^*}{P_{F,2}} c_2^* + g_2^* \right]^{1/\alpha}
\]

(79)

\[
n = \begin{cases} 
\pi & \text{if } \pi < z \\
1 & \text{if } \pi \geq z 
\end{cases}
\]

(80)

\[
n^* = \begin{cases} 
\pi^* & \text{if } \pi^* < z \\
1 & \text{if } \pi^* \geq z 
\end{cases}
\]

(81)

With relative prices as in (41), these are 8 equations in \(c_1, c_1^*, c_2, c_2^*, n, n^*, \pi \) and \(\pi^*\). They are derived as follows. (74) follows by substituting the labor supply schedule \(W_2/P_2 = \lambda c_2^\gamma\) and \(P_{H,2}(j)/P_{H,2} = n^{1/(\mu - 1)}\) into the optimal price setting equation (25). It also uses the expression (24) for \(y_2(j)\) that enters into the optimal price setting equation (25), after substituting (40) into the expression for \(y_2(j)\). (75) is the Foreign counterpart of (74). (76) follows from the intertemporal consumption Euler equation (13) after substituting the zero inflation monetary policy \(P_2 = P_1\). (77) is the Foreign counterpart.

(78) is an expression for available funds \(\pi = \pi_1 + \phi \pi_2/(1 + i)\). It is derived as follows. First, we derive \(\pi_1\), which is on the first line of the right hand side of (78). It is equal to

\[
\pi_1 = \frac{P_{H,1}}{P_1} y_1(j) - \frac{W_1}{P_1 A} \frac{1}{\gamma} y_1(j)^{1/\alpha}
\]

(82)
Using that $P_{H,1}(j) = P_{H,1}$, we have from (34) that $y_1(j) = c_{H,1} + c^*_H(j) + g_1$. Substituting $c_{H,1} = \psi(P_1/P_{H,1})c_1$ and $c_{H,1}^* = (1-\psi)(S_1P_1^*/P_{H,1})c_1^*$, and also using $P_1c_1 = S_1P_1^*c_1^*$ from (40), we have $y_1(j) = (P_1/P_{H,1})c_1 + g_1$. When we substitute this into (82), together with $W_1/P_1 = \lambda c_1^*$, we get the first line on the right hand side of (78). The second line is $\phi\pi_2/(1+i)$. We derive an expression for $\pi_2$ as follows. From (26) it is equal to $\pi_2(j) = \kappa\frac{1}{\lambda}(W_2/P_2)y_2(j)^{1/\alpha}$. We substitute $W_2/P_2 = \lambda c_2^*$ and the expression (24) for $y_2(j)$. In the expression for $y_2(j)$ we also substitute (40) and $P_{H,2}(j)/P_{H,2} = n^{1/(\mu-1)}$. This then delivers the second line on the right hand side of (78). (79) is the Foreign counterpart. Finally, (80) follows from the bankruptcy condition (31) and (81) is its Foreign counterpart.

The paper considers two special cases of this system of equations. In Sections 3.5 and 4.1-4.4 we assume $g_t = 0$ and in the vulnerability Section 5 we assume $\zeta = \alpha = 1$. We will now show that $g_t = 0$ allows us to summarize the equilibrium in the form of the 6 equations (42)-(47) and that $\zeta = \alpha = 1$ implies the symmetric equilibrium given by (55)-(57) in the vulnerability section.

Setting $g_2 = g_2^* = 0$ and taking (74)-(75) to the power $\alpha/(1-\alpha+\alpha\gamma)$, these two equations can be written as

$$\theta \left( \frac{P_2}{P_{H,2}} \right)^{\frac{1}{1-\alpha+\alpha\gamma}} c_2 = n^\zeta$$

$$\theta \left( \frac{P_2^*}{P_{F,2}} \right)^{\frac{1}{1-\alpha+\alpha\gamma}} c_2^* = n^{*\zeta}$$

with $\theta$ and $\zeta$ defined in Section 3.5. Substituting the expressions for relative prices from (41), this gives two equations in $c_2$ and $c_2^*$ that can be solved as a function of $n$ and $n^*$. Using that $c_1 = [\beta(1+i)]^{-1/\gamma}c_2$ and $c_1^* = [\beta(1+i^*)]^{-1/\gamma}c_2^*$ from the consumption Euler equations (76)-(77), we then have

$$c_1 = \frac{[\beta(1+i)]^{-1/\gamma}}{\theta} n^{(1-\delta)\zeta(1-\delta)\zeta}$$

$$c_1^* = \frac{[\beta(1+i^*)]^{-1/\gamma}}{\theta} n^{\delta(1-\delta)\zeta}$$

This corresponds to the equilibrium equations (42)-(43) in Section 3.5 for the case where monetary policy is $(1+i)\beta = (1+i^*)\beta = 1$. (44)-(45) follow directly from (78)-(79) after again setting $(1+i)\beta = (1+i^*)\beta = 1$ and $g_t = g_t^* = 0$. This monetary policy also implies $c_2 = c_1$ and $c_2^* = c_1^*$. We therefore replace second period with first-period consumption on the second lines of (78)-(79). We also use
that the second period relative prices are equal to the first period relative prices. This follows from (41), together with \( c_2 = c_1 \) and \( c_2^* = c_1^* \). Finally, (46)-(47) correspond exactly to (80)-(81).

In the vulnerability Section 5 we only consider symmetric equilibria, under the assumption that \( \alpha = \zeta = 1 \). All relative prices are then equal to 1. It then follows immediately from (74) that \( c_2 = n/\theta \). Together with the consumption Euler equation (76) this gives (55). (56) follows from (78) after substituting \( c_2 = n/\theta \), setting \( \alpha = \zeta = 1 \) and setting all relative prices equal to 1.

Finally, a couple of brief comments are in order about the flexible price equilibrium for the case where \( g_t = 0 \), discussed at the end of Section 3.5. In that case there are two additional variables to solve for, the nominal interest rates \( i \) and \( i^* \). There are also two additional equations, which are the period 1 analogues of (74)-(75), which follow from optimal price setting in period 1. Solving these equations for period 1, using the expression (41) for the relative price and the fact that the number of firms is 1 in period 1, gives \( c_1 = c_1^* = 1/\theta \). This in turn implies that \( \pi_1 = \pi_1^* = [\mu(1-\alpha) + \alpha]/(\mu \theta) \). Under Assumption 1, it follows that \( \pi_1 > z \), so that also \( \pi > z \) as \( \pi_2 > 0 \). Therefore no firms go bankrupt and \( n = 1 \). Similarly we also have \( n^* = 1 \). Solving for (74)-(75) with \( g_2 = 0 \) we then also have \( c_2 = c_2^* = 1/\theta \). First and second period consumption are therefore equal and it follows from the consumption Euler equations (76)-(77) that \( (1 + i)\beta = (1 + i^*)\beta = 1 \).

D. Proof of Proposition 2

We already know that both symmetric equilibria exist when \( \pi(\bar{n}) < z < \pi(1) \). We therefore focus on the existence of asymmetric equilibria. We will only consider the asymmetric equilibrium \((n, n^*) = (\bar{n}, 1)\) as the other asymmetric equilibrium, \((n, n^*) = (1, \bar{n})\), exists if and only if the first one exists.

From (42)-(43), setting \( n = \bar{n} \) and \( n^* = 1 \) gives \( c_1 = (1/\theta)\bar{n}^{(1-\delta)\zeta} \) and \( c_1^* = (1/\theta)\bar{n}^{\beta\zeta} \). Substituting these values for \( c_1 \) and \( c_1^* \) into (44)-(45) gives

\[
\hat{\pi}(\psi) = \frac{1}{\bar{n}} \bar{n}^{(1-\delta)\zeta} \left(1 - \frac{(\mu - 1)\alpha}{\mu} \bar{n}^{\alpha}\right) + \phi \beta \frac{\mu(1-\alpha) + \alpha}{\mu \theta} \bar{n}^{\zeta(1-\delta)-1}
\]

\[
\hat{\pi}^*(\psi) = (1 + \phi \beta) \frac{\mu(1-\alpha) + \alpha}{\mu \theta} \bar{n}^{\zeta\delta}
\]

where \( \hat{\pi}(\psi) \) and \( \hat{\pi}^*(\psi) \) are the values of \( \pi \) and \( \pi^* \) when \( (n, n^*) = (\bar{n}, 1) \) and \( \delta = (1 - \psi)/[(1 - \alpha + \alpha \gamma)(2\psi - 1) + 2(1 - \psi)] \). We will consider values of \( \psi \) between 0.5
and 1. The asymmetric equilibrium \((n, n^*) = (\bar{n}, 1)\) exists when \(\hat{\pi}(\psi) < z \leq \hat{\pi}^*(\psi)\). This is clearly the case for \(\psi = 1\) as \(\hat{\pi}(1) = \pi(\bar{n})\) and \(\hat{\pi}^*(1) = \pi(1)\).

Using the negative relationship between \(\psi\) and \(\delta\), it follows immediately from the expressions for \(\hat{\pi}\) and \(\hat{\pi}^*\) above that the derivative of \(\hat{\pi}\) with respect to \(\psi\) is negative and the derivative of \(\hat{\pi}^*\) with respect to \(\psi\) is positive for \(\psi\) between 0.5 and 1. We will also show that there is a value \(\bar{\psi} > 0\) for which \(\hat{\pi}(\bar{\psi}) = \hat{\pi}^*(\bar{\psi})\). These two results together imply the proposition. As we lower \(\psi\) below 1, \(\hat{\pi}\) rises and \(\hat{\pi}^*\) falls, until we reach a level \(\psi(z) > 0.5\) so that either \(\hat{\pi}(\psi(z)) = z\) or \(\hat{\pi}^*(\psi(z)) = z\). If this were not the case, then \(\hat{\pi}(\psi) < \hat{\pi}^*(\psi)\) for all \(\psi\) between 0.5 and 1, which is inconsistent with the finding that they are equal for \(\psi = \bar{\psi} > 0.5\). For values of \(\psi\) above \(\psi(z)\) we have \(\hat{\pi} < z\) and \(\hat{\pi}^* < z\), so that \((n, n^*) = (\bar{n}, 1)\) is an equilibrium. For values of \(\psi\) below \(\psi(z)\) we either have \(\hat{\pi} > z\) or \(\hat{\pi}^* < z\), so that \((n, n^*) = (\bar{n}, 1)\) is not an equilibrium.

We finally need to show that there is a value \(\bar{\psi} > 0.5\) for which \(\hat{\pi}(\bar{\psi}) = \hat{\pi}^*(\bar{\psi})\). Let the corresponding value of \(\delta\) be \(\bar{\delta}\). Equating the expressions above for \(\hat{\pi}\) and \(\hat{\pi}^*\) gives

\[
\frac{\bar{n}(1-2\delta)\zeta}{\mu} = \frac{(\alpha + \mu(1-\alpha))(1 + \phi\beta)}{\mu - (\mu - 1)\alpha\pi^p + \phi\beta(\mu(1-\alpha)+\alpha)}
\]  

(87)

It follows from \(\bar{n} < 1\) that the term on the right hand side is less than 1. Therefore it must be the case that \(\bar{\delta} < 0.5\), from which it follows that \(\bar{\psi} > 0.5\). It follows that there is a value \(\psi = \bar{\psi} > 0.5\) for which \(\hat{\pi}(\bar{\psi}) = \hat{\pi}^*(\bar{\psi})\), which completes the proof of Proposition 2.

In Section 6.5 we discuss an extension where \(\phi\) differs across countries, with \(\phi_F > \phi_H\). In order to evaluate whether an asymmetric equilibrium exists where there is only a panic in the Home country, we must replace \(\phi\) with \(\phi_F\) and \(\phi_H\) respectively in the expressions for \(\hat{\pi}^*(\psi)\) and \(\hat{\pi}(\psi)\). We can again find \(\bar{\psi}\) by setting \(\hat{\pi}^*(\psi) = \hat{\pi}(\psi)\). This still yields (87), with the \(\phi\) in the numerator and denominator replaced by respectively \(\phi_F\) and \(\phi_H\). \(\bar{\psi}\) will be larger than 0.5 \((\bar{\delta} < 0.5)\) when the term on the right hand side is less than 1. This can be rewritten as (69). This is a sufficient condition to assure that the asymmetric equilibrium with \(n = \bar{n}\) and \(n^* = 1\) does not exist for sufficient integration. It is not a necessary condition as either \(\hat{\pi}^*(\psi) < z\) or \(\hat{\pi}(\psi) > z\) may hold for sufficient integration even when (69) is not satisfied.

\(\text{E. Introducing Wage Rigidities}\)
In order to introduce wage rigidities we first introduce labor heterogeneity. Labor $L_t$ in the production function is now a CES index of labor supply by all households:

$$L_t = \left( \int_0^1 L_t(j)^{\frac{\omega - 1}{\omega}} dj \right)^{\frac{\omega}{\omega - 1}} \tag{88}$$

where $L_t(j)$ is labor by agent $j$. Given $L_t$, this specification leads to the following demand for individual labor:

$$L_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\omega} L_t \tag{89}$$

where $W_t(j)$ is the wage rate for labor supplied by agent $j$ and

$$W_t = \left( \int_0^1 W_t(j)^{1-\omega} dj \right)^{\frac{1}{1-\omega}} \tag{90}$$

Aggregate labor demand in period $t$ in the Home country is $n_{H,t}L_t$. Demand for labor supplied by agent $j$ is then

$$1 - l_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\omega} n_{H,t}L_t \tag{91}$$

We can now maximize agent $j$ utility with respect to $W_t(j)$. All households choose the same optimal $W_t(j)$, which will then be equal to $W_t$. We will replace the $\lambda l_t$ in the utility function with $\bar{\lambda} l_t$. Dropping the $j$, maximization of utility with respect to the individual wage rate gives

$$\frac{W_t}{P_t} = \lambda c_t^\gamma \tag{92}$$

where $\lambda = \bar{\lambda} \omega / (\omega - 1)$. With the redefined $\lambda$ this is the same as (18). Nothing else in the model changes.

When wages are rigid, they are set at the start of each period. This makes no difference for period 2 as there are no shocks during period 2. For period 1 the only shock is a self-fulfilling panic. We assume that the probability of a panic is infinitesimal. Then the right hand side of (92) needs to include the expectation of $c_1^\gamma$ at the start of period 1 giving infinitesimal weight to a panic occurrence. The expectation is therefore based on $c_1 = 1/\theta$, its value in the absence of a panic. When the real wage is set at the start of period $t$, it will then be set at $\lambda / \theta^\gamma$. If instead the nominal wage is set, it will be equal to $\hat{P} \lambda / \theta^\gamma$, where $\hat{P}$ is the price index in the non-panic state. This is equal to $P_{H1}$, the price set at the start of period 1 by all firms. The real wage will then be $(P_{H1}/P_1)(\lambda / \theta^\gamma)$, where $P_{H1}/P_1$ depends on $c_1^*/c_1$ as in (41).
References


* GDP, investment and consumption are from Datastream (growth over past 4 quarters). Non-U.S. G20 excludes Saudi Arabia for GDP and also China for consumption and investment. Corporate profits are net profits from Worldscope, aggregated over continuing firms within each country, divided by the GDP deflator and normalized at 100 in 2006:Q1. Non-US G7 is computed using relative PPP-adjusted GDP weights.
Figure 2 GDP Growth Forecasts Probabilities: Expectation and Variance*

*Data from Consensus Forecasts, based on one-year ahead forecast probabilities. See Appendix A for a description.

Non-US: Australia, China, Hong Kong, India, Indonesia, Malaysia, New Zealand, Singapore, South Africa, Taiwan, Thailand, Japan, Germany, France, U.K., Italy, Canada
Figure 3  Real GDP Growth During the Great Depression

*Source: Angus Maddison. Broken line is the U.S.; solid line is the non-U.S. G20 minus Saudi Arabia minus South Africa.
Figure 4 Multiple Equilibria in Generic Model
Figure 5 Coordination of Equilibria

No Panic in Foreign ($y^*=y^C$)

Panic in Foreign ($y^*=y^A$)

$\psi=1$ (autarky)
Figure 6  Symmetric Equilibria*
Figure 7  All Equilibria: Role of Trade Integration

- H, F: no panic
- H: no panic; F: panic
- H: panic; F: no panic

Graph showing different equilibria based on panic scenarios in trade integration.
Figure 8  Coordination of Equilibria*

* Solid lines: \( \psi = 1 \); Broken lines: \( \psi < 1 \).
Figure 9  Panic Vulnerability: Role of Credit
Figure 10  Panic Vulnerability: Role of Monetary Policy
Figure 11  Panic Vulnerability: Role of Fiscal Policy*

\[ g_1 = \bar{g} \]

\[ g_1 = \bar{g} - \Theta(c_1 - 1/\theta) \]
Figure A1 Two Symmetric Equilibria

Panel 1: $0.5 < 1 / f'(y^B) < \psi$

Panel 2: $1 / f''(y^B) < \psi < \overline{\psi}$

Panel 3: $\overline{\psi} < \psi < 1$

Panel 4: $\psi = 1$
Figure A2 Three Symmetric Equilibria

Panel 1: $0.5 < 1 / f'(y^B) < \psi$

Panel 2: $1 / f'(y^B) < \psi < \overline{\psi}$

Panel 3: $\overline{\psi} < \psi < \widetilde{\psi} < 1$

Panel 4: $\widetilde{\psi} < \psi < 1$

Panel 5: $\psi = 1$