Equity vs. Efficiency in Subsidies to Higher Education

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Abstract

Despite some empirical evidence to the contrary, government subsidies to higher education are usually presumed to be inequitable because college-educated workers earn more than less educated workers. Using a simple model of educational choice with endogenous wages and two worker types, I obtain strong results concerning this conflict between efficiency and equity – namely that equity and efficiency do not conflict unless there are borrowing constraints. Pre-existing distorting taxes or real externalities imply that the efficient subsidy is positive and that the efficient subsidy is also the subsidy which maximizes the net income of the unskilled. However, when tuition subsidies are used to overcome borrowing constraints, the efficient subsidy exceeds the subsidy which maximizes the net income of the unskilled. If borrowing constraints could be overcome with another policy, like student loans, efficiency and equity would not be in conflict. In a more complex model with a range of worker abilities there is no equity-efficiency trade-off only when the efficient subsidy is zero – that is, in the absence of real externalities, pre-existing taxes or borrowing constraints. The presence of any one of these three complications makes the efficient subsidy positive, while the subsidy that maximizes the net income of the unskilled is lower. In those cases, efficiency conflicts with equity.

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1 Introduction

Higher education in most advanced economies is heavily subsidized by government. Although these subsidies are often justified on efficiency grounds by borrowing constraints and externalities, the distributional consequences of higher education subsidies has been of particular concern because the recipients of higher education subsidies are, on average, wealthier than non-recipients.[1] For this reason, it is important to know what effect a system of higher education subsidies (and the taxes that finance them) has on the incomes of those who do not take advantage of the subsidies. Is there a trade-off between efficiency and equity ? Is the efficient subsidy Pareto preferred ? More generally, is there any non-zero level of subsidy which is a Pareto improvement over the no-subsidy case ?

Non-recipients might benefit from higher education subsidies for three major reasons – fiscal effects, production effects, and real externalities. Fiscal effects reflect the fact that college-educated workers earn more and hence pay higher taxes and draw fewer income-tested transfer payments from the government, benefitting non-recipients by either reducing their tax burden or increasing the supply of public goods. The size of the fiscal effect will depend on several factors: the extent to which subsidies encourage college enrollment, the effect of college on earnings and the sensitivity of taxes and transfers to earnings.

Production effects, on the other hand, arise when the number of college educated workers changes the marginal products of different types of labor within a firm, a result which is implied by any production function with less than perfect substitution among factors. Since these production effects are internal to the firm, it is well known that competitive labor markets will lead to efficient resource allocations; hence output maximization requires no subsidy to college education. However, as George Johnson (1984) showed, there may be distributional consequences of such subsidies; in fact, he showed the possibility of the counterintuitive result that non-college educated labor could gain from a tuition subsidy that increased the amount of college-educated labor if non-college labor and college labor are sufficiently complementary in production. The rise in non-college earnings could outweigh the cost of the subsidy to non-college workers leading non-college labor to prefer a positive college tuition subsidy. In Johnson's model, the college educated would prefer not to have a subsidy, since the subsidy increases the number of college-educated workers, reducing their wage.

The third effect, real production externalities, operates across firms and hence cannot be internalized by the firm. Competitive labor markets lead to less than the efficient investment in college because social benefits exceed private benefits. For example, if the total number of college-educated workers hired by one firm affected the

[1] The classic reference is Hansen and Weisbrod (1969) who argued that subsidies to public higher education were regressive. Recent work by Johnson(2006) challenges that conclusion.

productivity of labor in other firms, that real externality would imply that decentralized markets would not maximize total output. A tuition subsidy might be needed for efficiency reasons. And, of course, such a subsidy would have distributional consequences as well.

The general equilibrium microsimulation model of Heckman, Lochner and Taber (1999) concluded that tuition subsidies could raise the welfare of the least able workers through general equilibrium effects on the wages of the unskilled. Keane and Wolpin's (1997) microsimulation model concluded quite the opposite – that only the most able would benefit from a tuition subsidy. Because these models are very complex and depend on a myriad of assumptions, it is difficult to draw from them firm conclusions concerning when tuition subsidies might help the unskilled and when they might not. The models proposed in this paper are simpler — aiding understanding but, of course, sacrificing detail.

The three effects outlined above– production effects, real externalities, and fiscal effects – are all potential reasons that those who do not directly receive college tuition subsidies might nevertheless benefit from them even if they must pay for part of them. Part 2 of the paper lays out a simple general model with two types of workers, endogenous wages, and agents making educational choices with no borrowing constraints. The principal result of Part 2 is that there is no conflict between efficiency and equity; that is, subsidy policies which raise national income also raise the net incomes of those who do not attend college This general proposition is illustrated with specific examples of production effects, fiscal externalities and real externalities. Part 3 adds borrowing constraints to the model and shows that using tuition subsidies to overcome borrowing constraints can hurt the less skilled. However, if two policy tools are available – student loans to overcome borrowing constraints and tuition subsidies to offset fiscal or real externalities, then the result of Part 2 that equity and efficiency are not in conflict – becomes valid again. Part 4 looks at a more complex model with a range of worker abilities. Although analytic results are not possible with this model, numerical solutions of equilibria with plausible functional forms show that a version the basic result of Part 2 continues to hold: when the efficient tuition subsidy is zero, the subsidy which maximizes the net income of the unskilled is also zero. However, when the efficient subsidy is not zero (for example, when distorting taxes must be levied to finance other government spending), the conflict between efficiency and equity reappears.

2 The Simple Model with No Borrowing Constraints

2.1 General results

I first consider a simple model with only two types of agents, able and less able, and only two types of labor, skilled and unskilled. Low ability workers cannot invest in higher education and must be low-skilled workers. High ability workers are capable of investing in higher education. If they do so, they become high-skilled workers; otherwise, they are low-skilled workers. The ability composition of the labor force is exogenous but the skill composition of the labor force is endogenous because it depends on the investment decisions of high ability agents. There is no timing or discounting in the model; all activity, schooling and working, takes place in one period. Moreover, there is no labor supply decision here only a skill investment decision.[2]

Let the size of the labor force equal n, and let s denote the ratio of skilled to unskilled workers. The ratio s is endogenous to the model and is a function of the fraction of high ability individuals in the labor force and the proportion of high ability workers who get college educations and become skilled workers. The lower limit of s is, of course 0 and the upper limit is the ratio of high ability to low ability workers, $\overline{s}$. That is, $s \in [0, \overline{s}]$. For any value of s, the number of unskilled workers in the economy is $\frac{n}{1+s}$ while the number of skilled workers is $\frac{ns}{1+s}$. There are no other factors of production.

Let the resource cost of a worker's college education be a constant c. For simplicity I neglect the time cost in college investment, so c does not depend on the wage of unskilled workers. For any value of s the total resource cost of investing in skills is just the product of the number of skilled workers and c, or $\frac{nsc}{1+s}$. The policy instrument available to the government is a tuition subsidy, parameterized by x, the fraction of the total cost, c, paid by the government. Obviously, $x \in [0, 1]$. In equilibrium, the skill ratio in the economy will depend on the subsidy policy: $s(x)$.

Let y_u and y_s denote the net income of unskilled and skilled workers. For unskilled workers, net income will be the unskilled wage less taxes. For skilled workers, net income will be the skilled wage less taxes and the unsubsidized portion of tuition. We can write net income for each type of worker as a function of the skill ratio, s, the tuition subsidy rate, x, and the tax on that type of worker, t_u or t_s.

$$y_u(s, t_u) = w_u(s) - t_u \tag{1}$$

$$y_s(s, x, t_s) = w_s(s) - c(1 - x) - t_s \tag{2}$$

The skill ratio directly affects net income because the marginal product, and hence the wage, w_u or w_s, of each type of labor depends on the relative amounts of the two types of labor in the economy. The subsidy rate, x, directly affects only the net

[2] In George Johnson's(1984) model, there are two types of agents but three types of labor. Some agents are incapable of benefitting from higher education, while the others can benefit. If an agent of the latter type invests in higher education, he becomes the highest skilled type of labor. Otherwise he is a middle skilled type of worker. The first type of agent, who never invests, is the lowest skill type of all.

income of the skilled who must pay tuition. The tax burdens on each type of labor must satisfy the government budget constraint:

$$\frac{cxns}{1+s} = t_u \cdot \left(\frac{n}{1+s}\right) + t_s \cdot \left(\frac{ns}{1+s}\right)$$

or,

$$cxs = t_u + st_s \tag{3}$$

The government budget constraint, (3) and the structure of the tax system implicitly establish the tax burden on a worker as a function of s and x: $t_s(s,x)$ and $t_u(s,x)$. For example, if the tax structure were a head tax, then $t_u(s,x) = t_s(s,x) = \frac{cxs}{1+s}$.

Substituting these tax functions back into (1) and (2), we can write the net income of a worker as a function only of s and x:

$$y_u(s,x) = w_u(s) - t_u(s,x) \tag{4}$$

$$y_s(s,x) = w_s(s) - c(1-x) - t_s(s,x) \tag{5}$$

The fundamental equilibrium condition which determines s for any value of the policy variable, x, is the following. High ability workers will find it advantageous to invest in college as long as the net income of a skilled worker exceeds the net income of an unskilled worker. Hence, the equilibrium amount of human capital investment activity, which determines s , is the solution to

$$y_u(s,x) = y_s(s,x) \tag{6}$$

Equilibrium condition (6) , which holds for all values of x, implicitly defines the economy-wide skill ratio, s, as a function of the tuition subsidy, x.

We can now write total national income, Y , as the weighted sum of net income of each type of worker, where the weights are the numbers of workers of each type:

$$Y = \frac{ns}{1+s} \cdot y_s(s,x) + \frac{n}{1+s} \cdot y_u(s,x) \tag{7}$$

To find the efficient level of the tuition subsidy (the level that maximizes total national income), differentiate (7) with respect to x:

$$\frac{dY}{dx} = \frac{\frac{ds}{dx} \cdot n \cdot (y_s - y_u) + n \cdot (1+s)(s \cdot \frac{dy_s}{dx} + \frac{dy_u}{dx})}{(1+s)^2} \tag{8}$$

Since the equilibrium condition, eq. (6) , implies both that $y_u = y_s$ and that $\frac{dy_s}{dx} = \frac{dy_u}{dx}$, (8) becomes

$$\frac{dY}{dx} = n \cdot \frac{dy_s}{dx} = n \cdot \frac{dy_u}{dx} \tag{9}$$

Equation (9) implies that the effect of the tuition subsidy, x , on national income is in the same direction as the effect of x on the net income of an unskilled worker and the net income of a skilled worker. Most important, there is no conflict between equity and efficiency here; whatever tuition policy maximizes national income maximizes the net income of the unskilled. Note that this result holds regardless of the form of the two-factor production function and of the nature of the tax system.

Proposition 1 *If there are no borrowing constraints, so that all able agents can invest in education whenever it is financially advantageous to do so, then the tuition subsidy which maximizes national income also maximizes the income of the unskilled. There is no conflict between equity and efficiency.*

This discussion has assumed that the economy is at an interior solution, in which some but not all of the able agents acquire skills ($s < \overline{s}$). What if we are instead at a corner solution ? Suppose all able agents invest in college even with no subsidy ? Then, of course, tuition subsidies won't change any behavior ($\frac{ds}{dx} = 0$) so the efficient subsidy must be zero. Any positive subsidy merely redistributes from the unskilled to the skilled. Again, the efficient subsidy (no subsidy at all) is also the subsidy which maximizes the income of the unskilled.

2.2 Three examples

I now illustrate the implications of this proposition with three examples: production effects, fiscal externalities and real externalities.

2.2.1 Production Effects

Is there a plausible case for subsidizing college on the basis that unskilled workers' wages are an increasing function of the ratio of the number of skilled workers to the number of unskilled workers, as in neoclassical, two-factor production functions? Suppose output is produced by a linearly homogenous (constant returns to scale) production function with skilled and unskilled labor as inputs. The assumption of homogeneity of degree one allows writing total output divided by the number of unskilled workers as $f(s)$,where $f'(s) > 0$ and $f'' < 0$. The marginal product of skilled labor, w_s, is $f'(s)$, while the wage of unskilled labor, w_u, is $f(s) - sf^{'}(s)$. As expected, the wage of skilled workers decreases in s while the wage of unskilled workers rises with s . Hence, unskilled workers would seem to have an interest in increasing the number of skilled workers through tuition subsidies.

When there is no college subsidy, the equilibrium condition (6) can be rewritten as:

$$f(s) - sf^{'}(s) = f^{'}(s) - c \tag{10}$$

It is easy to verify that the value of s which satisfies (10), which reflects privately optimal schooling behavior when individuals pay the full cost of college, is also the value of s which maximizes the total output of the economy net of schooling costs. As expected, no government subsidy is necessary to achieve efficiency since there are no externalities or other market imperfections.

Although the tuition subsidy rate which maximizes total income is 0, is it possible that a positive tuition subsidy could increase the income of low ability workers (and necessarily hurt high ability workers since total national income net of college costs must fall) ? After all, the greater the number of skilled workers, the higher the marginal products of unskilled workers in this two-factor production function. If this positive wage effect outweighed the tax cost to unskilled workers of financing the tuition subsidy, it would seem that subsidizing tuition might improve the net incomes of less skilled workers. However, as plausible as this intuition might be, it turns out to be false.

National income less college costs can be written as output per unskilled worker times the number of unskilled workers, less college costs times the number of skilled workers. This is $f(s) \cdot (\frac{n}{1+s}) - c \cdot (\frac{ns}{1+s})$, since $f(s)$ is output per unskilled worker. Now consider the behavior of able agents. Future skilled workers pay net tuition of $(1-x)c$. If the subsidy is financed by a head tax on everyone, that head tax, t, must satisfy the government budget constraint:

$$tn = cx\left(\frac{ns}{1+s}\right) \tag{11}$$

The left hand side of (11) are tax revenues from the head tax, t, while the right hand side are expenditures on the college tuition subsidies of cx for the $\frac{ns}{1+s}$ skilled workers. The budget balancing head tax will be:

$$t = \frac{cxs}{1+s} \tag{12}$$

Now consider the net income of unskilled workers, which is the unskilled wage less an unskilled worker's share of college subsidies. With the budget balancing head tax given by (12), the net income of an unskilled worker is

$$f(s) - sf^{'}(s) - xc \cdot (\frac{s}{1+s}) \tag{13}$$

Since the net income of a skilled worker is the skilled wage less the unsubsidized portion of college tuition and the head tax, the equilibrium condition parallel to (10) with tuition subsidies financed by a head tax becomes

$$f(s) - sf^{'}(s) = f^{'}(s) - (1-x)c \tag{14}$$

Starting with the equilibrium condition (14), some algebra reveals that the net income of an unskilled worker (expression (13)) is equal to per capita national income less college cost, or $f(s)/(1+s) - cs/(1+s)$, or

$$f(s) - sf^{'}(s) - xc \cdot (\frac{s}{1+s}) = \frac{f(s)}{(1+s)} - \frac{cs}{(1+s)} \tag{15}$$

Equation (15) implies that when college subsidies are financed by a head tax, the subsidy rate which maximizes national income net of college cost is the subsidy rate which maximizes the net income of unskilled workers. With no externalities or other complications, the subsidy which maximizes national income less college cost is zero, as is the optimal subsidy rate from the point of view of the unskilled (and the skilled, for that matter).

What about a more realistic tax alternative to a head tax ? In this model workers differ on two dimensions, ability and skill. Only skill is observable (and therefore taxable) because high skill workers (who have been to college) earn more. Hence, the only way a tax system could deviate from a head tax is by distinguishing between high and low skill workers. Suppose that skilled and unskilled workers pay possibly different amounts toward the college subsidy. It is convenient to parameterize this more complex tax system in the following way. Let α denote the ratio of a skilled worker's tax bill relative to a head tax. Thus, a value of $\alpha = 1$ corresponds to a head tax, while $\alpha > 1$ indicates that skilled workers pay more tax than a head tax and therefore more than unskilled workers. Since the head tax that balances the government's budget is given by (12), the skilled worker's tax bill will be $\alpha xc(\frac{s}{1+s})$,which implies that the net income of a skilled worker, his wage less the unsubsidized part of college tuition less his tax bill, is $f^{'}(s) - (1-x)c - \alpha xc(\frac{s}{1+s})$. The tax burden of an unskilled worker, given values of α and x, is, by the government budget constraint, $sxc\left[\frac{1-(\alpha-1)s}{1+s}\right]$ Therefore, the net income of an unskilled worker can be written as

$$f(s) - sf^{'}(s) - xc \cdot (\frac{s}{1+s})\left[1 - (\alpha - 1)s\right] \tag{16}$$

As above, wages adjust until the net income of skilled workers equals the net income of unskilled workers, implying:

$$f^{'}(s) - (1-x)c - \alpha xc(\frac{s}{1+s}) = f(s) - sf^{'}(s) - xc \cdot (\frac{s}{1+s})\left[1 - (\alpha - 1)s\right] \tag{17}$$

Equation (17) implicitly defines equilibrium s given a subsidy rate, x, and a tax progressivity parameter, α. The equilibrium condition says that as the subsidy rate is changed, the net incomes of skilled and unskilled workers move in tandem. Moreover, as (9) shows, the effect of x on national income is in the same direction as the effect on the net incomes of a skilled or an unskilled worker.[3] Therefore, whatever raises national income net of college costs will raise the net income of an unskilled worker, and vice versa, regardless of the value of α,the relative tax burden on the skilled. Since we have already established that the optimal subsidy from the point of view of national income is zero, it follows that the subsidy which maximizes the net income of the unskilled is also zero, as is the optimal subsidy for the skilled.

Therefore, in a two factor model with no externalities, preexisting taxes or borrowing constraints, the optimal tuition subsidy from the point of view of both skilled and unskilled workers is zero, even though more skilled labor raises the wage of unskilled labor and the tax burden can be higher on the skilled than on the unskilled.

2.2.2 Fiscal Externalities

This section examines the possibility that a fiscal externality might generate the outcome that tuition subsidies make the unskilled better off. How does the situation change when there are non-lump sum taxes levied to finance other public goods ? Now taxes distort the schooling decision (because a worker's tax depends on his skill) and a subsidy may be needed to achieve efficiency. Will low ability workers prefer a higher subsidy than high ability workers ?

Suppose nG is the exogenous amount of public goods to be financed by taxes while $xcns/(1+s)$ is again the total cost of the tuition subsidy. Since per capita government spending is $G + xcs/(1+s)$, the tax on a skilled worker will be α times that amount or $\alpha(G + xcs/(1+s))$ while the tax on each unskilled worker is, from the government budget constraint, $[G + \frac{xcs}{1+s}](1 + s - \alpha s)$. The net wage of a skilled worker is $f'(s) - (1-x)c - \alpha(G + xcs/(1+s))$, while the net wage of an unskilled worker is $f(s) - sf'(s) - [G + \frac{xcs}{1+s}](1 + s - \alpha s)$. Again, assuming an interior solution in which neither all nor none of the able agents get college educations, the equilibrium condition equates the net wage of skilled and unskilled workers:

$$f'(s) - (1-x)c - \alpha(G + xcs/(1+s)) = f(s) - sf'(s) - [G + \frac{xcs}{1+s}](1 + s - \alpha s) \quad (18)$$

So, as before, the fortunes of skilled and unskilled workers ride together. Whatever subsidy maximizes national income less public good and college costs will again maximize the net income of each type of worker. Does the preexisting distortion – the fact that the tax which finances the public good discourages investment in education– imply that to maximize net national income the subsidy should not be zero ?

[3] This is not as trivial a proposition as it might seem. National income is the weighted average of the net income of a skilled worker and the net income of an unskilled where the weights correspond to the number of skilled and unskilled workers. It is conceivable that a tuition subsidy policy might reduce the net income of both types of workers but raise national income by increasing the relative number of the higher paid workers. That weighting possibility cannot happen here, but as will be seen, can happen when there are borrowing constraints.

The answer, not surprisingly, is yes. In fact, the optimal tuition subsidy exactly counteracts the distorting effect of the tax, yielding exactly the same equilibrium as a head tax with no tuition subsidy. This implies that the optimal tuition subsidy is an increasing function of G and of α. The greater the preexisting tax burden (G) and the more progressive the tax system (α), the greater the optimal tuition subsidy rate.[4]

2.2.3 True Externalities

Finally, consider the case of real externalities from schooling. One kind of real externality is evidenced in production – a worker's marginal product rises as the number of skilled workers in the economy rises, holding constant the number of skilled workers at the worker's firm.[5] This externality cannot be internalized – a firm does not hire enough skilled workers because the firm does not realize the benefit of skilled workers to production in other firms.

A simple model which illustrates this real externality adds an externality term, $\theta(s)$ to the production function. Now let output per unskilled worker in a firm be given by $f(s_f) \cdot \theta(s_e)$, where s_f represents the skill ratio at the firm and s_e is the skill ratio in the economy. The marginal product of a skilled worker at a firm is $\theta(s_e) \cdot f'(s_f)$ while the marginal product of an unskilled worker is $\theta(s_e)[f(s_f) - s_f \cdot f'(s_f)]$. In a competitive economy with many identical firms, the skill ratio at each firm will equal the skill ratio in the economy. At a subsidy rate of zero, when each skilled worker must pay full tuition, the equilibrium condition, which equates the net income of skilled and unskilled workers, becomes

$$\theta(s) \cdot f^{'}(s) - c = \theta(s)[f(s) - s \cdot f^{'}(s)] \tag{19}$$

The efficient level of skill in the economy is the value of s which maximizes national income. By an argument parallel to that used to derive equation (8) above, the optimal value of s is the solution to

$$\theta(s) \cdot f^{'}(s) - c = \theta(s)[f(s) - s \cdot f^{'}(s)] - (1 + s) \cdot \theta'(s) \cdot f(s) < \theta(s) \tag{20}$$

Since a real externality implies that $\theta'(s) > 0$, the last term on the right hand side of (20) is negative so efficiency requires a value of s high enough that the net income of skilled workers is less than the net income of unskilled workers. Private decision making will obviously not accomplish this, as the equilibrium condition (19) shows.

[4] Trostel(1996) also makes an efficiency argument for tuition subsidies based on distorting taxation.

[5] Acemoglu and Angrist(2000) and Morretti(2002) both estimate models of this type of production externality, but arrive at contrasting estimates of the size of the externality.

Comparing (19) and (20), it is easy to derive the optimal subsidy rate, x , as a function of the optimal skill ratio s^*, the value of s which solves (20). The optimal subsidy rate is given by

$$cx = \theta'(s^*) \cdot f(s^*) \cdot (1 + s^*) \tag{21}$$

The intuition behind condition (21) is straightforward. At the optimal subsidy rate, the total dollar subsidy received by a skilled worker is cx. The right side of (21) is proportional to the external effect of one more skilled worker on output per unskilled worker.

In the presence of real externalities, therefore, too little schooling is chosen to maximize national income; hence, the optimal tuition subsidy is positive. Since the goal of maximizing the net wage of the unskilled is identical to the goal of maximizing national income net of schooling costs, the efficient subsidy is also the subsidy which maximizes the income of the unskilled.

2.3 No distributional effects without borrowing constraints

The principal implication of this section is that distributional issues are not important if every able person gets higher education whenever it raises his net income. That result follows from the equilibrium condition which equates the net income of skilled and less skilled workers when able individuals are able to pursue privately optimal schooling investments. Hence, any policy which raises total income net of schooling costs raises the net income of an unskilled worker. Production effects alone do not justify tuition subsidies to achieve efficiency and therefore cannot justify subsidies to increase the income of the less skilled. When distorting taxes are used to finance public goods, discouraging investment in schooling below its optimal level, a tuition subsidy offsets the tax distortion and can raise total net income. In that case, the optimal subsidy exactly offsets the deviation of the tax system from a head tax. Optimal tuition subsidies are also positive if there are real externalities of education, since national income, and hence the wages of the unskilled, are increased with subsidies to education.

3 The Simple Model with Borrowing Constraints

The previous section argued that equity and efficiency are not in conflict when students can finance the investment in their own educations. Both fiscal externalities and real externalities would justify tuition subsidies from an efficiency standpoint and such subsidies would also enhance the net income of the unskilled. Production effects alone do not make a case for subsidy even though unskilled wages are an increasing function of the level of skill.[6] This section examines how these conclusions are altered if borrowing constraints prevent students from making profitable investments in college educations.

The effect of borrowing constraints is to invalidate equilibrium condition (6). Since all profitable investment in skill cannot be made, the net income of an unskilled worker will not be less than the net income of a skilled worker, even if $s < \overline{s}$. One way to parameterize borrowing constraints is to rewrite (6) as:

$$y_u(s, x) + k = y_s(s, x) \tag{22}$$

where k parameterizes the degree of borrowing constraint. If $k = 0$,then there is no borrowing constraint. Larger positive values of k denote increasingly effective constraints on borrowing, driving a bigger wedge between the net incomes of the less skilled and the skilled.

To see what a borrowing constraint does to the results in the previous section, consider the effect of tuition subsidies on national income as expressed by equation (8) above:

$$\frac{dY}{dx} = \frac{\frac{ds}{dx} \cdot n \cdot (y_s - y_u) + n \cdot (1+s)(s \cdot \frac{dy_s}{dx} + \frac{dy_u}{dx})}{(1+s)^2} \tag{23}$$

The new equilibrium condition with borrowing constraints (equation (22)) implies that, holding k constant, $y_s > y_u$, but $\frac{dy_s}{dx} = \frac{dy_u}{dx}$. As a result, the equation above can be rewritten as:

$$\frac{dY}{dx} = n\left[\frac{\frac{ds}{dx} \cdot k}{(1+s)^2} + \frac{dy_u}{dx}\right] \tag{24}$$

The first term in (24) will be positive if borrowing constraints are effective and subsidies induce more skill investment. Therefore at the level of x which maximizes national income (Y), where $\frac{dY}{dx} = 0$, $\frac{dy_u}{dx} < 0$. This last inequality says that at the efficient subsidy level, subsidies reduce the net income of unskilled workers. Hence reducing the subsidy from this level would reduce national income but help unskilled workers. There is now a trade-off between equity and efficiency. Note that nothing in equation (24) precludes the existence of fiscal or real externalities. In that case, (24) does not, of course, imply that the optimal subsidy from the point of view of the unskilled is zero; starting from $x = 0$, if an increase in x raises Y , $\frac{dy_u}{dx}$ could be positive. The implication is rather that the optimal subsidy from the point of view of national income will be greater than the optimal subsidy from the point of view of the unskilled.

Since $\frac{dy_s}{dx} = \frac{dy_u}{dx}$, we have the seemingly paradoxical outcome that the subsidy has the same effect on the net income of the skilled as on the net income of the unskilled.

[6] This result contrasts with that of G. Johnson's(1984) paper because, as mentioned above, G. Johnson's model has three types of labor.

How is it possible, then, for national income to be maximized at a subsidy rate which (locally) reduces the net incomes of both skilled and unskilled workers (an outcome that is possible if in (24) $\frac{dY}{dx} > 0$ while $\frac{dy_u}{dx} < 0$). The answer to the paradox is that the subsidy is inducing individuals to invest in schooling which raises their net income (since $y_s > y_u$) even though the subsidy may be reducing both y_s and y_u . Looking more closely at $\frac{dy_u}{dx}$,we have

$$\frac{dy_u}{dx} = \frac{\partial y_u}{\partial s} \cdot \frac{ds}{dx} + \frac{\partial y_u}{\partial x} \tag{25}$$

The first term on the right hand side of (25) is the production effect – the increase in unskilled net income that arises because subsidies raise the ratio of skilled to unskilled labor and thereby raise the unskilled wage. The second term is the negative direct effect of x – higher subsidies and more skilled workers means higher taxes for the unskilled. In the parallel expression for skilled workers, the signs are reversed – subsidies reduce the skilled wage but increase net income holding the wage constant.

Proposition 2 *When borrowing constraints prevent some who would profit from education from acquiring it, a tuition subsidy can increase national income even in the absence of true externalities or fiscal effects. The efficient subsidy is higher than the subsidy which would maximize the net income of an unskilled worker, though the subsidy which maximizes the income of the unskilled is not necessarily zero.*

Proposition 2 suggests that the equity efficiency trade-off arises because tuition subsidies are used to offset the effects of borrowing constraints without treating the root cause. A government with two policy instruments available to it – student loan policies and tuition subsidies – could use student loans to offset the borrowing constraints and tuition subsidies to overcome externalities. Though this is certainly not a novel suggestion, what this analysis suggests is that it would have the additional advantage of eliminating the trade-off between efficiency and equity. Efficient educational policies would also maximize the income of unskilled workers.

4 A More Complex Model

The results presented so far have used a particularly simple model of the workforce in which agents possess only two levels of ability and untrained able agents are equivalent in production to the less able, who by assumption, are unable to profit from schooling. In this section, we examine whether the conclusions of the previous sections of the paper continue to hold when the model is made somewhat more complex. In particular, we consider a model in which worker ability is distributed over an interval of values and workers of any ability can attend school (though some will find it not profitable to do so).

Let worker ability, a, be distributed on the closed interval $[0, b]$ according to the distribution function $G(a)$. A worker with ability a earns $a \cdot w_s$ if he acquires a college education and earns $a \cdot w_u$ if he doesn't. Hence ability and schooling are complements in the earnings function. Each individual chooses whether to go to college based on whether earnings net of college costs are increased by college. For a given w_s and w_u, and cost of college, c, an individual chooses to go to college if $a \cdot (w_s - w_u) > c$. In equilibrium, w_s will be no less than w_u since no-one would pay for the privilege of reducing their wage. Individual choice of optimal schooling implies that all workers with $a > \widehat{a}$ go to college and all workers with ability less than $\widehat{a}$ do not go. The critical level of ability, $\widehat{a}$, is defined by

$$\widehat{a} = \min[b, c/(w_s - w_u)] \tag{26}$$

At an individual level, college costs and the difference between the skilled and unskilled wage determine who chooses to go to college. At an economy wide level, however, those choices determine the skill level in the economy and hence the level of wages of both types of workers. Letting s again denote the ratio of skilled labor to unskilled labor in the workforce, a neoclassical production function with two factors of production makes w_s a decreasing function of s while the unskilled wage, w_u , is an increasing function of s . Since the total amount of skill in the economy is just the sum of the efficiency units of labor (i.e., the ability) of the workers who choose to get a college education[7], s can be written as:

$$s = \frac{\int_{\widehat{a}}^{b} a dG}{\int_{0}^{\widehat{a}} a dG} \tag{27}$$

Clearly, the lower is $\widehat{a}$, the larger will be the fraction of the population going to college, $1 - G(\widehat{a})$, the higher will be the skill ratio s , the lower will be the skilled wage w_s and the higher will be the unskilled wage w_u . The equilibrium for the economy is defined by the conditions (26) and (27) along with the production function relations that determine the skilled and unskilled wage rates :$w_s = f'(s)$ and $w_u = f(s) - sf'(s)$.

Our new model economy can be used to examine how tuition subsidy policies may affect total national income (efficiency) or the distribution of income (equity). In this model, total income is just the sum of the wage income of all the workers less the cost of college eduction:

$$Y = \int_{0}^{\widehat{a}} a \cdot w_u(s) dG + \int_{\widehat{a}}^{b} a \cdot w_s(s) dG - c \cdot (1 - G(\widehat{a})) \tag{28}$$

In (28) the first term on the right is the wage income of the unskilled, the second is the wage income of the skilled and the last term is the cost of college multiplied

[7] To simplify these expressions, the model assumes that the size of the population is 1 rather than n as in the simple model above.

by the fraction of the population choosing college. In measuring equity effects of a policy it is important to realize that policies will typically not only affect the relative wages of sklled and unskilled owrkers but will also change the size of the population that remains unskilled, by affecting the critical value, $\widehat{a}$.

4.1 Tuition Subsidy Financed by Proportional Taxation

One particularly important, and relatively simple, policy is a tuition subsidy financed by a proportional tax on all earnings. The tuition subsidy, x, reduces the private cost of attending college to $(1-x)\cdot c$. The government taxes all earnings at the rate, t , to finance the subsidy implying a government budget constraint:

$$x \cdot c \cdot [1 - G(\widehat{a})] = t \cdot [\int_0^{\widehat{a}} a \cdot w_u(s) dG + \int_{\widehat{a}}^{b} a \cdot w_s(s) dG] \quad (29)$$

We now have a five equation model, equations(26), (27) and (29) along with $w_s = f^{'}(s)$ and $w_u = f(s) - sf^{'}(s)$. Given a subsidy rate, x, we can solve for equilibrium values of $w_s, w_u, \widehat{a}, s$ and t. Analytic solutions are intractable with general production functions and distributions of a , so we turn to specific functional forms for those two functions and calculate equilibria numerically. The specific functions chosen are a uniform density for a and a Cobb-Douglas production function. That is,

$$g(a) = 1/b \text{ for } a \in [0, b] \quad (30)$$

$$f(s) = s^{\beta} \text{ where } 0 < \beta < 1 \quad (31)$$

The two crucial values are the b relative to college cost, c , and the value of β. To illustrate a particular equilibrium, suppose $c = 1$, $b = 2$, and $\beta = 0.5$. In the absence of a tuition subsidy, $\widehat{a} = 1.61$ implying that about 19.5% of the population goes to college. s is about 0.54, $w_u = .37$ and $w_s = .68$. National income net of college costs is .38.

Letting β vary from 0 to 1, and letting c vary from .25 to 2.0, the policy which maximizes efficiency in every case is a tuition subsidy of zero. This is to be expected since there are no externalities or preexisting taxes.

Is it possible that tuition subsidies raise the net income of the unskilled ? With proportional taxation, the hypothetical policies change each unskilled worker's net income by the same proportion since net income is $a \cdot (1-t) \cdot w_u$ and t and w_u are the same for everyone. Hence an obvious measure of the equity effects of a policy is whether the net (after-tax) unskilled wage rate rises or falls. If it rises, then all unskilled workers are better off.[8] Computing the equilibria for the full range of parameter values, the results of Proposition 1 continue to hold for this more complicated specification of the model, and these functional forms.

Proposition 3 *For the more general model described by equations (26) though (29), the efficient tuition subsidy is zero. The tuition subsidy which maximizes the net income of the unskilled is also zero. There is no conflict between efficiency and equity.*

4.2 When the Efficient Subsidy is not Zero

The argument above showed that part of the result derived for the simple model in Section 2 holds for the more complex, multi-ability model at least for certain specific functional forms. That is, there is no equity-efficiency trade-off when the efficient subsidy is zero. In this section, I examine the case when the efficient subsidy is not zero. One reason the efficient subsidy may not be zero is preexisting distorting taxation needed to finance other government spending. The distortion is this model arises because the after-tax gap between the skilled and unskilled wage will be lower than the pre-tax gap for most reasonable tax systems. This discourages investment in schooling below the efficient level. Tuition subsidies can offset the effects of distorting taxes and raise national income.

Again, I use the model parameterized by equations (30) to (31) except now the proportional taxation must pay for both tuition subsidies and the exogenous level of other government spending. Numerical computation of equilibria reveal that, as in the case of the simpler model studied in sections 2 and 3, the efficient subsidy is now positive because taxation discourages investment in schooling. However, the efficient subsidy level is no longer the subsidy which maximizes the income of the unskilled. The optimal subsidy from the point of view of the unskilled is still zero. Equity and efficiency are now in conflict.

The same result obtains when real externalities exist. I augment the model of equations (26) to (31) to include a real externality effect by making output per unskilled worker a Cobb-Douglas function of the skill ratio in the worker's firm, s_f , and the skill ratio in the entire economy, s_e : $f(s_f, s_e) = s_f^{\beta} \cdot s_e^{\gamma}$. When, for example, $c = .5$, $\beta = .5$, and $\gamma = .05$, the efficient subsidy is no longer zero but rather .66 requiring a proportional tax rate of .22. However, the tuition subsidy which maximizes the net income of the unskilled is zero; the efficient subsidy reduces the net income of the unskilled by 6.5% .Other parameter values reveal similar findings; the efficient subsidy is now larger than the equitable subsidy.

[8] As mentioned above, policy changes the population of unskilled workers by changing $\widehat{a}$. In this case, the tuition subsidy induces some workers to get a college education; they are obviously better off than they would have been had they not decided to become skilled. If the policy raises the net wage rate of the unskilled, therefore, it must raise the net incomes of every person who would have been unskilled in the absence of a tuition subsidy

Finally, as was shown with the simple model, borrowing constraints imply that it is efficient to subsidize tuition. In the context of the model described by equations (26) to (31) , the efficient subsidy is again greater than the equitable subsidy for the full range of parameter values.

Proposition 4 *In the more general model described by equations (26) to (31), pre-existing distorting taxes, real externalities and borrowing constraints all imply that it is efficient to subsidize tuition. However, in each of these cases, the optimal tuition subsidy from the point of view of unskilled workers is less than the efficient subsidy and may, in fact, be zero. Hence, there is a conflict between equity and efficiency.*

5 Conclusion

Despite some empirical evidence to the contrary, there is a strong presumption that government subsidy to higher education is inequitable because college educated workers earn more than less educated workers. A simple model of educational choice and wage determination yields strong results concerning this conflict between efficiency and equity – namely that there is no conflict between equity and efficiency unless there are borrowing constraints. This result is driven by the fact that in equilibrium the marginal unskilled worker is indifferent about getting education– the net incomes of the educated and less eduated are the same. Pre-existing distorting taxes or real externalities imply that the efficient subsidy is positive and that the efficient subsidy is also the subsidy which maximizes the net income of the unskilled. However, when tuition subsidies are used to overcome borrowing constraints, the efficient subsidy exceeds the subsidy which maximizes the net income of the unskilled. If borrowing constraints could be overcome with another policy, like student loans, efficiency and equity would not be in conflict. In a more complex model with a range of worker abilities, the equity-efficiency trade-off disappears only when the efficient subsidy is zero – that is, in the absence of real externalities, pre-existing taxes or borrowing constraints. The presence of any one of these three complications makes the efficient subsidy positive, while the subsidy that maximizes the net income of the unskilled is lower. Efficiency then conflicts with equity.

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