The Great Recession: A Self-Fulfilling Global Panic

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Abstract

The 2008-2009 financial crises, while originating in the United States, witnessed a steep decline in output, consumption and investment that was of similar magnitude in the rest of the world as in the United States. This is surprising both in the context of existing theory and past business cycle experience. Theory implies that perfect co-movement can only happen when countries are perfectly integrated, in sharp contrast to the large observed home bias in goods and financial markets. In order to shed light on this, we develop a two-country model that allows for self-fulfilling business cycle panics. We show that with only limited integration of goods and financial markets, such business cycle panics are necessarily perfectly synchronized across the two countries. The theory also implies that the world was particularly vulnerable to such global panics in 2008 as a result of the combination of factors: the zero-lower bound, tight credit and increased international integration. This sets the Great Recession apart from previous business cycles, which showed far less co-movement.
1 Introduction

The 2008-2009 Great Recession clearly had its origins in the United States, where an historic drop in house prices had a deep impact on financial institutions and markets. It is remarkable then, as illustrated in Figure 1, that the steep decline in output, consumption and investment during the second half of 2008 and beginning of 2009 was about the same in the rest of the world as in the United States.\(^1\) This is surprising both in the context of existing theory and historical experience. Transmission channels in existing models depend critically on trade and financial linkages. A recent literature has shown that it is possible to have one-to-one transmission of shocks if goods and financial markets are perfectly integrated and there are credit rather than technology shocks.\(^2\) But in reality goods and financial markets are far from perfectly integrated and there is significant home bias in both goods and asset trade. As illustrated in van Wincoop (2012), a model with credit shocks that captures the observed financial home bias will have partial transmission at best.\(^3\) Consistent with this, Rose and Spiegel (2010) and Kamin and Pounder (2010) find that there is little relation between financial linkages that countries have with the U.S. and the decline in their GDP growth and asset prices during 2008-2009.

The close co-movement of business cycles illustrated in Figure 1 is also unusual from an historical perspective. Figure 2 shows that during the Great Depression the decline in output in the rest of the world was much less than in the United States. Perri and Quadrini (2012) show that the co-movement during the 2008-2009 recession stands out significantly relative to previous recessions since 1965.

\(^1\)Even outside of Europe, which had by far the largest foreign exposure to U.S. asset backed securities, the business cycle decline was of similar magnitude.

\(^2\)Examples are Devereux and Sutherland (2011), Kollmann, Enders and Muller (2010) and Perri and Quadrini (2012). It is well known that with technology shocks output tends to be negatively correlated across countries even in models with perfect goods and financial market integration.

\(^3\)Dedola and Lombardo (2012) find that that perfect co-movement is possible even with portfolio home bias. But this relies on a setup that precludes arbitrage between risky and riskfree assets as only leveraged agents hold risky assets and face borrowing constraints. As shown in van Wincoop (2012), allowing for non-leveraged agents that can conduct such arbitrage, and calibrating the relative size of leveraged institutions in financial markets, transmission is limited. The 2008 crisis saw very large arbitrage between risky and low risk assets, with a large flight to quality that increased prices of low risk Treasuries.
This then leads to two questions that we aim to address in this paper. First, given the limited extent of goods and financial integration, how can we theoretically explain that the decline in business cycles was similar in the rest of the world as in the United States during the Great Recession? Second, what can explain the difference relative to previous recessions?

To answer these questions we develop a two-country, two-period Keynesian model that explains the recession as resulting from a self-fulfilling panic as opposed to an exogenous shock to fundamentals. The self-fulfilling beliefs are a result of several inter-linkages between the present (period 1) and the future (period 2). The future affects the present as beliefs of lower and riskier second period income leads to lower first period consumption and investment, which reduces output and firm profits. But the present also affects the future as lower profits leads to an expectation of more firm bankruptcies and greater sensitivity of firms to future shocks. This lowers expected future output and increases uncertainty about future output. Figure 3, which is based on survey data, shows that there was indeed a large drop in expected GDP growth and the increase in its perceived variance. Moreover, these changes in beliefs were of similar magnitude in the rest of the world as in the United States.

A key result of the model is that a self-fulfilling business cycle panic is necessarily synchronized across the two countries as long as there is a limited extent of trade and financial integration above a certain threshold. The drop in output, consumption and investment will then be of equal magnitude in the two countries. With very weak trade and financial linkages it is possible to have a business cycle panic that is limited to just one country. But this is no longer possible when there is sufficient economic integration. If the Foreign country does not panic, it then provides enough stability to the Home country to preclude a self-fulfilling panic limited to the Home country as an equilibrium outcome. A panic, if it happens, will then necessarily be global. The threshold level of economic integration to assure that this is the case does not need to be high. It is therefore possible to still have significant home bias in trade and asset holdings as seen in the data.

The model also provides an explanation for the difference relative to previous recessions. Limited co-movement of business cycles in open economy models is usually the result of partial transmission (through trade and financial linkages) of exogenous country-specific shocks. That may well be a good description for most business cycles. However, in our model the co-movement is not a result of
transmission but rather of a coordinated panic. A combination of distinct factors, all featured in the model, made the 2008 period particularly vulnerable to such a global self-fulfilling panic. First, we were in a liquidity trap. Without it the central bank would have a tool to stabilize the economy, which could avoid a self-fulfilling panic as an equilibrium outcome. Second, credit was tight. We show that when credit conditions are easier self-fulfilling panics are not feasible in equilibrium. Tight credit makes firms more susceptible to default when hit by a drop in demand that lowers profits. This is a critical element in our model of self-fulfilling beliefs. Third, the world has experienced a significant increase in both trade and financial integration over the past two decades. The model implies that panics are then more likely to be common across countries.

The crisis in U.S. financial markets plays a role in our theory of a global recession, but only as a trigger event for the self-fulfilling shift in beliefs. This stands in contrast to models in which the linkage between financial markets and the real economy operates through a credit shock or a decline in wealth. While credit was tight, it is hard to argue that there was a large credit shock. Chari, Christiano and Kehoe (2008) document an increase in both consumer and industrial bank credit in the second half of 2008. Adrian, Colla and Shin (2011) find that a decline in bank credit to firms in 2009 was replaced by an equal increase in bond financing. It is also hard to argue that a decline in wealth was responsible for the global recession. With the exception of some smaller European countries (Ireland and Spain) the sharp decline in housing wealth was a U.S. phenomenon rather than a global phenomenon. More generally, estimates of global household wealth (including both housing and financial wealth) show a far larger decline in the U.S. than in the rest of the world from 2007 to 2008.

The paper is related to some other recent work on self-fulfilling business cycle panics. The most important difference is that these are closed economy models and therefore do not address the co-movement question. Farmer (2012a,b) analyzes

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4 This is indeed something that can happen in our model as the need for credit increases when firm profits go down.

5 Also consistent with the absence of a large credit shock, Kahle and Stulz (2011) use firm level data to show that there was no relationship between the drop in investment by firms and their bank dependence. Helbling, Huidrom, Kose and Otrok (2010) estimate a global VAR to find that a global credit shock accounts for only 10% of the global drop in GDP in 2008-2009.

6 Give some numbers here. While stock markets declined significantly everywhere, they tend to be less important in most countries than in the United States.
models where self-fulfilling beliefs are associated with wealth. A belief of a lower value of financial wealth leads to lower consumption, which leads to lower firm profits, which justifies the drop in wealth. But as just pointed out, the decline in wealth was much smaller in the rest of the world than in the United States.

Heathcote and Perri (2012) also have a model where the decline in wealth is critical to self-fulfilling beliefs, although through a different mechanism. In their model lower housing wealth makes it possible to have self-fulfilling beliefs of higher unemployment. If agents find it less likely that they have a job tomorrow, and it is hard to borrow when their housing collateral is low, they will reduce consumption. This reduces output, which indeed leads to more unemployment. Both this paper and the Farmer papers rely on labor market rigidities rather than nominal rigidities to generate a link from demand to production.

Finally, Benhabib, Wang and Wen (2012) develop a model where business cycles are affected by market sentiments when production decisions need to be made in advance of knowing demand and agents receive imperfect information about aggregate demand. It has in common with the Farmer papers that the business cycle then depends on a market sentiment variable that can take on a continuum of values, as opposed to models such as Heathcote and Perri (2012) and ours where there is either a panic or not.\(^7\)

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 considers only symmetric equilibria where first period consumption, investment and output are equal across the two countries. This gives insight into conditions under which self-fulfilling panics are feasible and the mechanism behind it. Section 4 discusses all equilibria and shows that a limited degree of

\(^7\)Also related are Perri and Quadrini (2012), Bacchetta, Tille and van Wincoop (2012) and Bacchetta and van Wincoop (2012). Perri and Quadrini (2012) introduce a mechanism leading to self-fulfilling credit shocks. If the resale value of firms is expected to be low, credit will be tight. But tight credit makes it difficult for constrained firms to purchase assets from defaulting firms, which indeed makes the resale value low. While they have a two-country model with perfect business cycle co-movement, this is a result of perfect financial and goods market integration. Bacchetta and van Wincoop (2012) focus on the stock market rather than business cycles. Their model features self-fulfilling spikes in stock price risk and an associated sharp decline in stock prices. Bacchetta and van Wincoop (2012) extend this to an open economy framework. However, in contrast to the current paper, almost anything is possible regarding co-movement: a joint and equal panic in two countries, a panic limited to one country with no impact on the other, and anything in between.
integration is sufficient for panics to be necessarily synchronized across countries. Section 5 considers various extensions and Section 6 concludes.

2 The Model

In this section we describe the benchmark model. There are two countries, Home and Foreign, and two periods. There is partial integration of goods markets through trade. For now we assume that countries are in financial autarky, with financial assets (claims on firms, a bond and money) only held domestically. This will be relaxed in Section IV. For now we also assume that goods are only used for consumption, abstracting from investment. This will also be relaxed in Section IV. There are households, firms, a government and a central bank.

2.1 Households

Households make consumption and leisure decisions in both periods. Households in the Home country maximize

$$\frac{1}{1-\gamma}c_1^{1-\gamma} + \lambda l_1 + \beta E_1 \left( \frac{1}{1-\gamma}c_2^{1-\gamma} + \lambda l_2 \right)$$

(1)

where \(l_t\) is the fraction of time devoted to leisure in period \(t\) and \(c_t\) is the period \(t\) consumption index of Home and Foreign goods:

$$c_t = \left( \frac{c_{H,t}}{\psi} \right)^{\psi} \left( \frac{c_{F,t}}{1-\psi} \right)^{1-\psi}$$

(2)

where

$$c_{H,t} = \left( \int_0^{n_{H,t}} c_{H,t}(j) \frac{\mu - 1}{\mu} dj \right)^{\frac{\mu}{\mu - 1}}$$

(3)

$$c_{F,t} = \left( \int_0^{n_{F,t}} c_{F,t}(j) \frac{\mu - 1}{\mu} dj \right)^{\frac{\mu}{\mu - 1}}$$

(4)

Here \(c_{H,t}\) is the consumption index of Home goods and \(c_{F,t}\) the consumption index of Foreign goods. Consumption of respectively the Home and Foreign good \(j\) is \(c_{H,t}(j)\) and \(c_{F,t}(j)\). The number of Home and Foreign goods in period \(t\) is \(n_{H,t}\) and \(n_{F,t}\), which are equal to respectively the number of Home and Foreign firms. The elasticity of substitution among goods of the same country is \(\mu\), while the elasticity
of substitution between Home and Foreign goods is 1. There is a preference home bias towards domestic goods as we assume $\psi > 0.5$. The specification is symmetric for the Foreign country, with the overall consumption index denoted as $c_t^*$ and $c_{H,t}^*(j)$, $c_{F,t}^*(j)$ denoting the consumption of individual Home and Foreign goods consumption by Foreign households.

$\psi$ captures the degree of goods market integration, with the limit of $\psi = 0.5$ reflecting perfect goods market integration. As we will see, $\psi = 0.5$ implies that in equilibrium $c_t = c_t^*$, so that financial markets are complete even though there is no asset trade.\footnote{Financial market completeness implies that the ratio of marginal utilities of consumption across the two countries is equal to the real exchange rate, which is 1 when $\psi = 0.5$.} This is a feature that results specifically from the Cobb Douglas specification and is familiar from Cole and Obstfeld (1991). We can then think of $\psi = 0.5$ as perfect economic integration across the two countries.

In period 1 Home agents earn labor income $W_1(1 - l_1)$, where $W_1$ is the wage rate, earn a dividend $\Pi_1^C$ from firms and receive a transfer of $\bar{M}_1$ in money balances from the central bank. They use these resources to consume, pay a tax of $T_1$ to the government, buy Home nominal bonds with interest rate $i$ and hold money balances:

$$\int_0^{n_{H,1}} P_{H,1}(j)c_{H,1}(j) dj + \int_0^{n_{F,1}} S_1 P_{F,1}(j)c_{F,1}(j) dj + T_1 + B + M_1 = W_1(1 - l_1) + \Pi_1^C + \bar{M}_1$$

where $P_{H,t}(j)$ and $P_{F,t}(j)$ are the price of respectively Home and Foreign good $j$ in the Home and Foreign currency. $S_t$ is the nominal exchange rate in period $t$ (Home currency per unit of Foreign currency).

In period 2 Home agents earn labor income $W_2(1 - l_2)$, earn a dividend of $\Pi_2^C$ from firms, receive $(1 + i)B$ from bond holdings, carry over $M_1$ in money balances from period 1, and receive an additional money transfer of $\bar{M}_2 - \bar{M}_1$ from the central bank. These resources are then used to consume, pay a tax $T_2$ to the government and hold money balances $M_2$:

$$\int_0^{n_{H,2}} P_{H,2}(j)c_{H,2}(j) dj + \int_0^{n_{F,2}} S_2 P_{F,2}(j)c_{F,2}(j) dj + T_2 + M_2 = W_2(1 - l_2) + \Pi_2^C + (1 + i)B + M_1 + (\bar{M}_2 - \bar{M}_1)$$

We assume a cash in advance constraint, with the buyer’s currency being used
for payment:
\[
\int_0^{n_{H,t}} P_{H,t}(j) c_{H,t}(j) dj + \int_0^{n_{F,t}} S_t P_{F,t}(j) c_{F,t}(j) dj \leq M_t \tag{7}
\]

The constraint will always bind in period 2. It will bind in period 1 when the nominal interest rate \( i \) is positive. When \( i = 0 \), the constraint will generally not bind in period 1.

Households choose consumption and leisure to maximize (1). The first-order conditions are

\[
c_1^{-\gamma} = \beta(1 + i) E_1 \frac{P_1}{P_2} c_2^{-\gamma} \tag{8}
\]

\[
c_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\mu} c_{H,t} \tag{9}
\]

\[
c_{F,t}(j) = \left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\mu} c_{F,t} \tag{10}
\]

\[
c_{H,t} = \psi \frac{P_t}{P_{H,t}} c_t \tag{11}
\]

\[
c_{F,t} = (1 - \psi) \frac{P_t}{S_t P_{F,t}} c_t \tag{12}
\]

\[
W_t \frac{P_t}{P_t} = \lambda c_t^\gamma \tag{13}
\]

where

\[
P_{H,t} = \left( \int_0^{n_{H,t}} P_{H,t}(j)^{1-\mu} dj \right)^{1/(1-\mu)}
\]

\[
P_{F,t} = \left( \int_0^{n_{F,t}} P_{F,t}(j)^{1-\mu} dj \right)^{1/(1-\mu)}
\]

\[
P_t = P_{H,t}^{\psi} [S_t P_{F,t}]^{1-\psi}
\]

\( P_{H,t} \) and \( P_{F,t} \) are price indices of Home and Foreign goods that are denominated in respectively Home and Foreign currencies. \( P_t \) is the overall price index, denominated in the Home currency.

(8) is a standard intertemporal consumption Euler equation. (9)-(10) represent the optimal consumption allocation across goods within each country. (11)-(12) represent the optimal consumption allocation across the two countries. (13) represents the consumption-leisure trade-off. There is an analogous set of first-order
conditions for Foreign households. Other than for Home and Foreign prices and price indices, all we need to do is to add * superscripts to the variables and exchange $\psi$ and $1 - \psi$. The Foreign price index is $P^*_t = (P_{H,t}/S_t)^{1-\psi}P^\psi_{F,t}$.

2.2 The Government and the Central Bank

The government and central bank policy are analogous in the two countries. We therefore again only describe the Home country. The Home government only buys Home goods. The total government consumption index is analogous to the CES index for private Home consumption:

$$g_t = \left( \int_0^{n_{H,t}} g_t(j) \frac{\nu-1}{\nu} dj \right)^{-\frac{\mu}{\mu-1}}$$

(14)

We assume that overall government consumption is simply a constant:

$$g_t = \bar{g}$$

(15)

Optimal allocation of government spending across the different goods implies

$$g_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\mu} \bar{g}$$

(16)

We have $\int_0^1 P_{H,t}(j) g_t(j) dj = P_{H,t}g_t$. Since the timing of taxation across the two periods does not matter due to Ricardian equivalence, we simply impose the balanced budget condition

$$T_t = P_{H,t}\bar{g}$$

(17)

The central bank controls the money supply $\bar{M}_t$ in both periods. It uses its control over the second period money supply to achieve a zero inflation target from period 1 to period 2. While we assume that prices are preset at the start of each period, so that there are nominal rigidities, there will be no unexpected shocks during period 2, so that prices are essentially flexible. Equilibrium in period 2 is therefore neoclassical. The central bank can then control the price level through the cash in advance constraint, which holds as an equality in period 2: $P_2c_2 = \bar{M}_2$. The zero inflation target implies that $P_2 = P_1$.

For the first period we assume that monetary policy is impotent due to the zero lower bound. In both countries the nominal interest rate is zero. If the nominal interest rate were positive, so that the cash in advance constrain holds
with equality, the central bank could increase consumption and output in period 1 through expansionary monetary policy that lowers the interest rate. A self-fulfilling business cycle panic in period 1 is then impossible if the central bank intervenes to stabilize consumption and output. But this not feasible when the nominal interest rate is already zero. Then a rise in the money supply will not impact consumption and will only generate unused liquidity. This corresponds closely to the situation in the Fall of 2008.

### 2.3 Firms

The number of firms operating in period 1 is based on prior decisions and therefore taken as given. We normalize it at 1 for both countries, so \( n_{H,1} = n_{F,1} = 1 \). At the end of period 1 firms decide whether to continue to operate in period 2. We denote the number of period 2 firms by \( n_{H,2} = n(s) \) and \( n_{F,2} = n^*(s) \), where \( s \) represents the aggregate state of the economy that will be defined below. We do not allow new firms to enter.

We focus our description mainly on Home firms. Results are analogous for Foreign firms. Output of Home firm \( j \) in period \( t \) is

\[
y_t(j) = L_t(j)^\alpha
\]

where \( L_t(j) \) is labor input and \( 0 < \alpha \leq 1 \).

Firms set prices at the start of each period. As already discussed, this Keynesian assumption only bites for period 1 as no unexpected shocks happen after firms set prices at the start of period 2. But for period 1 a drop in consumption will lower demand for goods and therefore production. This Keynesian aspect is critical to the self-fulfilling business cycle panic in the model.

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\(^9\)When the cash in advance constraints bind we have \( M_1 = P_1 c_1 \) and \( M_1^* = P_1^* c_1^* \). As we will see, balanced trade implies \( P_1 c_1 = S_1 P_1^* c_1^* \). Therefore \( S_1 = M_1 / M_1^* \). From \( M_1 = P_1 c_1 \) and the definition of the price level we then have \( P_{H,1}^\psi_P F_{F,1} P_{1,1}^{1-\psi} c_1 = M_1^\psi (M_1^*)^{1-\psi} \). Since period 1 prices are preset, there is then a direct impact of the money supply on \( c_1 \), given Foreign monetary policy. A rise in the money supply will raise \( c_1 \), which will lower the interest rate through the consumption Euler equation (it will not affect second period consumption, which is determined in a neoclassical environment).

\(^{10}\)We could allow for entry under a fixed cost. If the fixed cost is large enough we revert to our current setup. Lower fixed costs that lead to limited entry, only partially replacing exiting firms, will only affect results quantitatively, not qualitatively.
Since prices in period 1 are preset, and their level does not matter for what follows, we simply assume that all firms set the same price of $P_{H1}$, so that $P_{H1}(j) = P_{H1}$. Similarly, for the Foreign firms $P_{F1}(j) = P_{H1}$. Labor demand in period 1 is set to satisfy demand for the goods, which gives

$$L_1(j) = \left(\psi S_1^{1-\psi} c_1 + \bar{g} + (1 - \psi) S_1^\psi c_1^*\right)^{1/\alpha}$$  \hfill (19)

In period 2 Home firm $j$ sets its price $P_{H,2}(j)$ to maximize profits

$$\Pi_2(j) = P_{H,2}(j)y_2(j) - W_2y_2(j)^{1/\alpha}$$  \hfill (20)

subject to

$$y_2(j) = c_{H,2}(j) + g_2(j) + c_{H,2}^*(j) = \left(\frac{P_{H,2}(j)}{P_{H,2}}\right)^{-\mu} \left[\psi \frac{P_2}{P_{H,2}} c_2 + \bar{g} + (1 - \psi) \frac{S_2 P_2^*}{P_{H,2}} c_2^*\right]$$  \hfill (21)

The optimal price is a markup $\mu/(\mu - 1)$ over the marginal cost:

$$P_{H,2}(j) = \frac{\mu - 1}{\mu - 1/\alpha} W_2y_2(j)^{1/\alpha}$$  \hfill (22)

Second-period profits are then

$$\Pi_2(j) = \kappa W_2y_2(j)^{1/\alpha}$$  \hfill (23)

where $\kappa = [\mu(1 - \alpha) + \alpha]/[(\mu - 1)\alpha]$ is a constant. Since all firms face the same demand and the same wage, they set the same price. From the definition of the Home price index we then have $P_{H,2} = P_{H,2}(j)n^{1/(1-\mu)}$.

Bankruptcy can occur at the end of period 1. The only difference across firms in period 1 is an ex post fixed cost. Total profits of Home firm $j$ in period 1, $\tilde{\Pi}_1(j)$, are equal to

$$\tilde{\Pi}_1(j) = \Pi_1 - P_1z(j) = P_{H,1}y_1 - W_1L_1 - P_1z(j)$$  \hfill (24)

where $z(j)$ are fixed costs that need to be paid by firm $j$ at the end of period 1. These costs capture all business costs other than wages. It is also useful to define $\Pi_1$ as period-1 profits before paying this cost. When firm $j$ is unable to fully pay the fixed cost, it is declared bankrupt and cannot produce in period 2. We assume that $z(j)$ does not affect aggregate resources and is paid to an agency. In case of bankruptcy, the agency seizes $\Pi_1$. The agency operates at no cost and
transfers its income to households.\textsuperscript{11} As described below, this cost will be affected by both firm-specific and aggregate shocks. We introduce it through an additive term in profits only because this simplifies the algebra. But results would not change fundamentally if instead we introduce shocks to firm productivity, which interact multiplicatively with $W_1 L_1$.

Firms face a borrowing constraint. Let $D(j)$ be borrowing by firm $j$ at the end of period 1. Since the nominal interest rate is zero, the firm then owes $D(j)$ in period 2. It is assumed that this can be no larger than a fraction $\phi$ of expected second period profits:

$$D(j) \leq \phi \Pi_2(j)$$ (25)

This standard borrowing constraint reflects that owners of the firm can seize at most a fraction $\phi$ of expected second period profits in case of non-payment. Note that second-period profits are positive and known at the end of period 1, when $z(j)$ is revealed.

The firm will go bankrupt if its debt limit is insufficient to cover negative profits in period 1, which is the case when

$$\Pi_1(j) + \phi \Pi_2(j) < 0$$ (26)

Another way to look at the bankruptcy condition is to define the real quantity of funds $\pi(s)$ available to pay for the fixed cost in state $s$:

$$\pi(s) \equiv \pi_1 + \phi \pi_2(s)$$ (27)

where $\pi_1 = \Pi_1/P_1$ and $\pi_2(s) = \Pi_2(s)/P_2(s)$. From (26), firm $j$ will go bankrupt when

$$z(j) > \pi(s)$$ (28)

For Foreign firms the fixed cost is $z^*(j)$ in real terms (dividing by the Foreign price index). We assume that these fixed costs have both an idiosyncratic and aggregate component:

$$z(j) = \varepsilon + u(j)$$ (29)
$$z^*(j) = \varepsilon^* + u^*(j)$$ (30)

\textsuperscript{11}One can for example think of an increase in $z(j)$ as representing higher fees charged for certain services, such as health care, security, maintenance, utilities, without any change in the quantity of services provided. One could endogeneize $z(j)$ by modeling the services sector or one could model the fixed costs as real costs to the economy overall, but these extensions would not provide any deeper insight.
where \( u(j) \) and \( u^*(j) \) are independent firm-specific shocks that are drawn from a uniform distribution over the interval \([a, b]\). \( \varepsilon \) and \( \varepsilon^* \) are aggregate shocks. Since we do not want to build in any exogenous co-movement across the two countries, it is assumed that \( \varepsilon \) and \( \varepsilon^* \) are uncorrelated. They can take on only two values, \( \bar{\varepsilon} \) and \(-\bar{\varepsilon}\), with any combination of these two values across the two countries having an equal probability of 0.25. This implies that there are four possible states in period 2: \((G, G)\), \((B, B)\), \((B, G)\) and \((G, B)\), where the letters indicate whether the aggregate shock in respectively the Home and Foreign country is good \((-\bar{\varepsilon})\) or bad \((\bar{\varepsilon})\).

Together with (28), this specification for the fixed costs determines the number of firms operating in period 2. It is conditional on the aggregate state \( s \). We have

\[
n(s) = \begin{cases} 
\frac{\pi(s) - a - \varepsilon}{b - a} & \text{if } a + \varepsilon < \pi(s) < b + \varepsilon \\
1 & \text{if } \pi(s) > b + \varepsilon \\
0 & \text{if } \pi(s) < a + \varepsilon 
\end{cases}
\]

The number of Foreign firms \( n^*(s) \) is given by the same expressions, with \( \pi(s) \) replaced by its Foreign equivalents \( \pi^*(s) \).

Let \( D(s) \) denote aggregate borrowing by firms in state \( s \), which is the aggregate of \(-\overline{\Pi}(j)\) for firms that borrow. Total dividends that households receive in period 1 and 2 from the firms and the service agency are then

\[
\Pi_1^C = \Pi_1 + D(s) \\
\Pi_2^C(s) = n(s)\Pi_2(s) - D(s)
\]

### 2.4 Market Clearing

For the Home country the market clearing conditions are

\[
y_t(j) = c_{H,t}(j) + g_t(j) + c^*_{H,t}(j) \quad t = 1, 2 \\
n_{H,t}L_t = 1 - l_t \quad t = 1, 2 \\
M_t = M_t \quad t = 1, 2 \\
B(s) = D(s)
\]

These represent respectively the goods markets clearing conditions, the labor market clearing condition, the money market clearing condition and the bond market.
clearing condition. There is an analogous set of market clearing conditions for the Foreign country.

If we substitute into the household budget constraints (5)-(6) the bond, money and labor market clearing conditions, along with the dividend expressions (32)-(33) and $T_t = P_{H,t}\bar{g}$, we get

$$P_{H,t}(c_{H,t} + \bar{g}) + S_t P_{F,t} c_{F,t} = \int_0^{\bar{y}_{H,t}} P_{H,t}(j)y_t(j) dj$$

This says that national consumption is equal to GDP. The trade balance is therefore zero. Indeed, multiplying the goods market clearing condition (34) by $P_{H,t}(j)$ and aggregating and substituting into the right hand side of (38), gives the balanced trade condition

$$S_t P_{F,t} c_{F,t} = P_{H,t} c^*_{H,t}$$

Using the expressions for $c_{F,t}$ and $c^*_{H,t}$, this can also be written as

$$P_t c_t = S_t P^*_t c^*_t$$

The nominal value of consumption is equal across the two countries. This does not imply that real consumption is equal as the real exchange rate $S_t P^*_t/P_t$ is not need to be 1 when $\psi > 0.5$.

### 2.5 Equilibrium

An equilibrium consists of values of $c_{H,t}(j)$, $c_{F,t}(j)$, $l_t$, $M_t$, $B$, $g_t(j)$, $L_t(j)$, $P_{H,2}(j)$, $D(j)$, $n(s)$ and their Foreign equivalents, that satisfies the household optimality conditions (8)-(11), the cash in advance constraint (7), the budget constraints (5), (6) and (17), optimal government consumption allocation (16), optimal second period price setting (22), the borrowing constraint (25), the expression (31) of the number of second-period firms, the market clearing conditions (34)-(37) and all their Foreign counterparts.

It is useful to summarize the equilibrium of the model in the form of 12 equations in 12 variables: $c_1$, $c^*_1$, $\pi^*_1$, $\pi_1$, $c_2(s)$ and $c^*_2(s)$ in the four states $s$. The Appendix lists the 12 equations for the general parameterization of the model. However, here we adopt some simplifying parameter assumptions that will be used in most of the analysis: $\alpha = \lambda = \beta = 1$ and $\mu = 1 = 1/\gamma$. The system of 12
equations is then

\[
\theta \left( \frac{P_2(s)}{P_{H,2}(s)} \right)^{1/\gamma} c_2(s) = n(s) \quad s = 1, \ldots, 4
\]  

(41)

\[
\theta \left( \frac{P_2^*(s)}{P_{F,2}(s)} \right)^{1/\gamma} c_2^*(s) = n^*(s) \quad s = 1, \ldots, 4
\]  

(42)

\[
c_1^{-\gamma} = \frac{1}{4} \sum_{s=1}^{4} c_2(s)^{-\gamma}
\]  

(43)

\[
(c_1^*)^{-\gamma} = \frac{1}{4} \sum_{s=1}^{4} c_2^*(s)^{-\gamma}
\]  

(44)

\[
\pi_1 = c_1 + \frac{P_{H,1}}{P_1} \tilde{g} - c_1^* \left( \frac{P_1}{P_{H,1}} c_1 + \tilde{g} \right)
\]  

(45)

\[
\pi_1^* = c_1^* + \frac{P_{F,1}}{P_1} \tilde{g} - (c_1^*)^\gamma \left( \frac{P_1}{P_{F,1}} c_1^* + \tilde{g} \right)
\]  

(46)

where \( \theta = (1 + \gamma)^{1/\gamma} \).

Equations (41) represents the optimal price set in period 2. It corresponds to (22) after substituting the labor supply schedule \( W_2/P_2 = \lambda c_2^2 \) and \( P_{H,2}(j)/P_{H,2} = n^{1/(\mu-1)} = n^\gamma \). (42) is its Foreign counterpart. There are four such equations for each country, one for each state. (43)-(44) are the Home and Foreign intertemporal consumption Euler equations, where we have set \( \beta = 1, i = 0 \) and \( P_2 = P_1 \) (zero inflation target). (45) is an expression for period 1 real profits excluding the fixed cost, \( \pi_1 = (P_{H,1} y_1(j) - W_1 y_1(j))/P_1 \). It follows by substituting \( W_1/P_1 = c_1^\gamma \) and \( y_1(j) = (P_1/P_{H,1}) c_1 + \tilde{g} \). The latter follows from the goods market clearing condition (34), using the expressions for optimal consumption, \( P_{H,1}(j) = P_{H,1} \) and (40). (46) is the Foreign counterpart of the period one real profit expression.

We need to substitute into these equations the expressions for the number of period 2 firms as a function first and second period profits (31). Using (23), together with \( W_2/P_2 = c_2^\gamma \) and (40), we can write second period profits as

\[
\pi_2(s) = \tilde{\kappa} \left( \frac{P_{H,2}(s)}{P_2(s)} \right)^{\mu} \left( \frac{P_2(s)}{P_{H,2}(s)} + \frac{\tilde{g}}{c_2(s)} \right)
\]  

(47)

where \( \tilde{\kappa} = (1 + \gamma)^{-\mu}/(\mu - 1) \). We also need to substitute the following expressions
for relative prices as a function of relative consumption:

\[
\frac{P_{H,1}}{P_1} = \left( \frac{c_1^*}{c_1} \right)^{\frac{1-\psi}{\rho-1}} 
\]

\[
\frac{P_{H,2}(s)}{P_2(s)} = \left( \frac{c_2^*(s)}{c_2(s)} \right)^{\frac{1-\psi}{\rho-1}} 
\]

These follow from (40) and the expressions of the Home and Foreign price indices. The Foreign relative prices are the reciprocal: \( P_{F,1} = P_1/P_{H,1} \) and \( P_{F,2} = P_2/P_{H,2} \).

### 3 Symmetric Equilibria

In this section we focus on symmetric equilibria in which first period consumption, output and profits are the same across the two countries. Considering such symmetric equilibria first has several advantages. First, the schedules that determine the equilibria can be characterized analytically. In the next section we will graphically illustrate all equilibria of the model, but this requires numerical computation. Second, it provides insight into conditions under which self-fulfilling panics are possible as we do not find any equilibria unless there are symmetric equilibria. Finally, knowing the symmetric equilibria immediately allows us to identify all equilibria in the model for the case where the two countries are not integrated at all (\( \psi = 1 \)).

#### 3.1 No Aggregate Risk

First consider the case without aggregate risk (\( \bar{\varepsilon} = 0 \)), which is the easiest. In that case the Home consumption Euler equation implies that \( c_1 = c_2 \). Using this, and using that the Home and Foreign equilibrium conditions are the same in the symmetric equilibrium, the equilibria are then characterized by two schedules in \( c_1 \) and \( \pi_1 \):

**schedule 1:**

\[
\pi_1 = \theta(b-a)c_1 - \phi \bar{\kappa} \left( 1 + \frac{\bar{g}}{c_1^*} \right) + a \quad \text{if } \pi \leq b
\]

\[
c_1 = \frac{1}{\bar{\theta}} \quad \text{if } \pi > b
\]

**schedule 2:**

\[
\pi_1 = (c_1 + \bar{g}) \left( 1 - c_1^* \right)
\]
Schedule 1 follows from (41) and schedule 2 from (45). Schedule 1 implies an upward sloping relationship between $\pi_1$ and $c_1$ until $\pi$ reaches $b$, after which $c_1$ is constant at $1/\theta$. Assuming $\gamma \geq 1$, schedule 2 gives a humped shaped expression for $\pi_1$ as a function of $c_1$. The shape of these curves can potentially lead to multiple equilibria. For example, with $\bar{g} = \phi = 0$ it is easy to verify that there are two equilibria as long as $b < \frac{1}{\frac{\gamma}{1+\gamma}}$, one where $n = 1$ and one where $n < 1$.

To illustrate the multiple equilibria, Figure 4 shows results for a particular set of parameters: $\phi = 0$, $\bar{g} = 0.3$, $\gamma = 2.5$, $a = 0.5$ and $b = 0.6$, which we will refer to as the benchmark parameterization. In addition results are shown for higher and lower values of the fixed costs through higher and lower values of $a$ and $b$, keeping $b - a = 0.1$. Schedule 2 does not depend on the values of $a$ and $b$, but schedule 1 does. From (50) an equal increase in $a$ and $b$ shifts up schedule 1 by this increase.

Figure 4 shows that there can be one, two or zero equilibria. When fixed costs are very low ($a = 0.2, b = 0.3$), profits will always be positive for all firms in period 1. No firms will go bankrupt so that $n = 1$. This will determine output in period 2, which affects consumption demand in period 1. There will be just one equilibrium. At the other extreme, when fixed costs are high ($a = 0.6, b = 0.7$), all firms will always go bankrupt. In that case there is no equilibrium.

The case of interest to us is an intermediate level of the fixed cost. As we can see from Figure 3, there are then two equilibria, A and B. Equilibrium A is a good one, which we refer to as the non-panic equilibrium. First period consumption and profits are high and no firms will go bankrupt ($n = 1$). Equilibrium B is the bad one, which we refer to as the panic equilibrium. First period consumption and profits are low and many firms go bankrupt. The number of firms in period 2 is $n = 0.44$.

The presence of two equilibria is a result of the possibility of self-fulfilling business cycle panics. This occurs due to reinforcing linkages between periods 1 and 2. This is illustrated in Figure 5. In general there are two reinforcing linkages, one associated with expected future income and the other with income risk. For now we focus on the linkage through expected income, illustrated on the left hand side of Figure 5, and turn to risk below. Low expected period 2 income leads to low period 1 consumption, which implies low firm profits. This in turn leads to a lot of bankruptcies and therefore a low number of firms in period 2. This implies low period 2 income, making the original belief self-fulfilling.

Figure 6 illustrates the role of the borrowing constraint and government spend-
ing. The first chart shows the impact of relaxing the borrowing constraint. Figure 4 shows that when we raise $\phi$ to 0.2, schedule 1 shifts down enough so that there is only one equilibrium. The more firms are able to borrow, the less fragile they are. They are better able to withstand a drop in demand that lowers first period profits. This in turn can make a self-fulfilling panic impossible. While it remains the case that conditions in period 2 affect consumption in period 1, the linkage in the other direction is broken. Even with low consumption in period 1, leading to low profits, firms can avoid bankruptcy by borrowing. With $\phi = 0.2$ the single equilibrium is in the vertical part of schedule 1, where $n = 1$. No firms will go bankrupt.

The role of credit is of particular relevance to the 2008 crisis as credit was known to be tight due to large losses experienced by banks and other financial institutions since early 2007, leading to deleveraging in the financial system. Figure 6 illustrates that such tight credit conditions can make an economy vulnerable to self-fulfilling panics, while they would not be with more relaxed credit conditions. Of course the precise values of $\phi$ in Figure 6 are only an illustration and will vary with other parameter assumptions.

The right hand side of Figure 6 illustrates the role of the government. Keeping $\phi = 0$, this only affects schedule 2. Figure 6 shows that when we raise $\bar{g}$ to 0.6, there is again just one equilibrium where no firm goes bankrupt. In this case it is impossible to have a self-fulfilling business cycle panic because of the stability that comes from government spending. Even if private consumption were to decline substantially, period-1 profits would remain relatively strong because of the stable government spending. This precludes any firms from going bankrupt, thus avoiding a self-fulfilling panic.

### 3.2 Aggregate Risk

In the results so far a decline in period 1 consumption was the result of lower expected period 2 income, which became self-fulfilling. Once we allow for aggregate shocks, so that $\bar{\sigma} > 0$, there is a second self-fulfilling connection between period 1 and 2, which is related to perceived uncertainty about second period income.\(^{12}\)

\(^{12}\)We have assumed a perfectly functioning labor market, so there is no idiosyncratic risk for workers. A realistic extension would be to introduce unemployment, in which case there would be uncertainty even without aggregate risk.
For convenience, we will assume that $\phi = 0$ in what follows.

The symmetric equilibrium is one where $c_1 = c^*_1$ and $\pi_1 = \pi^*_1$. But since the number of firms in the second period now depends on aggregate shocks, it is in general not the case that $c_2(s) = c^*_2(s)$ and relative prices in the second period are not equal to 1 in states $(G, B)$ and $(B, G)$. (41)-(42), together with the expressions for second period relative prices, can be used to solve for $c_2(s)$ as a function of the number of Home and Foreign firms:

$$c_2(s) = \frac{1}{\theta} n(s)^{1-\delta} n^*(s)^{\delta}$$  \(52\)

where $\delta = (1-\psi)/[\gamma(2\psi - 1) + 2(1-\psi)]$ is between 0 and 0.5. Together with the Euler equation (43) this implies

$$c_1 = \frac{1}{\theta} \left( \frac{1}{4} \sum_{s=1}^{4} [n(s)^{1-\delta} n^*(s)^{\delta}]^{-\gamma} \right)^{-1/\gamma}$$  \(53\)

Given the expressions for $n(s)$ and $n^*(s)$ as a function of $\pi_1$ this again implies a positive relationship between $c_1$ and $\pi_1$ up to the point where $\pi_1 - \bar{\varepsilon} = b$, similarly to schedule 1. For $\pi_1 - \bar{\varepsilon} > b$ we have $n(s) = n^*(s) = 1$ in all states, so that $c_1 = 1/\theta$. This corresponds to the vertical portion of schedule 1. The overall shape of schedule 1 therefore remains the same, while schedule 2 is not affected by $\bar{\varepsilon}$. Since the Figures representing the equilibria are very similar to those shown before, we will limit ourselves to a verbal description of the impact of aggregate shocks. In the next section we will describe all equilibria of the model with aggregate shocks.

Consider the solid schedules in Figure 4. In the non-panic equilibrium A the number of firms is 1 in period 2, while in the panic equilibrium B it is 0.44. Now consider introducing a marginal aggregate shock, so that $\bar{\varepsilon}$ is small. This will not change equilibrium A. The kink in schedule 1 shifts upward only by $\bar{\varepsilon}$. When $\bar{\varepsilon} < 0.047$, equilibrium A will remain the same. This means that the number of firms remains 1, independent of the aggregate shocks. The reason is that in the absence of aggregate shocks the profits of firms net of the fixed cost ($\pi_1 = 0.647$) is more than sufficient to cover the maximum fixed cost of 0.6. Firms will not go bankrupt even when faced with an additional fixed cost related to the aggregate shock of up to $\bar{\varepsilon} = 0.047$. This means that uncertainty associated with the aggregate shock does not translate into uncertainty about the number of firms, output, income or consumption.
Things are different in the panic equilibrium B. Since in the absence of aggregate shocks the number of firms is already less than 1, profits are not sufficient to cover the maximum cost. We have \( \pi_1 = 0.544 \), which only covers fixed costs up to 0.544. Since the fixed cost is uniformly distributed from 0.5 to 0.6, only 44% of firms will survive. The rest goes bankrupt. Adding aggregate shocks now has the effect of introducing uncertainty about the number of period 2 firms. A favorable aggregate shock reduces the number of firms that go bankrupt, while an unfavorable shock will increase the number of firms that go bankrupt. For example, under the benchmark parameterization together with \( \tilde{\varepsilon} = 0.01 \) and \( \psi = 0.8 \), in equilibrium B the number of firms is either 0.37 or 0.57, depending on the aggregate shock. This uncertainty about the number of firms translates into uncertainty about second period income and consumption.

As illustrated in Figure 5 (right hand side), income uncertainty generates a second self-fulfilling mechanism in the model. When agents believe that there is a lot of uncertainty about period 2 income, they will consume less in period 1. This in turn lowers period 1 profits, making firms more fragile. They will then be more sensitive to aggregate shocks, increasing uncertainty about the number of firms in period 2 and therefore period 2 income. The belief of income uncertainty then becomes self-fulfilling. In general, therefore, the self-fulfilling linkages between periods 1 and 2 are associated with both expected income and income risk.

### 3.3 Autarky

In the symmetric equilibria described so far the parameter \( \psi \) that measures the extent of trade integration enters only through the expressions for relative prices. But in the model without aggregate shocks we have \( c_t = c_t^* \) in both periods, so that the relative prices are always 1, independent on \( \psi \). Therefore the symmetric equilibria described in Section 3.1 do not depend on \( \psi \) and are the same when \( \psi = 1 \), so that the two countries are in autarky.\(^{13}\) We can therefore use the results of Section 3.1 to analyze the case where the two countries are in complete autarky, with \( \psi = 1 \).

When \( \psi = 1 \) the intersections of the two schedules describe not only symmetric

\(^{13}\)This is not the case with aggregate shocks, where the relative prices in period 2 will fluctuate with relative consumption, which is affected by aggregate shocks. As can be seen from (49), these relative prices will then depend on \( \psi \).
equilibria, but in fact describe all of the equilibria of a closed economy version of the model. Moreover, the equilibrium in one country has no impact on the equilibrium of another country when both economies are closed. In particular, one country may have a self-fulfilling business cycle panic (equilibrium B), while the other country is in the non-panic equilibrium A or the other way around. These are asymmetric equilibria.\textsuperscript{14} While in theory the two countries could panic simultaneously, there is no a priori reason why this should be so. There may be arguments outside of the our model why two countries would panic together. For example, if the trigger that sets off the panic is particularly frightening, the two countries may react together. But if this trigger event takes place in the Home country,\textsuperscript{15} it would seem odd that the Foreign country would react to it in the absence of any integration between the two countries.

In this section we have focused on symmetric equilibria where panics, if they occur, are global. However, with economies in autarky where $\psi = 1$ we see that asymmetric equilibria, where only one country panics, also exist. In the next section we examine whether asymmetric equilibria can also exist when the two countries are partially integrated, i.e. when $\psi < 1$. We will show that they do not exist when the countries are sufficiently integrated, so that $\psi$ lies somewhere between 0.5 and an upper threshold that may be quite large. In that case there are only symmetric equilibria and a panic is necessarily global.

\section{Numerical Results: All Model Equilibria}

In this section we examine all equilibria, not just symmetric equilibria. In this case the equilibrium schedules can only be derived numerically. We first describe the numerical method used in the solution. We then use the method to graphically illustrate the equilibria in the model and then provide intuition for the key result that there are only two equilibria, both symmetric, when the economies are sufficiently integrated. Business cycle panics are then perfectly synchronized across countries.

\textsuperscript{14}In the model with aggregate shocks the same result applies, but since the equilibria A and B will depend on $\psi$, we need to set $\psi = 1$ in order to compute the closed economy equilibria A and B.

\textsuperscript{15}An example is the bankruptcy of Lehman Brothers or more generally events surrounding U.S. financial markets in the Fall of 2008.
4.1 Solution Method

We find the solutions to the model by transforming the six equilibrium conditions (41)-(46) into two schedules in the space of \((c_1, c_1^*)\). The solutions are at the intersection of these schedules. The first schedule takes a value of \(c_1^*\) and then solves for all \(c_1\) such that all equations other than the Foreign profit expression (46) are satisfied. The second schedule takes a value of \(c_1\) and then solves for all \(c_1^*\) such that all equations other than the Home profits equation (45) are satisfied.

Consider the first schedule, which solves for \(c_1\) as a function of \(c_1^*\). Take particular values of \(c_1^*\) and \(c_1\). In Appendix B we show that we can then use (41), (42), (44) and (45) to derive unique solutions for \(c_2(s), c_2^*(s)\) and \(\pi_1\) if a solution exists. We then substitute the solutions for \(c_2(s)\) for all states \(s\), which depend on \(c_1\) and \(c_1^*\), into the Home consumption Euler equation (43). For a given \(c_1^*\), this gives an expression that only depends on \(c_1\). We then search numerically over values of \(c_1\) where this expression holds. This gives \(c_1\) as a function of \(c_1^*\), with possibly multiple solutions.

The other schedule, which gives \(c_1^*\) as a function of \(c_1\), is perfectly symmetric to the first in the 45 degree line. This is due to the symmetry of the model. Solutions for \(c_1\) and \(c_1^*\) are at the intersection of these two lines. Once we know their values, profits \(\pi_1\) and \(\pi_1^*\) follow immediately from (45)-(46). (41)-(42) can then be used to also for \(c_2(s)\) and \(c_2^*(s)\), as described in Appendix B, giving also \(n(s)\) and \(n^*(s)\).

4.2 Results

We first consider the benchmark case with aggregate risk examined in Section 3.2, where \(\bar{\varepsilon} = 0.01\) and \(\psi = 0.8\). Households then spend 80% of their budget on domestic goods and 20% on foreign goods. Imports as a fraction of GDP is even less than that since all of government spending is domestic. Imports in the non-panic symmetric equilibrium are then only 13% of GDP. The extent of integration is therefore quite limited.\(^{16}\)

Figure 7 shows the two schedules in the \((c_1, c_1^*)\) space. There are two equilibria, A and B, which are both symmetric equilibria.\(^{17}\) Note that while the two schedules converge at point A, so that A is an equilibrium, they do not cross. This is because

\(^{16}\)By comparison, US imports in 2011 was 18% of GDP.

\(^{17}\)It might look from the picture that there could be a third equilibrium with \(c_1 = c_1^* = 0\). But such an equilibrium does not exist since marginal utility is not well defined at zero.
the number of firms is at its maximum of 1 in equilibrium A, independently of the aggregate shock. Higher consumption cannot be achieved without increasing the number of firms.\textsuperscript{18}

The key finding from this picture is that are only symmetric equilibria, so that the analysis in Section 3 fully describe the results. There are no mixed equilibria where one country panics and the other does not. Either neither of the two countries panic or they experience a self-fulfilling business cycle panic at the same time. This implies that the drop in consumption, profits and output in period 1 will be equal across the two countries during a business cycle panic. This closely connects to what happened in 2008-2009 as we saw in Figure 1. It happens even though the two countries are only partially integrated.

In order to better understand this result it is useful to consider different degrees of trade integration as measured by $\psi$. This is done in Figure 8, which illustrates the equilibria for values of $\psi$ equal to 0.7, 0.8, 0.9 and 0.95. A higher value of $\psi$ corresponds to less trade, with the fraction of private consumption goods that is imported equal to $1 - \psi$. As we can see, there are two equilibria when $\psi$ is 0.7 or 0.8, while there are four equilibria for $\psi$ is 0.9 or 0.95. The cutoff from two to four equilibria occurs around $\psi = 0.84$. When $\psi$ is less than 0.84, there are 2 equilibria. When it is larger, there are 4 equilibria.\textsuperscript{19}

These results imply that when the two countries are sufficiently integrated ($\psi$ less than 0.84 in our example) a panic is perfectly synchronized across the two countries, while this is not generally the case with lower levels of integration. The additional equilibria that arise when $\psi$ is larger than 0.84 are mixed equilibria as in the autarky case, where one country panics while the other country does not. The country that does not panic is still affected through standard trade transmission channels, but its first period consumption and output falls much less than in the country that panics. For example, when $\psi = 0.9$, first period consumption of the country that panics drops by 66\% relative to the symmetric non-panic equilibrium, while that of the other country drops only by 7\%. When $\psi = 0.95$ these numbers are respectively 60\% and 2\%. When we shut down trade altogether ($\psi = 1$), as considered in Section 3.3, there are still four equilibria. In the asymmetric

\textsuperscript{18}If the number of firms were somewhat below 1 in equilibrium A, the two lines would cross as more consumption is then associated with more firms.

\textsuperscript{19}When we let $\psi$ get close to 0.5, the two schedules become very close to the 45 degree line, but the two equilibria remain the same.
equilibria the country that does not panic is then completely unaffected by the country that does panic.

4.3 Intuition

We have shown that a limited degree of integration is sufficient to make sure that countries panic together, so that the decline in consumption and output is perfectly synchronized across the two countries. In order to understand why panics are necessarily global, consider what would happen if instead there were only a panic in the Home country. In that case $c_1$ is much less than $c^*_1$. To see that this cannot be an equilibrium, we will consider the impact on the Home country of an increase in $c_1^*$ above $c_1$.

Figure 9 illustrates the impact of this increase in $c_1^*$. It affects the Home countries in different ways. On the positive side, it raises Home exports, which raises Home profits, the number of Home firms and therefore Home consumption. Also on the positive side, the increase in exports leads to a Home currency appreciation that improves the terms of trade for the Home country. For a given quantity of production $y$, the real value of revenue $(p_H/P)y$ will then rise, which raises the real value of profits, again increasing the number of period 2 firms and Home consumption. Even for a given number of Home firms the improvement in the terms of trade raises Home consumption. This is immediate from (41). Abstracting for a moment from risk, so that $c_1 = c_2$ and $c_1^* = c_2^*$ and therefore relative prices are equated across the two periods, it implies

$$c_1 = \frac{1}{\theta^*} \left( \frac{P_{H,1}}{P_1} \right)^{1/\gamma}$$  \hspace{1cm} (54)

For a given number of Home firms, Home consumption will rise as the terms of trade improvement (rise in $P_H/P$) leads to increased purchasing power.

These positive effects of an increase in $c_1^*$ on the Home country are only partially offset by a negative impact that operates through an expenditure switching effect. The appreciation of the Home currency raises the relative price of Home goods, which leads to substitution from Home to Foreign goods, lowering demand for Home goods and therefore Home profits.

Overall a rise in $c_1^*$ has a favorable effect on the Home economy. This weakens the linkages needed for self-fulfilling beliefs that are illustrated in Figure 5. Consider the left hand side of Figure 5. The first linkage is from a drop in expected
second period Home income to first period consumption demand faced by Home firms. This is weakened because a higher level of $c^*$ raises export demand. The second linkage is from lower consumption to lower profits. This linkage is weakened because the favorable terms of trade effect due to the Home appreciation raises the real value of Home profits. The last linkage is from a lower number of Home firms to lower expected period 2 income. This is weakened because the favorable terms of trade effect raises real income for a given number of Home firms. These counterbalancing forces will avoid a self-fulfilling panic altogether when trade is above a certain threshold. A limited amount of trade is sufficient to provide enough stability to the Home country to avoid a panic that is limited to only the Home country.

One might think that perhaps with a limited amount of trade there can already be a lot of transmission, so that we do not need our self-fulfilling panics to explain the close co-movement of business cycles. This is not the case though. To illustrate this, consider the benchmark parameterization leading to the equilibria in Figure 7, where $\psi = 0.8$. One way to consider the magnitude of transmission in the model is to introduce an anticipated asymmetric shock at the start of period 1 by shifting upwards the distribution of the fixed cost. Starting from the non-panic equilibrium $A$, assume that this shock happens in the Foreign country and the increase in the fixed cost is 0.1. We then find that the drop in Home output in period 1 is only 0.12 times the drop in Foreign output. As a result of the limited trade (13% of GDP), transmission is not large. But the positive interconnectedness is nonetheless sufficient to assure that panics will be perfectly synchronized.

Figure 10 illustrates the role of the government in these results. The shaded area shows combinations of $\bar{g}$ and $\psi$ under which there are only two equilibria, so that panics are always perfectly synchronized. The figure on the left hand side is under the assumptions that the other parameters correspond to the benchmark parameterization, but without aggregate shocks. The figure on the right hand side is for lower values of $a$ and $b$, which has the advantage that an equilibrium exists even when $\bar{g} = 0$. In both cases we see that an increase in government spending raises the threshold for $\psi$. The higher the level of government spending, the less integrated the two countries need to be in order to assure that panics are always

\[^{20}\]If we change the benchmark parameterization to set $\bar{g} = 0$, no equilibria exist. Demand is then too low, so that profits are insufficient for any firms to operate given the magnitude of the fixed cost.
global. This can again be understood with the help of Figure 9. The only negative impact of an increase in $c^*_t$ on the Home country operates through an expenditure switching effect due to the Home currency appreciation. Since the government only buys domestic goods and its demand is stable, the relative importance of this expenditure switching effect is weaker the larger the government.

5 Extensions

In this section we will consider three extensions. The first extension allows for risksharing, which leads to further integration across the two countries. The second extension allows for a non-unitary elasticity of substitution between Home and Foreign goods. The last extension adds investment to the model. We know from Figure 1 that not only consumption and output, but also investment dropped by similar magnitude in the rest of the world as in the U.S. during the Great recession.

5.1 Financial Integration

In the model so far the two countries trade goods but are in financial autarky. We have seen that a limited degree of goods market integration is sufficient to guarantee that a business cycle panic is global. We now add to this financial integration. We only consider the extreme on the opposite side of financial autarky, which is full risksharing. We do not consider intermediate cases as partial financial integration can be accomplished in many ways and is not necessarily captured well through one parameter in a way that is analogous to $\psi$ for goods market integration.

We will only consider the model without aggregate shocks to the cost of firms. Nonetheless there is still a need for risksharing as business cycle panics are a shock as well and may be limited to one country. Under complete markets the ratio of marginal utilities of consumption is equal to the real exchange rate:

$$\frac{c_t^{-\gamma}}{(c^*_t)^{-\gamma}} = \frac{P_t}{S_t P^*_t}$$

(55)

This replaces the condition $P_t c_t = S_t P^*_t c^*_t$ under financial autarky. As long as $\gamma$ is different from 1 these two conditions will differ.\footnote{We assume that only households share risk. Firms do not have access to risksharing because of standard principle agents problems that also lead to borrowing constraints.}
Two changes need to be made to the model equations. First, the expressions for relative prices (48)-(49) change. All expressions for relative prices as a function from relative consumption can be derived from (55). Second, the expressions for profits (45)-(46) change. Home profits is

\[\pi_1 = \left(\psi c_1 + (1 - \psi)\frac{S_1 P^*_1}{P_1} c^*_1 + \frac{P_{H,1}}{P_1} \bar{g}\right)\left(1 - \frac{P_1}{P_{H,1}} c^*_1\right)\]  

(56)

This same expression also holds under financial autarky, but previously we substituted \(S_t P^*_t / P_t = c_t / c^*_t\), which is now replaced by (55).

Figure 11 illustrates the impact of risksharing. Analogous to Figure 10, the shaded areas indicate combinations of \(\psi\) and \(\gamma\) for which there are only two equilibria, a global panic equilibrium and a global non-panic equilibrium. The picture shows the results for both the case of financial autarky and perfect risksharing. The case where \(\gamma = 2.5\) is that of the benchmark parameterization.\(^{22}\) The lowest value of \(\gamma\) reported is 2 because somewhat below that the two symmetric equilibria become close together and then disappear (no equilibria exist).\(^{23}\)

Figure 11 tells us that further integration through risksharing makes it even more likely that a panic will be global. More financial integration implies that less trade integration is needed to assure that this will happen. For most values of \(\gamma\), \(\psi = 0.9\) or less is sufficient to assure the existence of only two equilibria, corresponding to a fraction 10% of private consumption being imported. Under financial autarky more trade is needed to assure that panics are global, as seen from lower threshold values \(\psi\) that are mostly between 0.8 and 0.85. Figure 11 also shows that the difference between financial autarky and perfect risksharing is somewhat larger for larger values of \(\gamma\), which makes sense as the optimal risksharing condition deviates further from the balanced trade condition \(P_t c_t = S_t P^*_t c^*_t\) for larger values of \(\gamma\). We can conclude that what matters is the overall degree of integration across countries. If we add financial integration, then less trade integration is needed to assure a coordination of panics across countries.

In order to understand why financial integration, just like trade integration, contributes to the coordination of panics across countries, consider again the case

\(^{22}\)In the benchmark parameterization we assume \(\mu = 1 + (1/\gamma) = 1.4\). In Figure 11 we keep \(\mu = 1.4\) fixed when changing \(\gamma\).

\(^{23}\)The absence of equilibria for low \(\gamma\) is because of the relatively high wage rate implied by a low \(\gamma\), which reduces profits and leads to bankruptcy of all firms. Of course this can be avoided by changing other parameters.
where a panic is limited to the Home country only, so that \( c_1 < c_1^* \). We saw that with partial trade integration a higher level of \( c_1^* \) positively affects the Home economy and weakens several of the linkages that give rise to self-fulfilling panics. Adding financial integration leads to an even more favorable impact of a higher \( c_1^* \) on the Home country. This provides even more stability and further weakens the self-fulfilling linkages of Figure 5.

To see this, first consider what would happen if the exchange rate appreciated just as much as under financial autarky, sufficient to generate balanced trade and therefore \( P_c = S^P c^* \). In that case we would have

\[
\frac{c_t}{(c_t^*)^{-\gamma}} > \frac{P_t}{S_t P_t^*}
\]

when \( \gamma > 1 \). This therefore would lead to a transfer to the Home country. The Home country will then be able to run a trade deficit. This will occur through a combination of higher Home consumption and a larger Home appreciation. As illustrated in Figure 5, the latter has both a negative expenditure switching effect and a positive terms of trade effect. The latter dominates, which further raises Home profits and consumption.

The self-fulfilling links in Figure 5 become even weaker than with only trade integration. First, a drop in expected second period income now has even less effect on consumption demand because of a net transfer to Home agents associated with risksharing. Second, the larger appreciation leads to an even more favorable terms of trade effect, which raises Home profits and therefore the number of Home firms. Finally, the same terms of trade improvement also raises real income in period 2 for a given number of firms by further improving Home purchasing power.

### 5.2 Elasticity of Substitution

Throughout the paper so far we have assumed a unitary elasticity of substitution between Home and Foreign goods. We now relax this assumption by adopting a CES specification with an elasticity of substitution of \( \nu \) between Home and Foreign goods:

\[
c_t = \left[ \psi^{1/\nu} c_{H,t}^{\nu-1} + (1 - \psi)^{1/\nu} c_{F,t}^{\nu-1} \right]^{\nu/(\nu - 1)}
\]

The specification for \( c_t^* \) is analogous, with the weights \( \psi \) and \( 1 - \psi \) switched.

The equations (41)-(46) that describe the equilibrium of the model remain unchanged. The only change is that the expressions for relative prices as a function
of relative consumption no longer correspond to (48)-(49). These are derived from
the balance trade condition $S_t P_{F,t} c_{F,t} = P_{H,t} c_{H,t}^*$. With a unitary elasticity this
implies $P_t c_t = S_t P_t^* c_t^*$. With an elasticity $\nu$ this generalizes to
$$
\left( \frac{S_t P_{F,t}}{P_{H,t}} \right)^{\nu^{-1}} \left( \frac{S_t P_t^*}{P_t} \right)^{\nu} = \frac{c_t}{c_t^*}
$$
(59)
The left hand side is a function of the relative price $S_t P_{F,t}/P_{H,t}$, so this gives an
implicit solution of the relative price as a function of $c_t/c_t^*$. It is well known that
for sufficiently low elasticities of substitution (in our case below 0.5), this balanced
trade condition has more than one solution for the relative price. Since that is an
entirely separate form of multiplicity, we will only consider values of $\nu > 0.5$ where
this does not arise.\footnote{See Bodenstein (2010) for a detailed analysis of this type of multiplicity.}

Figure 12 shows the values of $\psi$ and $\nu$ for which the model only exhibits two
equilibria (the symmetric panic and non-panic equilibria). All parameters other
than $\nu$ correspond to the benchmark parameterization without aggregate shocks.
For elasticities lower than 1, even higher values of $\psi$ are sufficient to guarantee that
a panic is necessarily global. For $\nu = 0.6$, the estimated trade elasticity for the
U.S. by Hooper, Johnson and Marquez (2000), $\psi = 0.96$ is sufficient to guarantee
that a panic is global. Only 4% of private consumption is then imported. The
opposite is the case for elasticities higher than 1.

The intuition behind this finding can be understood by returning to Figure 9
and is similar to the role of government spending. The only negative impact of
higher Foreign consumption on the Home economy operates through the expen-
diture switching effect. Similar to higher government spending, the expenditure
switching effect is weakened when we lower the elasticity of substitution between
Home and Foreign goods. This guarantees a more favorable effect of higher For-
eign consumption on the Home economy. The stability generated by the Foreign
country then makes it more difficult to have a panic limited to the Home country.

5.3 Investment

As shown in Figure 1, not only consumption and output, but also investment
d eclined sharply during the Great Recession. And the decline was again of similar
magnitude in the rest of the world as in the United States. To capture this, we now
consider an extension that allows for investment. We briefly describe the model with investment, leaving details to the Appendix.

To simplify, we only consider the model with $\phi = 0$ (no borrowing by firms) and no aggregate shocks. We change the production function to

$$y_t(j) = L_t(j)^\alpha K_t(j)^{1-\alpha}$$

(60)

The capital stock in period 1 is given and equal to $K_1$ for all firms (both Home and Foreign). The capital stock in period 2 is equal to investment in period 1. We model the capital good in a way analogous to consumption. The output of each firm can now be used for both consumption and capital. An index of Home and Foreign capital is created using the same CES index of the different goods as for consumption. Home and Foreign capital are then aggregated into a capital stock index that takes the same Cobb Douglas form as for aggregate consumption. Because international trade in period 1 is now needed for both consumption and investment, the previous balanced trade condition $P_1c_1 = S_1P_1^*c_1^*$ is replaced by $P_1(c_1 + I_1) = S_1P_1^*(c_1^* + I_1^*)$, where $I_1 = K_1$ is the same index of Home and Foreign goods as for consumption of the Home country.\(^{25}\)

After setting the optimal price in period two, the real value of second-period profits of Home firm $j$ takes the form

$$\pi_2(j) = h(c_2, S_2P_2^*/P_2)K_2(j)^{\omega_k}$$

(61)

where $h(., .)$ is a function specified in the Appendix and $0 < \omega_k < 1$ depends on parameters $\alpha$ and $\mu$. Period 1 profits equal revenue, minus labor cost, investment and the fixed cost. Let $\hat{\pi}_1$ be the real value of revenue minus labor cost. Then the real value of total profits is

$$\tilde{\pi}_1 = \hat{\pi}_1 - I_1(j) - u(j) = \pi_1 - u(j)$$

(62)

where $\pi_1$ is first period profit without subtracting the fixed cost $u(j)$. $\hat{\pi}_1$ is written in the Appendix as a function of aggregate Home consumption and investment and the first period relative price.

The unconstrained optimal level of investment without a binding borrowing constraint maximizes $\pi_1(j) + \pi_2(j)$ and therefore equates the marginal product of

\(^{25}\)Because the model has only two periods, there is no investment in period 2, so that the balanced trade condition for period 2 remains $P_2c_2 = S_2P_2^*c_2^*$. 

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capital to 1 (the gross interest rate):

\[
\frac{\partial \pi_2(j)}{\partial I_1(j)} = 1 \tag{63}
\]

Firm \( j \) goes bankrupt at the end of period 1 when \( u(j) > \hat{\pi}_1 - I_1(j) \). If the unconstrained optimal level of investment is such that \( b < \hat{\pi}_1 - I_1(j) \), so that the borrowing constraint does not bind even for the largest possible value of the fixed cost, then (63) indeed describes optimal investment.

When this is not the case, so that there is a possibility that the borrowing constraint will bind, optimal investment is derived as follows. Let \( f(u) \) be the distribution of the idiosyncratic fixed cost shock and \( F(u) \) the cumulative distribution. Then investment is determined by maximizing the value of the firm

\[
V = \int_a^{\hat{\pi}_1 - I_1(j)} (\hat{\pi}_1 + \pi_2(j) - I_1(j) - u)f(u)du
\]

\[
= (\hat{\pi}_1 + \pi_2(j) - I_1(j))F(\hat{\pi}_1 - I_1(j)) - \int_a^{\hat{\pi}_1 - I_1(j)} uf(u)du \tag{64}
\]

Taking the derivative with respect to \( I_1(j) \), we have

\[
\frac{\partial \pi_2(j)}{\partial I_1(j)} = 1 + \frac{f(\pi_1)}{F(\pi_1)}\pi_2(j) \tag{65}
\]

The marginal product of capital will now be larger than 1 because of the possibility that the borrowing constraint may bind. The firm can reduce this probability by investing less in period 1, which leads to a larger period 1 cash flow.

We will adopt a somewhat different distribution for the idiosyncratic fixed cost \( u(j) \) than the uniform distribution adopted before. The disadvantage of the uniform distribution is that it generates a discontinuity. As can be seen from (65), with \( f(u) \) equal to a constant \( 1/(b - a) \) the marginal product of capital will be discretely above 1 as long as there is a possibility of the borrowing constraint binding (\( \pi_1 < b \)). But it drops discontinuously to 1 once \( \pi_1 > b \) and the borrowing constraint has no chance of binding. The reason is that even when it is very unlikely that the constraint binds, a drop in investment still has a large effect on the marginal probability of default.

In the application we will assume that the density declines linearly with \( u \) and approaches 0 when \( u \to b \): \( f(u) = 2(b - u)/(b - a)^2 \). Therefore \( F(u) = 1 - (b - u)^2/(b - a)^2 \). In that case it is immediate from (65) that the constrained
optimal level of investment is increasing in first period profits \( \pi_1 \) and the optimal level of investment approaches its unconstrained level when \( \pi_1 \rightarrow b \), so that the probability of a default goes to zero. Higher profits raises investment for two reasons. First, it reduces the probability of default and therefore makes it more likely that the firm actually gets to use the capital in period 2. This is captured through an increase in \( F(\pi_1) \). Second, it reduces the marginal increase in the probability of default resulting from a rise in investment. This is captured by the drop in \( f(\pi_1) \).

Leaving details to the Appendix, the system of equilibrium equations is an extended version of the 6-equation system (41)-(46) plus the Euler equations for investment in both countries. The solution method is along similar lines as described before. We can again condense the entire set of equilibrium equations to two schedules in \( c_1 \) and \( c_1^* \) that can be graphed.

Figure 13 provides an illustration. We have kept some parameters the same as in the benchmark parameterization \((a, b, \bar{g}, \gamma)\) but modified some others in order to incorporate investment.\(^{26}\) Figure 13 shows that there are again only two symmetric equilibria. The right hand side of the Figure shows that first period consumption, investment and profits, as well as the number of period 2 firms, are all substantially lower in the panic equilibrium B than in the non-panic equilibrium A. The fact that investment is now also lower in the panic equilibrium is a result of the relationship between investment and profits discussed above. Lower first-period profits increases the probability of bankruptcy, which lowers investment.

Figure 13 is drawn for \( \psi = 0.8 \) (trade is 20% of GDP). Even though the model is significantly extended, the picture looks quite similar to Figure 7, which was also drawn for \( \psi = 0.8 \). The key conclusion remains the same. As long as there is a limited extent of economic integration, a business cycle panic will necessarily be perfectly coordinated across the two countries. With investment we find that we need \( \psi \) to be at least 0.82 for this to be the case (trade at least 18% of GDP), although we have already seen that the precise cutoff depends on various other model assumptions and may even be much lower.

\(^{26}\)In particular, we can no longer adopt the simplification that \( \alpha = 1 \) as it gives zero weight to capital in the production function. We have also lowered \( \lambda \), which reduces wages and raises profits, in order to compensate for the fact that first period profits are now reduced by investment.
6 Conclusion

The paper is motivated by evidence of close business cycle co-movement during the Great Recession. Even though the housing and financial shock originated in the United States, business cycles in the rest of the world were impacted to a similar extent. Given limited trade and financial integration across countries this is surprising as standard models with exogenous shocks and limited integration generate only partial transmission. It is also surprising given the much lower co-movement of business cycles during prior recessions.

We have developed a model with self-fulfilling business cycle panics to shed light on this. We find that limited economic integration is sufficient to assure that a panic, when it occurs, is necessarily perfectly synchronized across countries. Such global business cycle panics should not be considered a normal outcome. We have argued that several factors made the 2008 episode particularly vulnerable to such a global panic: the zero lower bound, tight credit and increased economic integration across countries. The combination of these conditions separates the 2008 episode from previous recessions.
Appendix

A. Model Equilibrium

Under the benchmark parameterization we assumed that $\alpha = \lambda = \beta = 1$ and $\mu - 1 = 1/(\alpha \gamma)$. More generally, without making these restrictive assumptions, the system of equations in section 2.5 that describes the equilibrium of the model becomes (define $\omega = \alpha/(\alpha + \mu(1 - \alpha))$)

$$
\frac{\mu}{\mu - 1} \frac{\lambda}{\alpha} c_2(s)^\gamma \frac{P_2(s)}{P_{H,2}(s)} \left( \frac{P_2(s)}{P_{H,2}(s)} c_2(s) + \bar{g} \right)^{\frac{1-\alpha}{\alpha}} = n(s)^{\frac{1}{(\mu-1)\omega}} s = 1, ..., 4 \quad (66)
$$

$$
\frac{\mu}{\mu - 1} \frac{\lambda}{\alpha} c_2^*(s)^\gamma \frac{P_2^*(s)}{P_{F,2}^*(s)} \left( \frac{P_2^*(s)}{P_{F,2}^*(s)} c_2^*(s) + \bar{g} \right)^{\frac{1-\alpha}{\alpha}} = n^*(s)^{\frac{1}{(\mu-1)\omega}} s = 1, ..., 4 \quad (67)
$$

$$
c_1^{-\gamma} = \beta \frac{1}{4} \sum_{s=1}^{4} c_2(s)^{-\gamma} \quad (68)
$$

$$
(c_1^*)^{-\gamma} = \beta \frac{1}{4} \sum_{s=1}^{4} c_2^*(s)^{-\gamma} \quad (69)
$$

$$
\pi_1 = c_1 + \frac{P_{H,1}}{P_1} \bar{g} - \lambda c_1^* \left( \frac{P_1}{P_{H,1}} - c_1 + \bar{g} \right)^{1/\alpha} \quad (70)
$$

$$
\pi_1^* = c_1^* + \frac{P_{F,1}}{P_1} \bar{g} - \lambda(c_1^*)^\gamma \left( \frac{P_1^*}{P_{F,1}}c_1^* + \bar{g} \right)^{1/\alpha} \quad (71)
$$

with relative prices as in (48) and (49) and the number of Home firms as in (31), with second period profits equal to

$$
\pi_2 = \bar{\kappa} \lambda^{\omega(1-\mu)} \left( \frac{P_{H,2}}{P_2} \right)^{\mu \omega/\alpha} \left( \frac{P_2}{P_{H,2}} c_2 + \bar{g} \right)^{\omega/\alpha} c_2^{\omega(1-\mu)} \quad (72)
$$

where $\bar{\kappa} = [\mu/(\alpha(\mu - 1))]^{\omega(1-\mu)}[\mu(1 - \alpha) + \alpha]/\mu$. The expressions for the number of Foreign firms and Foreign profits are analogous.

B. Model Solution

In the description of the model solution in Section 4.1 it was stated that for given values of $c_1$ and $c_1^*$ one can use (41), (42), (44) and (45) to derive unique solutions for $c_1(s)$ and $c_1^*(s)$ if a solution exists. In this Appendix we provide a proof of that statement. First, it is immediate from (45) that we know $\pi_1$ as it only depends on $c_1$ and $c_1^*$ once substituting the expressions for relative prices.
Next consider (41), where we need to substitute expressions for the relative prices and \(n(s)\). The left hand side depends positively on \(c_2(s)/c_2^*(s)\) and \(c_2(s)\). The right hand side, \(n(s)\), depends on \(\pi_1\), which has been solved already, and also depends on both \(c_2(s)/c_2^*(s)\) and \(c_2(s)\) with a slope less or equal to 0. From this we can solve for \(c_2(s)/c_2^*(s)\) as a negative function of \(c_2(s)\), which in turn implies a solution of \(c_2(s)\) as a function of \(c_2^*(s)\) with a slope that is positive and less than 1.

Next consider (42), where we need to substitute expressions for the relative prices and \(n^*(s)\). The left hand side depends negatively on \(c_2(s)/c_2^*(s)\) and positively on \(c_2^*(s)\). The right hand side, \(n^*(s)\), depends on \(\pi_1^*\) with a slope greater or equal to 0, on \(c_2(s)/c_2^*(s)\) with a slope greater or equal to 0 and on \(c_2^*(s)\) with a slope less than or equal to 0. From this we can solve for \(c_2(s)/c_2^*(s)\) as a function of \(c_2^*(s)\) with a positive slope and \(\pi_1^*\) with a slope less than or equal to 0. This gives \(c_2(s)\) as a function of \(c_2^*\) with a slope that is positive and greater than 1 and \(\pi_1^*\) with a slope less than or equal to 0. When we combine this with the solution of \(c_2(s)\) from (41) as a function of \(c_2^*(s)\) with a slope that is positive and less than 1, we obtain a solution for both \(c_2(s)\) and \(c_2^*(s)\) as a function of \(\pi_1^*\) with a slope larger than or equal to 0.

Next we substitute the solution of all \(c_2^*(s)\) as a function of \(\pi_1^*\) with a non-negative slope into the Foreign consumption Euler equation (44). Assuming that a solution exists, there are now two possibilities. One is that this gives an internal solution for \(\pi_1^*\). In that case we also know \(c_2(s)\) and \(c_2^*(s)\) for all states since they are non-negative functions of \(\pi_1^*\). The other is that the solution is at a corner where none of the \(c_2^*(s)\) depend positively on \(\pi_1^*\). This is the case when \(n^*(s) = 1\) for all states. In that case \(c_2(s)\) and \(c_2^*(s)\) are at the level when \(\pi_1^*\) is at least as high as a threshold above which \(c_2(s)\) and \(c_2^*(s)\) no longer depend on \(\pi_1^*\).

C. Model Equilibrium in Extension with Investment

In period 2 Home firm \(j\) sets its price \(P_{H,2}(j)\) to maximize profits
\[
\Pi_2(j) = P_{H,2}(j)y_2(j) - W_2y_2(j)^{1/\alpha}K_2(j)^{\alpha-1} \quad (73)
\]
subject to
\[
y_2(j) = c_{H,2}(j) + g_2(j) + c_{H,2}^*(j) = \left(\frac{P_{H,2}(j)}{P_{H,2}}\right)^{\mu} \left[\psi \frac{P_2}{P_{H,2}}c_2 + \bar{g} + (1 - \psi)S_2P_2^*c_2\right] \quad (74)
\]
Maximizing profits with respect to \( P_{H,2}(j) \), we have
\[
P_{H,2}(j) = \frac{\mu}{\mu - 1} \alpha W_2 y_2(j) \frac{1-\alpha}{\alpha} K_2(j) \frac{\alpha-1}{\alpha} \tag{75}
\]
Period 2 real profits is then
\[
\pi_2(j) = \kappa \frac{W_2}{P_2} y_2(j) \frac{1}{\alpha} K_2(j) \frac{\alpha-1}{\alpha} \tag{76}
\]
with \( \kappa \) as defined in the text.

It is useful to rewrite the profit expression as follows. First, using that \( P_2 c_2 = S_2 P_2^* c_2^* \) still holds (there is no investment in period 2), we have
\[
y_2(j) = \left( \frac{P_{H,2}(j)}{P_{H,2}} \right)^{-\mu} \left[ \frac{P_2}{P_{H,2}} c_2 + \bar{g} \right] \tag{77}
\]
Second, from (75) we have
\[
\frac{P_{H,2}(j)}{P_{H,2}} = \frac{\mu}{\mu - 1} \frac{\lambda}{\alpha} \frac{P_2}{P_{H,2}} \gamma y_2(j) \frac{1-\alpha}{\alpha} K_2(j) \frac{\alpha-1}{\alpha} \tag{78}
\]
Combining these last two equations and substituting into (76) gives
\[
\pi_2(j) = h(c_2, P_2/P_{H,2}) K_2(j)^{\omega_k} \tag{79}
\]
where
\[
h(c_2, P_2/P_{H,2}) = \eta c_2^{-\gamma/\kappa} \left( \frac{P_2}{P_{H,2}} c_2 + \bar{g} \right) \frac{1}{\alpha+\mu(1-\alpha)} \left( \frac{P_2}{P_{H,2}} \right)^{\frac{-\mu}{\alpha+\mu(1-\alpha)}}
\]
\[
\eta = \kappa \lambda \left( \frac{\mu}{\mu - 1} \frac{\lambda}{\alpha} \right)^{\frac{-\gamma}{\alpha+\mu(1-\alpha)}}
\]
\[
\omega_k = \frac{(1 - \alpha)(\mu - 1)}{\alpha + \mu(1 - \alpha)}
\]

Since goods are aggregated into an index of capital in the same way as they are aggregated into an index of consumption, and first-period prices are the same for all goods of the same country, we have that
\[
y_1(j) = \left[ c_{H,1} + I_{H,1} + \bar{g} \right] + \left[ c_{H,1}^* + I_{H,1}^* \right] = \\
\psi \frac{P_1}{P_{H,1}} [c_1 + I_1] + (1 - \psi) \frac{S_1 P_1^*}{P_{H,1}} [c_1^* + I_1^*] + \bar{g}
\]

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where \( I_{H,1} \) is investment demand of Home goods by Home firms and \( I_{H,1}^* \) is investment demand of Home goods by Foreign firms.

The balanced trade condition is

\[
S_1 P_{F,1} (c_{F,1} + I_{F,1}) = P_{H,1} (c_{H,1}^* + I_{H,1}^*)
\]

(80)

Using the expressions for consumption demand and the analogous expressions for investment demand, this can be written as

\[
P_I (c_1 + I_1) = S_1 P_I^* (c_1^* + I_1^*)
\]

(81)

The expression for \( y_1(j) \) then becomes

\[
y_1(j) = \frac{P_I}{P_{H,1}} (c_1 + I_1) + \bar{g}
\]

(82)

First period profits is \( \bar{\Pi}_1 = P_{H,1} y_1(j) - W_1 Y_1(j)^{1/\alpha} K_1^{(\alpha-1)/\alpha} - P_I I_1(j) - P_I u(j) \).

Using the results above, its real value can be written as

\[
\bar{\pi}_1(j) = c_1 + I_1 + \frac{P_{H,1}}{P_I} \bar{g} - \lambda c_1^\gamma \left( \frac{P_I}{P_{H,1}} (c_1 + I_1) + \bar{g} \right)^{1/\alpha} K_1^{\alpha-1} - I_1(j) - z(j)
\]

(83)

which we also write as \( \bar{\pi}_1(j) = \hat{\pi}_1 - I_1(j) - z(j) = \pi_1(j) - u(j) \). Firm \( j \) goes bankrupt when \( u(j) > \pi_1(j) \). If the firm goes bankrupt, its value will be zero. Otherwise its value is \( \bar{\pi}_1(j) + \pi_2(j) \). In the text we derived the Euler equations for investment that are based on maximizing the expected value of the firm.

The system of equations that summarize the model equilibrium are then

\[
\omega_k h(c_2, P_2/P_{H,2}) I_1^{\omega_k-1} - 1 = \ell(\pi_1) \frac{2(b - \pi_1)}{(b - a)^2 - (b - \pi_1)^2} \pi_2
\]

(84)

\[
\omega_k h(c_2^*, P_2^*/P_{H,2}^*) (I_1^*)^{\omega_k-1} - 1 = \ell(\pi_1^*) \frac{2(b - \pi_1^*)}{(b - a)^2 - (b - \pi_1^*)^2} \pi_2^*
\]

(85)

\[
\frac{\mu}{\mu - 1} \frac{\lambda}{\alpha} c_1^\gamma I_1^{\alpha-1} = \frac{P_2}{P_{H,2}} \left( \frac{P_2}{P_{H,2}} c_2 + \bar{g} \right)^{1-\alpha/\alpha} = \bar{n}^k
\]

(86)

\[
\frac{\mu}{\mu - 1} \frac{\lambda}{\alpha} (c_2^*)^{\gamma} (I_1^*)^{\alpha-1} = \frac{P_2^*}{P_{H,2}^*} \left( \frac{P_2^*}{P_{H,2}^*} c_2^* + \bar{g} \right)^{1-\alpha/\alpha} = (\bar{n}^*)^k
\]

(87)

\[c_1 = c_2 \]

(88)

\[c_1^* = c_2^* \]

(89)

\[
\pi_1 = c_1 + \frac{P_{H,1}}{P_I} \bar{g} - \lambda c_1^\gamma \left( \frac{P_I}{P_{H,1}} (c_1 + I_1) + \bar{g} \right)^{1/\alpha} K_1^{\alpha-1}
\]

(90)

\[
\pi_1^* = c_1^* + \frac{P_{F,1}}{P_I} \bar{g} - \lambda (c_1^*)^\gamma \left( \frac{P_I^*}{P_{F,1}} (c_1^* + I_1^*) + \bar{g} \right)^{1/\alpha} K_1^{\alpha-1}
\]

(91)
(84) follows from the investment Euler equations with and without binding borrowing constraints. The indicator \( \tau(\pi_1) \) is equal to 1 when \( \pi_1 \leq b \) and 0 when \( \pi_1 > b \). (85) is the counterpart Foreign investment Euler equation. (86) follows from (78) after substituting \( P_{H,2}(j) = n^{1/(\mu-1)}P_{H,2} \) and (77). (87) is the Foreign counterpart. (88)-(89) are the consumption Euler equations without aggregate shocks. (90) follows from (83), using that \( I_1(j) = I_1 \) is the same for all firms in equilibrium. (91) is the Foreign counterpart.

We need to substitute into these equations the expressions for relative prices. For the second period we still have \( P_2c_2 = S_2P_2^*c_2^* \), so that \( P_{H,2}/P_2 \) is still given by (49). For period 1 we now have that \( P_1(c_1 + I_1) = S_1P_1^*(c_1^* + I_1^*) \), from which it follows that \( P_{H,1}/P_1 = [(c_1^* + I_1^*)/(c_1 + I_1)]^{(1-\psi)/(2\psi-1)} \).

The solution method is analogous to that in the model without investment, although we need to now incorporate the investment Euler equations to derive the two schedules in \( c_1, c_1^* \) space. We can immediately substitute the Euler equations \( c_2 = c_1 \) and \( c_2^* = c_1^* \) into the other equations, which leaves 6 equilibrium conditions. In order to derive the Home schedule \( f(c_1, c_1^*) = 0 \) we use all 6 equilibrium equations with the exception of the expression (91) of Foreign profits. For a given \( c_1^* \) this gives us a solution for \( c_1, \pi_1, n^*, I_1 \) and \( I_1^* \). There may be multiple solutions, but for a given \( c_1 \) the other variables are unique if a solution exists. We therefore only need to search over values of \( c_1 \).

To see the uniqueness of the other variables, consider first the Home investment Euler equation. The left hand side depends negatively on \( I_1 \). The right hand side depends positively on \( I_1 \). First, \( \pi_2 \) depends positively on \( K_1 = I_1 \). Second, the right hand side depends negatively on \( \pi_1 \), which itself depends negatively on \( I_1 \) from (90), both directly and indirectly through the relative price \( P_{H,1}/P_1 \). Therefore the solution to \( I_1 \) is unique for a given \( c_1^* \) and \( c_1 \). The same is the case for Foreign investment \( I_1^* \). This then in turn determines \( \pi_1 \) uniquely from (90) and \( n^* \) from (87) if a solution \( n^* \leq 1 \) exists. For a given \( c_1^* \) we are then on the schedule \( f(c_1, c_1^*) = 0 \) if (86) also holds, with \( n = (\pi_1 - a)/(b - a) \). This requires us to search numerically over values of \( c_1 \), with possibly more than one solution. The Foreign schedule \( f(c_1, c_1^*) = 0 \) is again perfectly symmetric and is based on all equilibrium conditions other than the Home profit expression (90).
References


* Source: Datastream. Growth over past 4 quarters. Broken line is the U.S.; solid line is the non-U.S. G20 minus Saudi Arabia. Consumption and investment also do not include China.
Figure 2 Real GDP Growth During the Great Depression

*Source: Angus Maddison. Broken line is the U.S.; solid line is the non-U.S. G20 minus Saudi Arabia minus South Africa.*
Figure 3 One-year ahead GDP Growth Forecasts: Average Expectation and Variance*

*Data from Consensus Forecasts. Non-US: Australia, China, Hong Kong, India, Indonesia, Malaysia, New Zealand, Singapore, South Africa, Taiwan, Thailand, Japan, Germany, France, U.K., Italy, Canada
Figure 4 Symmetric Equilibria*

\[ \pi_1 \]

Schedule 1 (a=0.2, b=0.3)
Schedule 1 (a=0.5, b=0.6)
Schedule 1 (a=0.6, b=0.7)
Schedule 2

\[ \hat{\gamma} = 2.5, \phi = 0, \bar{\varepsilon} = 0, \bar{g} = 0.3 \]
Figure 5 Mechanisms Generating Self-Fulfilling Beliefs

A. Expected Income

1. Expect low period 2 income
2. Low period 1 consumption
3. Low number period 2 firms
4. Low period 1 firm profits

B. Income Risk

1. High uncertainty about period 2 income
2. Firms sensitive to aggregate shocks
3. Low period 1 consumption
4. Low period 1 firm profits
Figure 6 Symmetric Equilibria: Role of Borrowing Constraint and Government*

Unless otherwise indicated, parameters are $\gamma = 2.5, a = 0.5, b = 0.6, \bar{c} = 0, \phi = 0, \bar{g} = 0.3$
Figure 7 All Equilibria in Two-Country Model*

Panic equilibrium (B): 
\( n_B = 0.37; \ n_G = 0.57; \ c_1 = 0.27 \)

Non-panic equilibrium (A): 
\( n_B = 1; \ n_G = 1; \ c_1 = 0.61 \)

*Parameters are \( \gamma = 2.5, \ a = 0.5, \ b = 0.6, \bar{\epsilon} = 0.01, \phi = 0, \bar{g} = 0.3, \psi = 0.8 \)
Figure 8 Equilibria in Two-Country Model: Different Degrees of Trade Integration*

* Other parameters same as Figure 7
Figure 9 Interconnectedness Through Trade
Figure 10 Parameter Region Generating Only Two Equilibria: Role of Government Spending
Figure 11 Parameter Region Generating Only Two Equilibria: Role of Risksharing
Figure 12 Parameter Region Generating Only Two Equilibria: Role of Elasticity of Substitution $\nu$
Figure 13 Equilibria in Model with Investment*

panic equilibrium (B):
\[ c_1 = 0.29, I_1 = 0.06, \]
\[ n = 0.82, \pi_1 = 0.58 \]

non-panic equilibrium (A):
\[ c_1 = 0.53, I_1 = 0.21 \]
\[ n = 1, \pi_1 = 0.69 \]

\[^*\alpha = 0.5, \gamma = 2.5, \mu = 2, \lambda = 0.3, k_1 = 0.5, a = 0.5, b = 0.6, \bar{g} = 0.3, \psi = 0.8, \phi = 0\]