

Information Frictions and International Asset Reallocation*

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Abstract

The topic of international portfolio allocation has received significant attention in the literature in the context of portfolio home bias. The focus of the literature has been mainly on the cross-sectional characteristics of portfolio holdings, considering the allocation of portfolios across countries at a point in time. Not much work has been done on the time series dimension of international portfolio allocation, largely as a result of data limitations. This has changed due to the recent availability of carefully constructed monthly data on equity positions of U.S. investors across over 40 countries. This paper investigates to what extent the time-variation in portfolio allocation of U.S. investors can be accounted for by time-varying risk. This involves both changes in the variance of U.S. and foreign equity returns and their covariance. This time-varying risk is estimated through a multivariate GARCH model. The portfolio allocation model adopts a simple mean-variance framework, modified to include information frictions that generate the observed portfolio home bias. I find that time-varying risk can account for 38% of the variance of the portfolio share invested abroad by U.S. investors based on monthly data. The model can account for most of the large drop in the share invested abroad during the 2008 financial crisis.

JEL classification: C32; D81; F37; G11; G15

Keywords: International portfolio choice; Home bias; Private information; GARCH

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1 Introduction

The volatility of stock returns is time-varying. Since the pioneering work of Engle (1982), the finance literature has studied this phenomenon extensively and applied it to many areas, including pricing options, predicting stock returns, and calculating optimal portfolio shares. Very few studies have looked at the implications of time-varying volatility for portfolio choice in the international context.¹ There are two main reasons for this. First, as pointed out in Ahearne et al. (2004), Bertaut and Tryon (2007) and Thomas et al. (2006), data on international portfolio holdings is difficult to obtain. Second, while the Markowitz mean-variance portfolio optimization theory is still widely used in the portfolio choice literature, matching its implications for a time-varying portfolio with the actual data is not straightforward. A Markowitz portfolio model is known to produce extreme portfolio weights that are unstable and volatile (which is not observed in the data) when estimated moments are plugged into the model.² This drawback has prevented researchers from analyzing rigorously the impacts of return distributions on observed portfolio choice of investors.

When making a decision about investing abroad, investors face many challenges about understanding foreign stocks, such as language, culture, or regulatory systems. The literature on international portfolio choice has incorporated these challenges (information frictions) to explain the home-bias phenomenon, in which investors invest a larger share in their own country's assets than what the mean-variance portfolio suggests. The focus of the literature has been mainly on the cross-sectional characteristics of portfolio holdings, considering the allocation of portfolios across countries at a point in time. Not much work has been done on the time series dimension of international portfolio allocation.

The goal of this paper is to fill these gaps in the literature by studying the effects of time-varying second moments and information frictions on how international portfolio holdings evolve over time. I consider a two-period partial equilibrium set-up of two assets, Home (H) and Foreign (F), where a home investor is assumed to maximize an objective function whose

¹Papers such as Chang and Hung (2000), Ng (2004) and Mo and Wu (2007) study only the predictability of stock returns.

²See Aït-Sahalia and Hansen, eds (2009) or Frank J. Fabozzi (2007) for the literature review on this matter.

expected value is described fully by the mean and variance of the portfolio return.³ I assume that a home investor makes the investment decision from a private signal and the information implied by an econometric model. Additionally, I also assume that expected returns are equal to each other and therefore the portfolio share in the F asset depends only on second moments of asset returns and the inverse variances (precisions) of private signals.

I perform the Taylor approximation of this portfolio choice solution and obtain the expression of how portfolio choice deviates from the trend value.⁴ I will then simulate the model results to explore how holdings change over time with different private signals. In my study, the data obtained from Bertaut and Tryon (2007) are HP-filtered, and the trend values of portfolio shares in foreign asset are used to infer about private signals.⁵ In particular, using this trend data and the estimated variance-covariance matrix every period, one can form a unique relationship between inverse variances (precisions) of private signals about F and H asset returns, τ_{SF} and τ_{SH} . In each simulation, τ_{SH} is kept constant throughout the period 1995-2008 while τ_{SF} is allowed to change so that the model portfolio solution at the trend value is matched with the HP-filtered data of foreign-asset positions. Varying the value of τ_{SH} itself generates different simulations of how portfolio allocations change over time.⁶ Comparing data on changes of actual holding and simulated data based on their standard deviations, one can find the most (or at least the range of) plausible value of τ_{SH} .⁷ This approach helps determine two unknown beliefs, τ_{SF} and τ_{SH} , using two sources of information: home bias (mean) and the dynamic of the observed portfolio allocation (standard deviation).

The contribution of this paper is two-fold. First, it is the first study that incorporates time-varying second moments and information frictions to explain the international portfolio

³The solution of portfolio choice obtained here can also be developed based on Taylor-series approximation of utility function in other forms (this can be traced back to early works by Samuelson (1970) and Pulley (1981)).

⁴There are two reasons for me to work with Taylor approximation. First, the correlation between the Taylor approximation and exact solution in my models is fairly high (0.86). Second, the Taylor approximation decomposes the sources (volatility and covariance) of portfolio changes in an intuitive way that is not easy to be achieved otherwise.

⁵One can also obtain the trend of this data by specifying it as a cubic function of time. My results, however, do not change significantly between either of trending methods.

⁶Intuitively, given a same change in estimated covariance, a higher value of either τ_{SF} or τ_{SH} represents stronger prior belief of home investor and as a result generates less variable portfolio allocations.

⁷In this paper, I do not use a formal approach to find the optimal values of τ_{SF} and τ_{SH} but rather look at one criteria which is standard deviation. Opting for a more formal approach is left for future research development.

reallocations. The main benefit of this approach is that the model can match both long run and short run behavior of the portfolio holdings. Second, it helps to gain better understanding of how introducing information frictions changes the way estimated second moments influence investors' decisions. In particular, an increase in the covariance between H and F asset returns causes two opposing influences on the F asset share in the home investor's portfolio: the diversification effect and the information effect. When the covariance between H and F asset returns increases, the diversification effect implies that it is less beneficial to invest abroad. As a result a home investor will reduce his foreign exposure. However, if a home investor incorporates his prior belief with the public information, an increase in the covariance implies that he now knows more about the F asset and wants to increase his foreign exposure. The model suggests that the information effect does play an important role and the simulation results support this argument.

The paper is organized as follows: Section 2 discusses the relevant literature. Section 3 describes the empirical approach. Section 4 presents a model of one home and one foreign asset (1H-1F) and section 5 presents a model of one H and N F assets (1H- N F). Section 6 concludes the paper. The appendix provides technical details.

2 Literature review

2.1 International portfolio holdings

Home bias is a striking phenomenon in the international portfolio investment in which domestic assets are heavily weighted relative to foreign assets in investors' portfolios.⁸ Differences in regulations, institutions, cultural conditions are claimed to increase the transaction costs and prevent investors from fully diversifying their portfolios as the standard portfolio choice theory suggests. Empirical works that seem to successfully support this argument use indirect measures such as distance (Faruqee et al. (2004), Portes and Rey (2005), Portes et al. (2001)), capital control (Buch et al., 2010), or financial market development (Baele et al. (2007), Forbes (2010),

⁸This phenomenon has been documented as early as in French and Poterba (1991), Tesar and Werner (1995) and Baxter and Jermann (1997)

Heathcote and Perri (2004)) as proxies for transaction costs rather than using direct measures such as commissions or fees (as pointed out by Glassman and Riddick (2001)).

Another important factor that causes the home bias is information frictions.⁹ The seminal work of Brennan and Cao (1997) shows that the less foreign investors are informed about local markets, the more they respond to news about it compared to local investors. One important implication drawn from their model is that investors tend to chase return, i.e. buy foreign assets when their returns are high and sell them when their returns are low. This return chasing behavior is documented in many other works including Albuquerque et al. (2009), Bohn and Tesar (1996), Brennan et al. (2005), and Froot and Ramadorai (2008).¹⁰ Some other studies suggest that investors choose to invest in the assets that they are more familiar with or have a better understanding about. Ahearne et al. (2004) show that US investors' portfolios are biased toward countries which have more companies listed in the US. Kho et al. (2009) apply the theory of corporate ownership to argue that the home bias fell during the period of 1994-2004 as a result of corporate governance's improvement in the destination countries. Although the greater integration of financial markets and the development of technology allow investors to obtain information about foreign assets much easier than before, prior belief about certain assets can still play a crucial role. Nieuwerburgh and Veldkamp (2009) show that when home investors have better knowledge (no matter how small) about home assets than about foreign assets, they will choose to learn about home assets given their limited capacity. Consequently, their knowledge about home assets is amplified and their investment is biased toward home assets.

A very small part of the literature of international portfolio focuses on the diversification motivation, which is determined by second moments of returns. On one hand, some people argue that stock return correlation and foreign portfolio positions are negatively related as suggested by the standard mean-variance portfolio theory. For example, Timmermann and Blake (2005) examine changes in second-order moments and foreign equity positions (in Japan, North America, Europe and Asia-Pacific) of 247 UK pension funds from 1991 to 1997. They find

⁹A review on this can be found in Guidolin (2003), Guidolin and Timmermann (2008), Hatchondo (2008), Karolyi and Stulz (2003), Kang and Stulz (1997) .

¹⁰A recent study by Curcuro et al. (2009), however, suggests otherwise.

a strong evidence of a negative correlation between fund position and conditional covariance. On the other hand, Ramos and von Thadden (2008) find an opposite result. They investigate how stock exchanges compete when financial markets become more highly integrated. When the diversification benefit decreases as the financial market correlation increases, stock exchanges reduce transaction costs to attract company trading. The results are that cross-border trading activities increase and home biases decrease. The study of Portes and Rey (2005) is the only work that compares the role of information frictions (proxied by distance) with that of diversification motivations to explain international portfolio allocations. They regress net flows on covariance to conclude that the information frictions play a more important role than the diversification motivation.

This paper is related to that of Okawa and van Wincoop (2010) who develop a theoretical framework for an international portfolio choice model under information frictions. The information asymmetries are introduced so that investors can have different perspectives about variances of home and foreign asset returns. Although they show that these information asymmetries have been reduced over the period 1997-2006, foreign risk was still perceived to be about 50 times higher than domestic risk for US investors.¹¹

2.2 Shrinkage estimation

The plug-in mean-variance portfolio is highly volatile due to the volatility of the estimated returns and variance-covariance matrix. If one imposes a certain prior belief on expected returns and/or covariance matrix, the (posterior) estimation of these variables will be more steady around the prior values. Hence, the method is called shrinkage estimation.¹²

Many studies show that the measurement errors are one of the causes of highly-volatile estimated inputs of plug-in portfolios. Those associated with estimated returns play a more important role than those associated with variance-covariance matrix. Therefore, the majority of the literature are devoted to fix the measurement errors of estimated returns but not of estimated variance-covariance matrices.

¹¹Glassman and Riddick (2001) and Li, 2004 also use home bias data to derive the implications about investor's prior belief about foreign returns.

¹²See extensive surveys in Frank J. Fabozzi (2007), Meucci (2005) and Connor et al. (2010)

In studies that focus on the shrinkage estimation of variance-covariance matrix, the final estimated matrix is a weighted average of a target and the sample covariance matrix. For instance, in Ledoit et al. (2003), the shrinkage estimator is found by averaging the sample covariance matrix and the covariance matrix estimated under the assumption of single index (factor). They then minimize the loss function as the Frobenius norm¹³ of the difference between the true covariance matrix and the shrinkage estimator to obtain the final estimated matrix.

This paper differentiates from other studies in this branch of literature by forming the posterior variance-covariance matrix using the Bayesian rule. Although a shrinkage estimator requires some types of prior belief, the posterior variance covariance is merely obtained as the arithmetic of prior and estimated variance-covariance matrix.¹⁴ Consequently, although the “shrinkage” second moments fluctuate much less than the estimated second moments, the dynamics of the prior and the posterior matrices are identical. While this method conveniently produces portfolio positions that are not too much volatile, the dynamic of “shrinkage” portfolio is identical to that of “plug-in” portfolio, except for the different scale. In my set-up, an increase in the covariance contributes to the diversification effect (decrease foreign exposure) but at the same time leads to the decrease in posterior variance of foreign asset (increase Foreign exposure). The conditional covariance matrix and the optimal portfolios do not necessarily have the same dynamic as the “simple-plug-in” portfolios.

3 Empirical approach

3.1 Data

U.S. holdings of foreign assets and foreign holdings of U.S. assets are obtained from Bertaut and Tryon (2007). U.S. holdings of domestic asset are calculated as U.S. stock market capitalization, which is obtained from World Federation of Exchanges, minus foreign holdings of U.S. assets.

¹³The Frobenius norm of a matrix is defined as the square root of the sum of the squares of its elements.

¹⁴The idea of shrinkage estimator approach is partly similar to that of Bayesian updating when the prior is normally distributed. In both set-ups, the posterior expected return is the arithmetic mean of prior expected return and public expected return. However, the Bayesian posterior var-cov matrix is the harmonic mean while the “shrinkage” var-cov matrix is the arithmetic mean of prior var-cov and public var-cov matrices.

Figure 1: Foreign assets share in US investors' portfolio

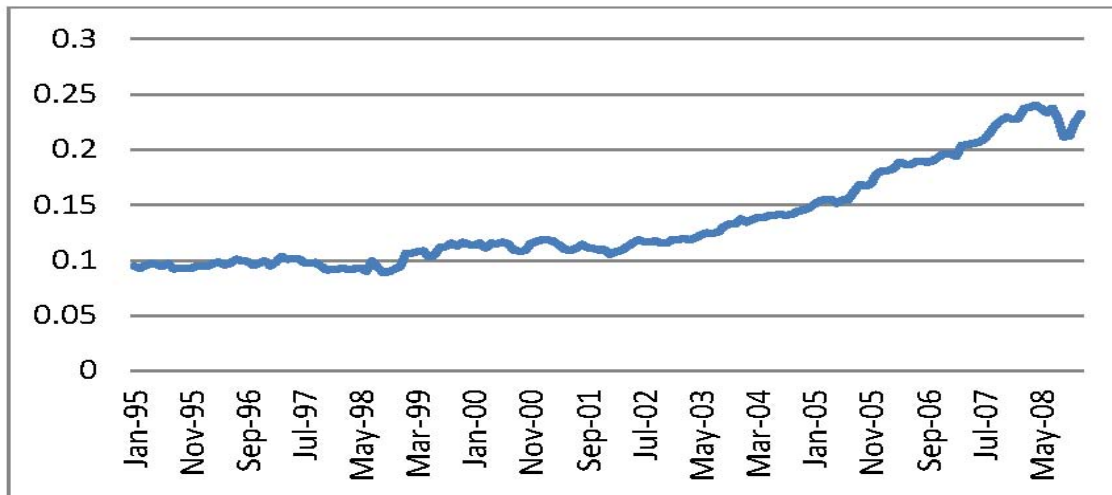


Table 1: 21 countries with the largest U.S. interests (December 2008)

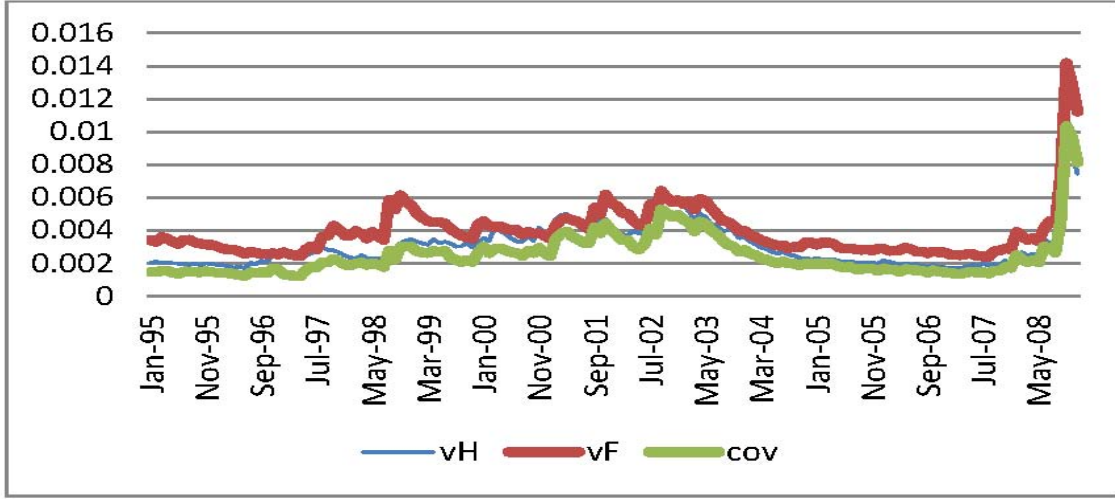
Countries	Share	Countries	Share	Countries	Share
Belgium	1%	Spain	2%	Canada	7%
Finland	1%	Sweden	1%	Korea	2%
France	8%	Switzerland	8%	Taiwan	1%
Germany	6%	United Kingdom	14%	Brazil	3%
Ireland	1%	Australia	2%	Mexico	2%
Italy	2%	Hong Kong	1%	Israel	1%
Netherlands	3%	Japan	3%	Singapore	1%

Figure 1 show the data on foreign assets shares of U.S. investors from January 1995 to December 2008. Stock returns are calculated as log-difference of Morgan Stanley Capital International (MSCI) index for the period from January 1988 to December 2008. For the two-country model, F asset returns are obtained from the MSCI index for the rest of the world. Data for the N -country model pertain to the 21 countries with the largest U.S. interests. Table 1 shows the list of these countries with their relative shares out of total foreign portfolio of U.S. investors as of December 2008.

3.2 Estimation

The literature of estimating second moments is large, one can refer to Andersen et al. (2006, 2009) or Poon and Granger (2003) for extensive surveys. To ensure the robustness of the results, I use two widely-used methods of estimating Multivariate-GARCH (M-GARCH): BEKK-

Figure 2: Estimated second moments - BEKK-GARCH method



GARCH (Engle and Kroner (1995))¹⁵ and DCC-GARCH (Engle (2002)). In addition to studying at monthly frequency, I also investigate the model implication at quarterly frequency.¹⁶

In general, model's implications are similar for the two methods of estimation and two frequencies of data. For brevity, only the monthly results from the BEKK-GARCH approach are described here.

As applied to the vector $r_t \equiv (r_{H,t}, r_{F,t})'$ or monthly domestic and foreign returns, the BEKK-GARCH model is represented as,

$$\begin{pmatrix} r_{H,t} \\ r_{F,t} \end{pmatrix} = H_t^{1/2} z_t = \begin{pmatrix} H_{1,t} & H_{2,t} \\ H_{2,t} & H_{3,t} \end{pmatrix} \begin{pmatrix} z_{H,t} \\ z_{F,t} \end{pmatrix}, \quad (1)$$

where

$$H_t = C^* C^* + A' r_{t-1} r_{t-1}' A + B' H_{t-1} B$$

$$Cov(z_{H,k}, z_{H,q}) = Cov(z_{F,k}, z_{F,q}) = Cov(z_{H,k}, z_{F,k}) = 0, \forall k, q,$$

and where disturbances $\{z_{H,t}, z_{F,t}\}_{t=1}^T$ are i.i.d as $N(0, 1)$.

A , B , and C^* are 2x2 matrices but C^* is upper triangular.

The BEKK-GARCH model is estimated by the maximum likelihood method. The results

¹⁵Figure 2 shows the estimated second moments using the BEKK-GARCH method.

¹⁶To estimate the GARCH model at quarterly frequency, one approach is to use quarterly returns directly. An alternative approach suggested by Hlouskova et al. (2009) is to use monthly data to estimate quarterly variance-covariance matrix. I investigated both and find that the second approach does a slightly better job in matching the model with data.

provide a series of estimated second moments, which will be used as inputs for the model simulations in the section 4 and 5. One feature of the GARCH model is that the future second moments are deterministic. One might be interested in understanding the impact of estimation errors on the results. I follow the bootstrap procedure in Pascual et al. (2006), which is reproduced below for convenient purpose, to obtain prediction intervals of future second moments.

The goal of this procedure is to estimate the distribution of H_{t+1} , $\forall t = 1, \dots, T$ conditionally at each time t . Based on estimated parameters, $\hat{\theta} = (\hat{A}, \hat{B}, \hat{C})$, one can calculate the residuals $\hat{z}_t = r_t \hat{H}_t^{-1/2}$, $t = 1, \dots, T$, where $\hat{H}_t = \hat{C}' \hat{C} + \hat{A}' r_{t-1} r_{t-1}' \hat{A} + \hat{B}' \hat{H}_{t-1} \hat{B}$, $t = 2, \dots, T$, and \hat{H}_1 is the unconditional matrix.

- Step 1: Simulate bootstrap returns $r^* = \{r_1^*, \dots, r_T^*\}$ as follows:

$$\hat{H}_t^* = \hat{C}' \hat{C} + \hat{A}' r_{t-1}^* r_{t-1}^{*'} \hat{A} + \hat{B}' \hat{H}_{t-1}^* \hat{B}$$

$$r_t^* = \left(\hat{H}_t^* \right)^{1/2} z_t^*, t = 1, \dots, T,$$

where z_t^* are random draws with replacement from the residuals, \hat{z}_t , calculated above, and $\hat{H}_1^* = \hat{H}_1$.

- Step 2: Obtain estimated bootstrap parameters, $\hat{\theta}^* = (\hat{A}^*, \hat{B}^*, \hat{C}^*)$, to simulate bootstrap forecasts of future values:

$$\hat{H}_{t+1}^* = \hat{C}^{*'} \hat{C}^* + \hat{A}^{*'} r_t^* r_t^{*'} \hat{A}^* + \hat{B}^{*'} \hat{H}_{t-1}^* \hat{B}^*$$

- Step 3: Repeat step 1 & 2 B times. For each t , B values of \hat{H}_{t+1}^* are combined to create a bootstrap distribution of future second moments predictions.

4 One home - one foreign models

This section investigates different ways to incorporate home bias in a portfolio choice model of two assets, H and F. I begin with a model where information frictions play as costs associated with investing abroad. Then in section 4.2, I allow the home investor to have private information

about H asset only. Finally, in section 4.3, I allow private information about both classes of assets.

In all of the models considered in this section, I assume that a home investor maximizes an objective function that is described fully by mean and variance of the portfolio return:

$$U = E(R_p) - \frac{\gamma}{2} \text{var}(R_p) \quad (2)$$

$$R_p = (1 - k) R_H + k R_F \quad (3)$$

where γ is the coefficient of risk aversion.

Assuming that the two returns have the same expected value, the solution for the foreign share k is:

$$k = \frac{\text{var}(R_H) - \text{cov}(R_H R_F)}{\text{var}(R_H) + \text{var}(R_F) - 2\text{cov}(R_H R_F)} \quad (4)$$

where k is the portfolio share in the foreign-country asset.

4.1 One home - one foreign - iceberg cost

This sub-section presents a model where an information cost, τ , is introduced as an iceberg cost occurred when investing abroad.¹⁷ The simulated results will be discussed later in section 4.4. The portfolio return is:

$$R_p = (1 - k) R_{H,1} + k (R_{F,1} - \tau)$$

and the share of investment in foreign asset is:

$$k = \frac{\text{var}(R_H) - \text{cov}(R_H R_F) - \tau/\gamma}{\text{var}(R_H) + \text{var}(R_F) - 2\text{cov}(R_H R_F)}.$$

Based on the HP-trend data at each period, the above equation will be used to infer about

¹⁷If information cost, τ , is introduced as a fixed cost, it does not have any effect on the portfolio choice and the model does not produce home bias as seen in the data.

the information cost at each period. The Taylor expansion around the trend value is as follows:

$$k - \bar{k} = \hat{k} = \frac{1}{\hat{V}_{er}} \left[\underbrace{(1 - \bar{k}) \hat{V}_H}_{C_1} - \underbrace{\bar{k} \hat{V}_F}_{C_2} - \underbrace{(1 - 2\bar{k}) \widehat{cov}}_{C_3} \right] \quad (5)$$

where \hat{k} is the deviation from the trend value, \bar{k} ; the hats of second moments are deviations from their unconditional mean; $V_{er} = Var(r_H - r_F)$.

C_1 is called V_H component, C_2 is called V_F component, C_3 is called *cov* component in the 1H-1F-no-prior model.

The deviation of portfolio from the trend value can also be written as follows:

$$k - \bar{k} = \frac{1}{\hat{V}_{er}} \left[0.5 (\hat{V}_H - \hat{V}_F) + (0.5 - \bar{k}) \hat{V}_{er} \right] \quad (6)$$

where $V_{er} = V(r_H - r_F) = V_H + V_F - 2cov$.

The first term captures the difference in relative risk between H and F assets. When H asset becomes relatively more risky than the F asset then the home investor allocates more of his wealth abroad. The second term captures the diversification effect. An increase in \hat{V}_{er} implies that the diversification benefit increases and therefore has a positive sign.¹⁸

4.2 One home - one foreign - information frictions – one prior

From here on, exogenous cost will not be considered. Instead, asymmetry in prior belief about H and F asset will be introduced to the model. In this section, I assume that a home investor has some prior belief about H asset but not about F asset. His conditional belief will be updated based on the Bayesian rule:¹⁹

$$\hat{\Sigma} = \begin{pmatrix} \frac{V_H V_S}{V_H + V_S} & \frac{cov V_S}{V_H + V_S} \\ \frac{cov V_S}{V_H + V_S} & V_F - \frac{cov^2}{V_H + V_S} \end{pmatrix} \quad (7)$$

¹⁸This is similar to the decomposition in Tille and van Wincoop (2010).

¹⁹Section 4.3 consider a more general case when the home investor has priors both about H and F asset. The posterior variance can be derived directly from its result and therefore the information structure is skipped here. Although this section is less general, it provides same intuitions on the results while requires less algebra.

where V_S is the variance of prior belief about H asset.

Appendix B, Eq. (26), shows that the portfolio choice is:

$$k = \frac{V_S V_H - cov V_S}{V_S V_H - 2cov V_S + V_F (V_S + V_H) - cov^2} = \frac{A}{B} \quad (8)$$

Eq. (8) is used as the basis to calibrate the value of V_S . The HP-filtered data on stock holdings is used as the trend value.

Taking Taylor expansion around the trend value of Eq. (8), we have:

$$k - \bar{k} = \underbrace{\frac{\partial k}{\partial V_H} |_{\bar{a}} (V_H - \bar{V}_H)}_{B_1} + \underbrace{\frac{\partial k}{\partial V_F} |_{\bar{a}} (V_F - \bar{V}_F)}_{B_2} + \underbrace{\frac{\partial k}{\partial cov} |_{\bar{a}} (cov - \bar{cov})}_{B_3} \quad (9)$$

$$\frac{\partial k}{\partial V_H} |_{\bar{a}} = \frac{1}{B} [\bar{V}_S - (\bar{V}_S + \bar{V}_F) \bar{k}] = A_1 \quad (10)$$

$$\frac{\partial k}{\partial V_F} |_{\bar{a}} = -\frac{1}{B} [\bar{V}_S + \bar{V}_H] \bar{k} = A_2 \quad (11)$$

$$\frac{\partial k}{\partial cov} |_{\bar{a}} = \frac{1}{B} [-\bar{V}_S + 2(\bar{V}_S + \bar{cov}) \bar{k}] = A_3 \quad (12)$$

A_1 is called V_H component, A_2 is called V_F component, A_3 is called cov component in the 1H-1F-1 prior model.

4.2.1 The sign of A1, A2, and A3

Appendix B, Eq. (34), shows that:

$$Sign(A_1) = Sign(V_F - cov)$$

The sign is ambiguous, depending on the relative size of V_F and cov . However, with the data in studied, we can conclude that the sign of A1 is always positive ($V_H=0.0015$, $V_F =.002$, $cov=0.0012$).

When k is positive, the sign of A2 is always negative.

An increase in estimated covariance will have 2 effects: less diversification benefit and

knowing more about F stocks. How much the second effect will offset the first depends on the strength of prior belief that home investor is born with. Appendix B, Eq. (36), shows that the sign of A3 is always negative and therefore it implies that the second effect can not offset the first effect entirely in any case.

4.2.2 Alternative decomposition of portfolio holding changes

Appendix B, Eq.(39), shows that:

$$k - \bar{k} = \frac{1}{B} \left\{ 0.5\bar{V}_S (\hat{V}_H - \hat{V}_F) + \bar{V}_S (0.5 - \bar{k}) \hat{V}_{er} - \bar{k}\Delta (V_H V_F - cov^2) \right\} \quad (13)$$

The first two terms are similar to the case of without information frictions (Eq. 6). The last term represents the change in the determinant of the variance-covariance matrix. Loosely speaking, when V_{er} increases (due to an increase in V_F or V_H , or a decrease in cov), the last term also increase (in absolute term). Ignore the changes in V_F or V_H for simplicity, then a decrease in covariance have two effects: increase in V_{er} (the diversification effect, which increases foreign portfolio) and increase the last term (the information effect, which decreases foreign portfolio).

4.2.3 The relative size of V_H , V_F , and cov components

Appendix B, Eq. (37) and (38), shows that $0 < \frac{A_1}{-A_2} < \frac{C_1}{-C_2}$, and $0 < \frac{-A_3}{-A_2} < \frac{-C_3}{-C_2}$.

The change in portfolio allocation is made up of three components: due to V_H , V_F , and cov . The above ratios show that under the information friction model, the coefficients of V_H -component and cov -component become relatively smaller than that of V_F -component. It means that given the same change in estimated second moments, V_H and cov components contribute less while V_F -components contribute more to the final change in portfolio holdings when information frictions are embedded in the model. This can be understood algebraically if one compares how differently the conditional var-cov matrix and unconditional matrix changes when the estimated second moments change. Recall from 7 that:

$$\hat{\Sigma} = \begin{pmatrix} \frac{V_H V_S}{V_H + V_S} & \frac{cov V_S}{V_H + V_S} \\ \frac{cov V_S}{V_H + V_S} & V_F - \frac{cov^2}{V_H + V_S} \end{pmatrix}$$

An increase in estimated V_H increases the conditional V_H and V_F . The first effect increases F asset holding while the second effect decreases it. A same situation happens when the estimated cov decreases (the conditional covariance decreases while the conditional V_F increases). Only the change in V_F does not have this kind of second effect under the information model. As a result, under the information friction model, V_H and cov components contribute less to the total change in portfolio allocation than under the model without information frictions.

4.3 One home - one foreign - information frictions – two priors

In this section, I assume that the home investor makes the investment decision based on information inferred from an econometric model and his private signal.

$$\text{Private signal: } \begin{pmatrix} r_H \\ r_F \end{pmatrix} \sim N \left(\begin{matrix} S_H & 1/\tau_{SH} & 0 \\ S_F & 0 & 1/\tau_{SF} \end{matrix} \right) \equiv r \sim N(S, \Sigma_S)$$

$$\text{Econometric model: } \begin{pmatrix} r_H \\ r_F \end{pmatrix} \sim N \left(\begin{matrix} r_{H,0} & V_H & cov \\ r_{F,0} & cov & V_F \end{matrix} \right) \equiv r \sim N(r_0, \Sigma)$$

where τ_{SF} and τ_{SH} are inverse variances (precisions) of prior belief about F and H asset returns.

Assuming a non-informative prior about realized return, the posterior belief of the home investor is updated using Bayesian rule:

$$\begin{pmatrix} r_H \\ r_F \end{pmatrix} \sim N \left((\Sigma^{-1} + \Sigma_S^{-1})^{-1} (\Sigma^{-1} S + \Sigma_S^{-1} r_0), (\Sigma^{-1} + \Sigma_S^{-1})^{-1} \right) \equiv r \sim N(\tilde{r}, \tilde{\Sigma})$$

In addition, assuming that both the econometric model and the prior belief suggest that expected returns are equal, the portfolio share in F asset is:

$$k = \frac{\tilde{V}_H - c\tilde{ov}}{\tilde{V}_H + \tilde{V}_F - 2c\tilde{ov}}$$

where
$$\begin{pmatrix} \tilde{V}_H & c\tilde{ov} \\ c\tilde{ov} & \tilde{V}_F \end{pmatrix} = (\Sigma^{-1} + \Sigma_S^{-1})^{-1}.$$

Appendix C, Eq. 41, shows that this is equivalent to:

$$k = \frac{V_H + \tau_{SF}(V_H V_F - cov^2) - cov}{V_H + V_F + (\tau_{SF} + \tau_{SH})(V_H V_F - cov^2) - 2cov} = \frac{\bar{A}}{\bar{B}} \quad (14)$$

The coefficients of V_H , V_F , cov -components in the Taylor expansion of the above solution around the trend are:

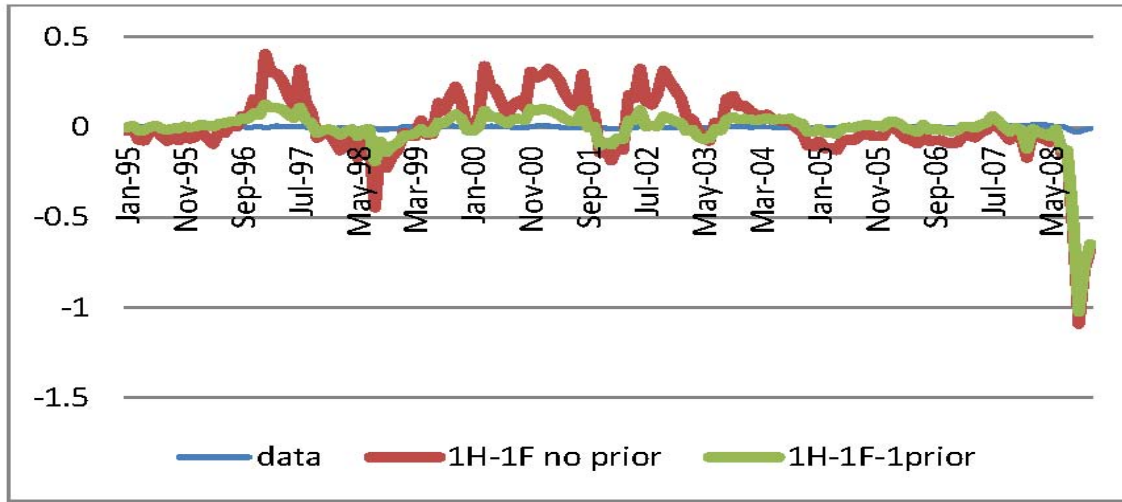
$$\frac{\partial k}{\partial V_H} \Big|_{\bar{bar}} = \frac{1}{\bar{B}} [1 + \tau_{SF}\bar{V}_F - (1 + (\tau_{SF} + \tau_{SH})\bar{V}_F)\bar{k}] = \bar{A}_1 \quad (15)$$

$$\frac{\partial k}{\partial V_F} \Big|_{\bar{bar}} = -\frac{1}{\bar{B}} [\tau_{SF}\bar{V}_H - (1 + (\tau_{SF} + \tau_{SH})\bar{V}_H)\bar{k}] = \bar{A}_2 \quad (16)$$

$$\frac{\partial k}{\partial cov} \Big|_{\bar{bar}} = \frac{1}{\bar{B}} [-(2c\bar{ov}\tau_{SF} + 1) + 2(c\bar{ov}(\tau_{SF} + \tau_{SH}) + 1)\bar{k}] = \bar{A}_3 \quad (17)$$

Eq. (14) provides the linear relationship between τ_{SF} and τ_{SH} . This allows me to study the implication of different values of τ_{SF} and τ_{SH} on how portfolio changes over time while at the same time ensures that the trend value of home bias is matched with the HP-filtered data. In each simulation, I allow τ_{SF} to change while keeping τ_{SH} constant in all periods. From 1995 to 2008, the home bias decreased and as a result, the inferred values of τ_{SF} increase accordingly. This can be interpreted as more information about F assets being available during this period, which led the home investor's prior belief about F assets became more accurate. For monthly simulations, I choose $1/\tau_{SH}$ to be .000075 (1/20 of \bar{V}_H , .0015). With this selection, the value of $1/\tau_{SF}$ decreased about 2 times, from .0005 to .00025, during 1995-2008.

Figure 3: Actual data vs. model: 1H-1F (no prior & 1 prior)



4.4 Results

Figure 3 shows the comparison of the deviation of foreign-asset shares from the HP-trend data (the vertical axis represents percentage points) and results simulated under two models as in Eq.4 and Eq.5. The correlations between actual data and results simulated by the 1H-1F-1-prior model is about 57%. Simulated results fluctuate much more than the actual data, although the 1H-1F-1-prior model does a slightly better job than the 1H-1F-no-prior model. This is understandable given the differences in the nature of two models. The information costs in the 1H-1F-no-prior model are imposed to match the home bias but do not have any influence on how estimated second moments affect the portfolio choice. The 1H-1F-1prior model uses the prior belief about the second moments' structure and therefore produces less variable portfolio changes.

Figure 4 shows the comparison of actual data and results simulated under the 1H-1F model with prior beliefs about H and F asset returns. Note that since I control for the strength of prior signals, the holding changes implied by the model do not fluctuate as much as in the case of 1H-1F-one-prior. The correlation between actual data and data generated by 1H-1F-2-prior model is 63%.²⁰

Figures 5 and 6 display three components, referred to as V_F , V_H and *cov*-components,

²⁰A simple OLS regression of actual data on model results implies that the R^2 is 0.4.

Figure 4: Actual data vs. model: 1H-1F (2 priors)

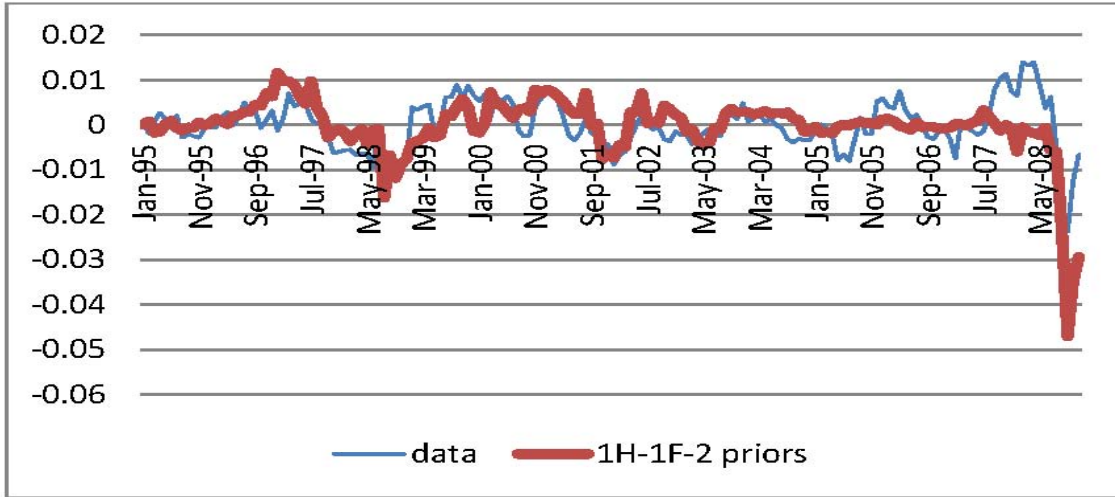


Figure 5: Components: 1H-1F (no prior)

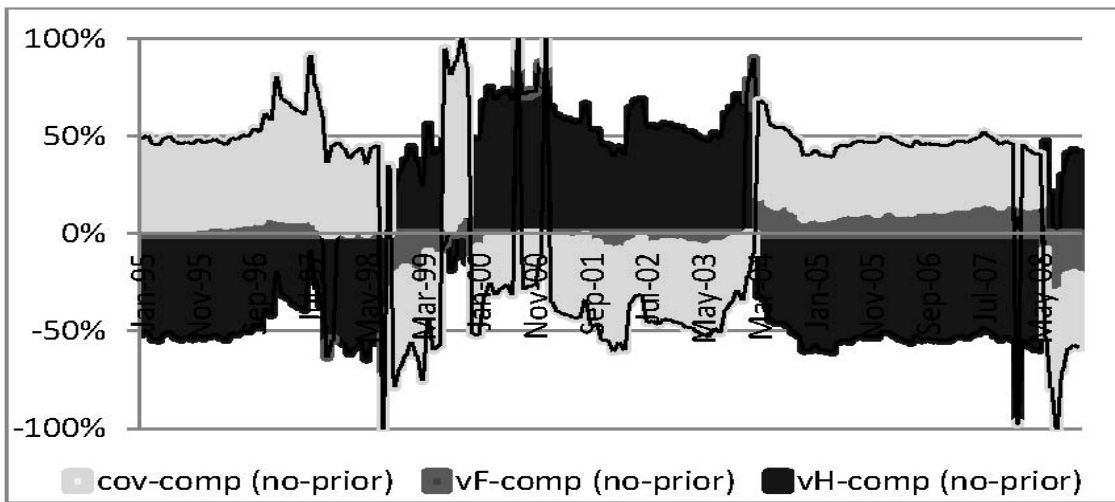
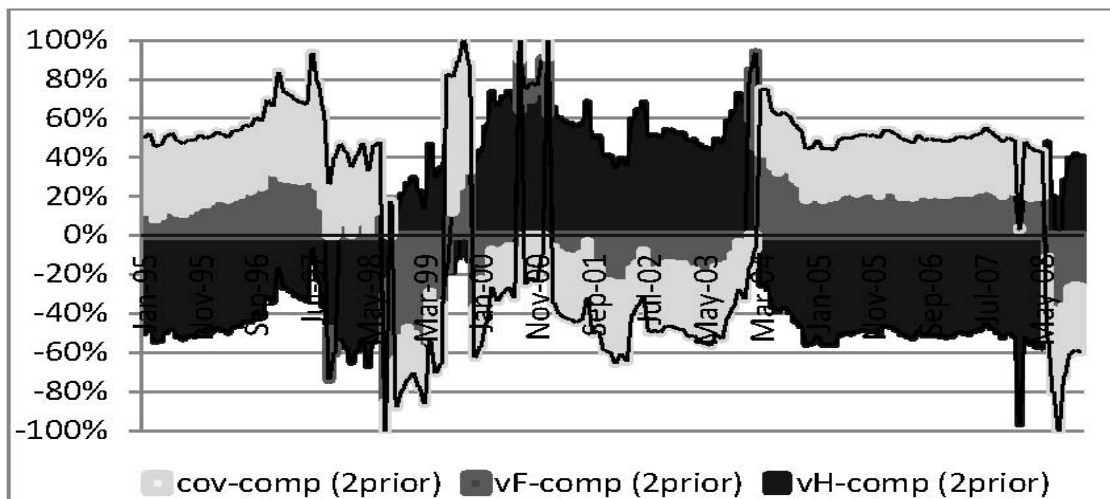


Figure 6: Components: 1H-1F (2 priors)



that contribute to the portfolio changes under different set-ups. There are two key differences between these figures. First, the share of the *cov*-component under the information friction model is significantly smaller than that under the no information model. This is as expected from the explanation in the previous section: introducing information friction creates another force which is opposite to the diversification effect. This opposite force does not offset the diversification entirely but reduces it rather significantly. Second, the V_F -component's share under the information friction model is significantly bigger than that under the no information model. These are matched with the models' implications discussed in the previous section.

It is interesting to recognize that, as seen in the data, the information friction model produces the significant drop in the 2008-foreign asset share. It is common in a financial crisis that both the volatility and covariance of asset returns increase significantly and the 2008 financial crisis is not an exception. While an increase in the volatility of H asset return causes an increase in the F asset share, the increase in the volatility of F asset return and covariance has an opposite effect. In other words, the F asset share of the home investor does not change significantly because both H and F asset become riskier during the financial crisis. It changes because the diversification benefit decreases considerably as the covariance increases.

A last point to be considered is that the information effect does play an important role. This argument is plausible because results simulated under the 1H-1F-2 prior model have a stronger correlation with the actual data than that under the 1H-1F-no-prior model.

Figure 7: Prediction intervals: 1H-1F-2-priors

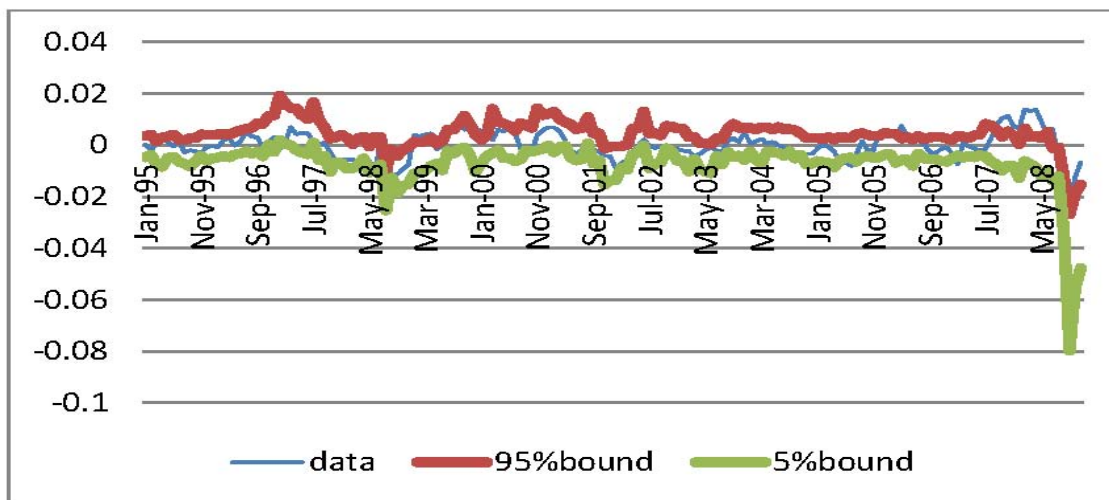


Figure 7 shows the lower (5%) and upper (95%) bounds of theoretical foreign portfolio holding changes. These bounds are obtained by simulating the holding changes based on bootstrap distributions of estimated second moments. Actual data are mostly within the prediction levels suggested by the model.

4.5 The role of expected returns

The mean-variance analysis of the portfolio choice pioneered by Markowitz (1952) has been an important framework in the finance literature. In the present paper I exclude expected returns from the analysis for two main reasons. First, ignoring expected returns is not something new. Many studies adopt the global minimum variance portfolio model, which focuses on second moments only. One can consult this literature by reading works such as Chan et al. (1999), Chopra and Ziemba (1993), and Jagannathan and Ma (2003). Second, in a mean-variance portfolio, the variation in mean has a dominant influence on the portfolio choice.²¹ I have estimated the expected returns (using past returns and dividend yields) and found that when the risk aversion coefficient is less than 10, the effect of second moments on the portfolio choice is essentially zero. This result does not depend on the strength of the private signals. The reason is because while stronger signals imply smaller variation of posterior mean, posterior

²¹See, for example, Chopra et al. (1993), Best and Grauer (1991).

second moments also vary less proportionately in my model.

To match the model with data, one can increase the risk aversion coefficient to reduce the effect of expected returns. In fact, the risk aversion coefficient needs to be at a significant high level (around 400) to make the simulated results display the data fluctuation level. An alternative approach is to use different subjective priors on mean and second moments but one needs to justify how to select those priors. This paper's approach is to infer prior beliefs about variances of H and F assets from trend data of foreign asset share and standard deviation around the trend. Therefore, while adding more priors is interesting, it is left for future research.

5 One home - N foreign models

This part extends the model developed in the previous part by allowing for N F assets to be included in the home investor's portfolio. I assume that these N F asset returns are generated by a single-factor (foreign market factor) model. Home investor, in this section, is assumed to have prior belief about H asset only.

5.1 One home - N foreign - information frictions - one prior

One H asset: $r_H \sim N(\alpha, \sigma_H^2)$

NF assets: $r_i = \beta_i r_F + \epsilon_i; r_F \sim N(\eta, \sigma_F^2)$, where $\epsilon_i \sim N(0, \sigma^2)$;

$V(r_i) = \beta_i^2 V_F + \tilde{V}$

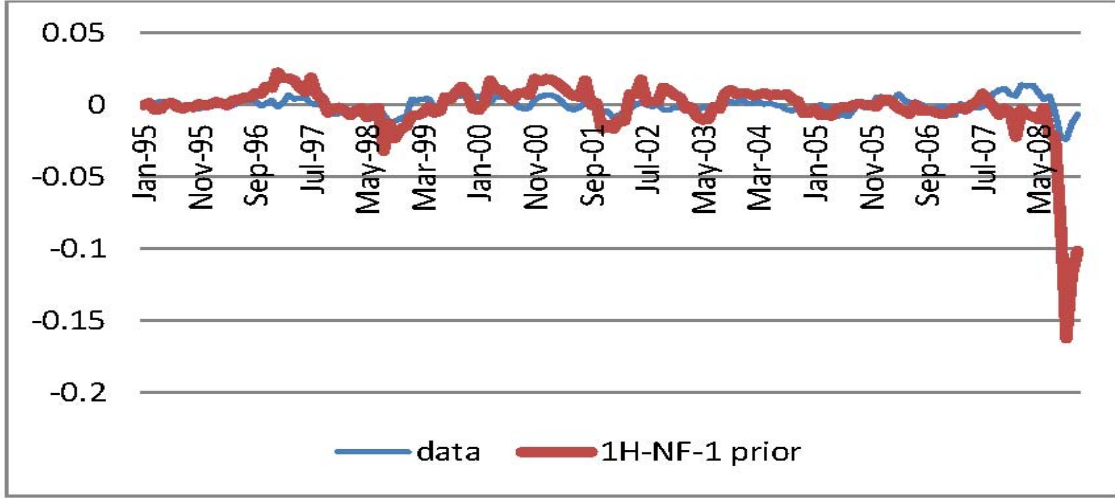
Notation: $\beta = \sum_i \beta_i; \beta^2 = \beta \cdot \beta; k = \sum_i k_i; \tilde{\beta}k = \sum_i \beta_i k_i; \beta^2 = \sum_i \beta_i^2$;

Home investor maximizes the following:

$$\begin{aligned}
 E(R_p) - 0.5\gamma var(R_p) &= (1 - \sum_i k_i) E(R_H) + (\sum_i \beta_i k_i) E(R_F) \\
 -0.5\gamma [(1 - \sum_i k_i)^2 V_H &+ (\sum_i \beta_i k_i)^2 V_F + 2(1 - \sum_i k_i) (\sum_i \beta_i k_i) cov + (\sum_i k_i^2) V_i]
 \end{aligned} \tag{18}$$

Taking Taylor expansion around its trend value (Appendix E, Eq. (66) to Eq. (68)):

Figure 8: Actual data vs. model: 1H-NF (1 prior)



$$\begin{aligned} \frac{\partial k}{\partial V_H} \Big|_{\bar{b}} &= \frac{1}{Q} \left\{ \left[\bar{V}_S \tilde{V} n - (\beta^2 - n\tilde{\beta}^2) \bar{V}_S \bar{V}_F \right] - \left[\tilde{V} (\tilde{\beta}^2 \bar{V}_F + n\bar{V}_S + \tilde{V}) - \bar{V}_S \bar{V}_F (\beta^2 - n\tilde{\beta}^2) \right] \bar{k} \right\} \\ \frac{\partial k}{\partial V_F} \Big|_{\bar{b}} &= \frac{1}{Q} \left\{ -(\beta^2 - n\tilde{\beta}^2) \bar{V}_H \bar{V}_S + \left[\bar{V}_S \bar{V}_H (\beta^2 - n\tilde{\beta}^2) - \tilde{V} \tilde{\beta}^2 (\bar{V}_S + \bar{V}_H) \right] \bar{k} \right\} \\ \frac{\partial k}{\partial cov} \Big|_{\bar{b}} &= \frac{1}{Q} \left\{ 2c\bar{ov} (\beta^2 - n\tilde{\beta}^2) \bar{V}_S - \tilde{V} \beta \bar{V}_S - 2 \left[c\bar{ov} (\beta^2 - n\tilde{\beta}^2) \bar{V}_S - \tilde{V} (\tilde{\beta}^2 c\bar{ov} + \beta \bar{V}_S) \right] \bar{k} \right\} \end{aligned}$$

5.2 Results

Figure 8 shows the comparison of actual data and theoretical data simulated under the 1H-NF model when home investors have prior belief about H asset returns. The model does not match with data well compared to the 1H-1F-2prior model does but it is a significant improvement of the 1H-1F-no prior model. Recall that in the 1H-1F-2prior model, the precision about H asset (which is fixed over all of the periods in each simulation) determines the level of fluctuation in holding reallocation. Essentially, two parameters (precisions of prior belief about H and F asset returns) need to be calibrated. It is interesting to recognize that in the 1H-NF-1prior model, however, only one parameter needs to be calibrated (precision of prior belief about H asset returns) while the variation of model portfolio is significant reduced.

This can be understood as follows. Given the actual data and imposing neither information frictions nor ice-berg costs, the 1H-1F model implies that the average F asset share is .34 while

the 1H-NF model implies that it is .70 .²² The precisions of private signals therefore need to be stronger in the latter than in the former to match the model’s result with actual data. The conditional var-cov matrix in the latter is therefore “shrunk” more than that in the former. A similar change in the estimated var-cov matrix causes a smaller change in the conditional var-cov matrix in the 1H-NF model than in the 1H-1F model. As a result, the implied holding changes in the 1H-NF model are of smaller scale than that in the 1H-1F model.

6 Conclusion

I have incorporated information frictions in a portfolio choice model to study their implications for the observed home bias and international portfolio rebalancing from the perspective of US investors. The model can match both long run and short run behavior of the portfolio holdings and there are two methods that have proved successful. The first method is to assume that the home investor makes the decision with prior beliefs in H and F assets. The relationship of these beliefs is unique in each period given the HP-filtered trend holding data. Fixing this relationship for each period allows me to choose different values of prior beliefs which in turn influence the fluctuations of portfolio positions over periods. The second method is to model N F assets and assume that the home investor is born with prior beliefs only in Home asset. The advantage of this approach is that while the prior beliefs are calibrated so that the model produces results matched only with the home bias data, the portfolio choices over periods display fluctuations similar to those observed in the data.

The model also helps to gain better understanding of how introducing information frictions changes the way estimated second moments influence investors’ decisions. Any change in portfolio choice is made up of three components: V_H, V_F , and *cov*-components. When information frictions are included in the model, the size of each component changes. In particular, V_H and *cov*-components become less important while the V_F -component becomes more important. In other words, when the home investor has some beliefs biased toward H asset, a change in V_F

²²Intuitively, when more F assets are included in the portfolio, the diversification benefit between H and F assets will be replaced by the diversification benefit between Foreign assets themselves. The share of H asset will decrease as a result.

alters his decision more significantly than in the case that he is neutral about either of the assets. The coefficient of *cov*-component has a negative sign even when information frictions are included, and it means that the diversification motivation dominates the information effect.

The results of the study indicate that second moments combined with the asymmetric prior belief explain about 38% of the movements around the trend of the foreign asset positions of US investors. The model also account for most of the large drop in the share invested abroad during the 2008 financial crisis. Without trying to fully explain these movements, but rather studying the effects of second moments, future research can improve this paper's results by focusing on other factors such as expected returns, higher moments, and changes in foreign countries' regulations or economic fundamentals.

Appendix

A 1H-1F - without information

Solution

A Home investor is assumed to maximize a utility function which is described fully by mean and variance of the portfolio return:

$$U = E(R_p) - \frac{\gamma}{2} \text{var}(R_p) \quad (19)$$

$$R_p = (1 - k) R_{H,1} + k(R_{F,1} - \tau) \quad (20)$$

This is equivalent to maximizing:

$$E[(1 - k) R_{H,1} + k(R_{F,1} - \tau)] - \frac{\gamma}{2} \text{Var}[(1 - k) R_{H,1} + k(R_{F,1} - \tau)]$$

The portfolio share in the foreign asset is:

$$k = \frac{(-R_{H,1} + R_{F,1} - \tau) / \gamma + \text{var}(R_H) - \text{cov}(R_H R_F)}{\text{var}(R_H) + \text{var}(R_F) - 2\text{cov}(R_H R_F)}$$

Assuming equalization of expected returns, it can be written as:

$$k = \frac{-\tau/\gamma + V_H - \text{cov}}{V_H + V_F - 2\text{cov}} = \frac{-\tau/\gamma + V_H - \text{cov}}{V_{er}} \quad (21)$$

where τ is the exogenous cost incurred for investing abroad. In this section, τ is included so that the model produces home bias as seen in data.

Taylor expansion

Taking Taylor expansion around the trend value of Eq. (21), we have:

$$\frac{\partial k}{\partial V_H} \Big|_{\bar{b}ar} = \frac{1}{\bar{V}_{er}} [1 - \bar{k}] = C_1 \quad (22)$$

$$\frac{\partial k}{\partial V_F} \Big|_{\bar{b}ar} = -\frac{1}{\bar{V}_{er}} \bar{k} = C_2 \quad (23)$$

$$\frac{\partial k}{\partial cov} \Big|_{\bar{b}ar} = \frac{1}{\bar{V}_{er}} [-1 + 2\bar{k}] = C_3 \quad (24)$$

B 1H-1F - with information - 1 prior

Solution

From this section onward, the exogenous cost, τ , will not be considered. Substituting $\tau = 0$ into Eq. (21), we have:

$$k = \frac{\text{var}(R_H) - \text{cov}(R_H R_F)}{\text{var}(R_H) + \text{var}(R_F) - 2\text{cov}(R_H R_F)}$$

Conditional second moments of returns are formed using Bayes law:

$$\hat{\Sigma} = \begin{pmatrix} \frac{V_H V_S}{V_H + V_S} & \frac{\text{cov} V_S}{V_H + V_S} \\ \frac{\text{cov} V_S}{V_H + V_S} & V_F - \frac{\text{cov}^2}{V_H + V_S} \end{pmatrix} \quad (25)$$

$$\Rightarrow k = \frac{\frac{V_S V_H}{V_S + V_H} - \frac{\text{cov} V_S}{V_S + V_H}}{\frac{V_S V_H}{V_S + V_H} - \frac{2\text{cov} V_S}{V_S + V_H} + \frac{V_F (V_S + V_H)}{V_S + V_H} - \frac{\text{cov}^2}{V_S + V_H}} = \frac{V_S V_H - \text{cov} V_S}{V_S V_H - 2\text{cov} V_S + V_F (V_S + V_H) - \text{cov}^2} = \frac{A}{B} \quad (26)$$

$$\Rightarrow k [V_S V_H - 2\text{cov} V_S + V_F (V_S + V_H) - \text{cov}^2] = V_S V_H - \text{cov} V_S$$

$$\Rightarrow V_S = \frac{k (V_H V_F - \text{cov}^2)}{(V_H - \text{cov}) - k (V_H - 2\text{cov} + V_F)} \quad (27)$$

Taylor expansion

Taking Taylor expansion around the trend value of Eq. (26), we have:

$$\frac{\partial k}{\partial V_H} \Big|_{\bar{b}ar} = \frac{\bar{V}_S B - (\bar{V}_S + \bar{V}_F) A}{B^2} = \frac{1}{B} [\bar{V}_S - (\bar{V}_S + \bar{V}_F) \bar{k}] = A_1 \quad (28)$$

$$\frac{\partial k}{\partial V_F} \Big|_{\bar{b}ar} = \frac{-A (\bar{V}_S + \bar{V}_H)}{B^2} = -\frac{1}{B} [\bar{V}_S + \bar{V}_H] \bar{k} = A_2 \quad (29)$$

$$\frac{\partial k}{\partial cov} \Big|_{\bar{b}ar} = \frac{-\bar{V}_S B + 2(\bar{V}_S + c\bar{ov}) A}{B^2} = \frac{1}{B} [-\bar{V}_S + 2(\bar{V}_S + c\bar{ov}) \bar{k}] = A_3 \quad (30)$$

The sign of B

From Eq. (26):

$$B = V_S V_H - 2covV_S + V_F (V_S + V_H) - cov^2 = V_S V_{er} + V_H V_F - cov^2 > 0$$

The sign of A1

Because $B > 0$, therefore the sign of A_1 , Eq. (28), is the sign of $[\bar{V}_S - (\bar{V}_S + \bar{V}_F) \bar{k}]$. Combining this with Eq. (26):

$$\begin{aligned} \Rightarrow \bar{V}_S - (\bar{V}_S + \bar{V}_F) \bar{k} &= \bar{V}_S - (\bar{V}_S + \bar{V}_F) \frac{\bar{V}_S \bar{V}_H - c\bar{ov} \bar{V}_S}{\bar{V}_S \bar{V}_H - 2c\bar{ov} \bar{V}_S + \bar{V}_F (\bar{V}_S + \bar{V}_H) - c\bar{ov}^2} \\ &= \bar{V}_S - (\bar{V}_S + \bar{V}_F) \frac{\bar{V}_S (\bar{V}_H - c\bar{ov})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\ &= \frac{\bar{V}_S (\bar{V}_S \bar{V}_{er} + \bar{d}et) - (\bar{V}_S + \bar{V}_F) \bar{V}_S (\bar{V}_H - c\bar{ov})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\ &= \bar{V}_S \frac{\bar{V}_S \bar{V}_{er} + \bar{d}et - (\bar{V}_S + \bar{V}_F) (\bar{V}_H - c\bar{ov})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\ &= \bar{V}_S \frac{\bar{V}_S (\bar{V}_F - c\bar{ov}) - c\bar{ov}^2 + \bar{V}_F c\bar{ov}}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\ &= \bar{V}_S \frac{(\bar{V}_S + c\bar{ov}) (\bar{V}_F - c\bar{ov})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \end{aligned} \quad (31)$$

$$\Rightarrow \text{Sign}[A_1] = \text{Sign}(\bar{V}_F - c\bar{ov}) \quad (32)$$

When $V_F > cov$, then $\text{Sign}[A_1] > 0$.

The sign of A2

Because $B > 0$, therefore the sign of A_2 , Eq. (29), is the sign of $-(\bar{V}_S + \bar{V}_H) \bar{k}$. Combining this with Eq. (26):

$$\begin{aligned}
-(\bar{V}_S + \bar{V}_H) \bar{k} &= -(\bar{V}_S + \bar{V}_H) \frac{\bar{V}_S \bar{V}_H - c\bar{v} \bar{V}_S}{\bar{V}_S \bar{V}_H - 2c\bar{v} \bar{V}_S + \bar{V}_F (\bar{V}_S + \bar{V}_H) - c\bar{v}^2} \\
&= -\bar{V}_S \frac{(\bar{V}_S + \bar{V}_H) (\bar{V}_H - c\bar{v})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)}
\end{aligned} \tag{33}$$

$$\Rightarrow Sign[A_2] = -Sign(\bar{V}_H - c\bar{v}) \tag{34}$$

When $V_H > cov$, then $Sign[A_2] < 0$.

The sign of A3

Because $B > 0$, therefore the sign of A_3 , Eq. (30), is the sign of $[-\bar{V}_S + 2(\bar{V}_S + c\bar{v}) \bar{k}]$.

Combining this with Eq. (26):

$$\begin{aligned}
\Rightarrow -\bar{V}_S + 2(\bar{V}_S + c\bar{v}) \bar{k} &= -\bar{V}_S + 2(\bar{V}_S + c\bar{v}) \frac{\bar{V}_S \bar{V}_H - c\bar{v} \bar{V}_S}{\bar{V}_S \bar{V}_H - 2c\bar{v} \bar{V}_S + \bar{V}_F (\bar{V}_S + \bar{V}_H) - c\bar{v}^2} \\
&= -\bar{V}_S + 2(\bar{V}_S + c\bar{v}) \frac{\bar{V}_S (\bar{V}_H - c\bar{v})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\
&= \bar{V}_S \frac{-\bar{V}_S \bar{V}_{er} - \bar{d}et + 2(\bar{V}_S + c\bar{v}) (\bar{V}_H - c\bar{v})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\
&= \bar{V}_S \frac{-\bar{V}_S (\bar{V}_H - 2c\bar{v} + \bar{V}_F) - (\bar{V}_H \bar{V}_F - c\bar{v}^2) + 2\bar{V}_S (\bar{V}_H - c\bar{v}) + 2c\bar{v} (\bar{V}_H - c\bar{v})}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\
&= \bar{V}_S \frac{\bar{V}_S \bar{V}_H - \bar{V}_S \bar{V}_F - \bar{V}_H \bar{V}_F - c\bar{v}^2 + 2c\bar{v} \bar{V}_H}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\
&= \bar{V}_S \frac{\bar{V}_S (\bar{V}_H - \bar{V}_F) + \bar{V}_H (\bar{V}_H - \bar{V}_F) - \bar{V}_H^2 - c\bar{v}^2 + 2c\bar{v} \bar{V}_H}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)} \\
&= \bar{V}_S \frac{(\bar{V}_S + \bar{V}_H) (\bar{V}_H - \bar{V}_F) - (\bar{V}_H - c\bar{v})^2}{\bar{V}_S \bar{V}_{er} + \bar{d}et(\Sigma)}
\end{aligned} \tag{35}$$

When $V_F > V_H > cov$,

$$\Rightarrow Sign[A_3] < 0 \tag{36}$$

The relative size of A1&A2 vs. that of C1&C2

This part compares the relative strength of V_H and V_F on portfolio choice under two types of setup: with and without information frictions.

Combining Eq. (31) and Eq. (33), we have:

$$\frac{A_1}{-A_2} = \frac{(\bar{V}_S + c\bar{ov}) (\bar{V}_F - c\bar{ov})}{(\bar{V}_S + \bar{V}_H) (\bar{V}_H - c\bar{ov})} > 0$$

Combining this with Eq. (27), we have:

$$\begin{aligned} \frac{A_1}{-A_2} &= \frac{\left(\frac{\bar{k}(\bar{V}_H\bar{V}_F - c\bar{ov}^2)}{(\bar{V}_H - c\bar{ov}) - \bar{k}(\bar{V}_H - 2c\bar{ov} + \bar{V}_F)} + c\bar{ov} \right) (\bar{V}_F - c\bar{ov})}{\left(\frac{\bar{k}(\bar{V}_H\bar{V}_F - c\bar{ov}^2)}{(\bar{V}_H - c\bar{ov}) - \bar{k}(\bar{V}_H - 2c\bar{ov} + \bar{V}_F)} + \bar{V}_H \right) (\bar{V}_H - c\bar{ov})} \\ &= \frac{\left(\frac{\bar{k}(\bar{V}_H\bar{V}_F - c\bar{ov}^2) + c\bar{ov}(\bar{V}_H - c\bar{ov}) - \bar{k}c\bar{ov}(\bar{V}_H - 2c\bar{ov} + \bar{V}_F)}{\bar{k}(\bar{V}_H\bar{V}_F - c\bar{ov}^2) + \bar{V}_H(\bar{V}_H - c\bar{ov}) - \bar{k}\bar{V}_H(\bar{V}_H - 2c\bar{ov} + \bar{V}_F)} \right) (\bar{V}_F - c\bar{ov})}{(\bar{V}_H - c\bar{ov})} \\ &= \frac{\left(\frac{\bar{k}(\bar{V}_H\bar{V}_F + c\bar{ov}^2 - c\bar{ov}\bar{V}_H - c\bar{ov}\bar{V}_F) + c\bar{ov}(\bar{V}_H - c\bar{ov})}{\bar{k}(-c\bar{ov}^2 - \bar{V}_H^2 + 2c\bar{ov}\bar{V}_H) + \bar{V}_H(\bar{V}_H - c\bar{ov})} \right) (\bar{V}_F - c\bar{ov})}{(\bar{V}_H - c\bar{ov})} \\ &= \frac{\left(\frac{\bar{k}(\bar{V}_F - c\bar{ov})(\bar{V}_H - c\bar{ov}) + c\bar{ov}(\bar{V}_H - c\bar{ov})}{-\bar{k}(\bar{V}_H - c\bar{ov})^2 + \bar{V}_H(\bar{V}_H - c\bar{ov})} \right) (\bar{V}_F - c\bar{ov})}{(\bar{V}_H - c\bar{ov})} \\ &= \frac{\left(\frac{\bar{k}(\bar{V}_F - c\bar{ov}) + c\bar{ov}}{\bar{V}_H - \bar{k}(\bar{V}_H - c\bar{ov})} \right) (\bar{V}_F - c\bar{ov})}{(\bar{V}_H - c\bar{ov})} = \frac{\bar{k}\bar{V}_F + c\bar{ov}(1 - \bar{k})}{\bar{V}_H(1 - \bar{k}) + c\bar{ov}\bar{k}} \frac{(\bar{V}_F - c\bar{ov})}{(\bar{V}_H - c\bar{ov})} \end{aligned}$$

Combing Eq. (22) and Eq. (23) with Eq. (21):

$$\Rightarrow \frac{C_1}{-C_2} = \frac{1 - \bar{k}}{\bar{k}} > 0$$

$$\text{Since } V_S = \frac{\bar{k}(\bar{V}_H\bar{V}_F - c\bar{ov}^2)}{(\bar{V}_H - c\bar{ov}) - \bar{k}(\bar{V}_H - 2c\bar{ov} + \bar{V}_F)} > 0 \Rightarrow (\bar{V}_H - c\bar{ov}) - \bar{k}(\bar{V}_H - 2c\bar{ov} + \bar{V}_F) > 0$$

$$\Leftrightarrow (\bar{V}_H - c\bar{ov})(1 - \bar{k}) - \bar{k}(\bar{V}_F - c\bar{ov}) > 0$$

$$\Rightarrow \begin{cases} \frac{\bar{V}_H - c\bar{ov}}{\bar{V}_F - c\bar{ov}} > \frac{\bar{k}}{1 - \bar{k}} \\ (\bar{V}_H - c\bar{ov})(1 - \bar{k}) > \bar{k}(\bar{V}_F - c\bar{ov}) \end{cases} \Rightarrow \begin{cases} \frac{\bar{V}_F - c\bar{ov}}{\bar{V}_H - c\bar{ov}} < \frac{1 - \bar{k}}{\bar{k}} \\ c\bar{ov}\bar{k} + \bar{V}_H(1 - \bar{k}) > c\bar{ov}(1 - \bar{k}) + \bar{V}_F\bar{k} \end{cases}$$

$$\Rightarrow \frac{A_1}{-A_2} < \frac{C_1}{-C_2} \quad (37)$$

The relative size of A3&A2 vs. that of C3&C2

This part compares the relative strength of V_H and V_F on portfolio choice under two types of setup: with and without information frictions.

Combining Eq. (35) and Eq. (33), we have:

$$\frac{A_3}{A_2} = -\frac{\bar{V}_S \bar{V}_H - \bar{V}_S \bar{V}_F - \bar{V}_H \bar{V}_F - c\bar{ov}^2 + 2c\bar{ov} \bar{V}_H}{(\bar{V}_S + \bar{V}_H)(\bar{V}_H - c\bar{ov})}$$

Recall from Eq. (26) that:

$$k = \frac{V_S V_H - cov V_S}{V_S V_H - 2cov V_S + V_F (V_S + V_H) - cov^2}$$

$$\begin{aligned} \Rightarrow \frac{1-2k}{k} &= \frac{V_S V_H - 2cov V_S + V_F (V_S + V_H) - cov^2 - 2V_S V_H + 2cov V_S}{V_S V_H - cov V_S} \\ \Rightarrow \frac{1-2k}{k} &= \frac{V_S (V_F - V_H) + V_F V_H - cov^2}{V_S (V_H - cov)} \\ \Rightarrow \frac{1-2\bar{k}}{\bar{k}} - \frac{A_3}{A_2} &= \frac{\bar{V}_S (\bar{V}_F - \bar{V}_H) + \bar{V}_F \bar{V}_H - c\bar{ov}^2}{\bar{V}_S (\bar{V}_H - c\bar{ov})} + \frac{\bar{V}_S \bar{V}_H - \bar{V}_S \bar{V}_F - \bar{V}_H \bar{V}_F - c\bar{ov}^2 + 2c\bar{ov} \bar{V}_H}{(\bar{V}_S + \bar{V}_H)(\bar{V}_H - c\bar{ov})} \\ \Rightarrow \left[\frac{1-2\bar{k}}{\bar{k}} - \frac{A_3}{A_2} \right] (\bar{V}_H - c\bar{ov}) \bar{V}_S (\bar{V}_S + \bar{V}_H) &= \left\{ \begin{array}{l} (\bar{V}_S + \bar{V}_H) [\bar{V}_S (\bar{V}_F - \bar{V}_H) + \bar{V}_F \bar{V}_H - c\bar{ov}^2] \\ + \bar{V}_S [(\bar{V}_S + \bar{V}_H)(\bar{V}_H - \bar{V}_F) - (\bar{V}_H - c\bar{ov})^2] \end{array} \right\} \\ &= \left\{ (\bar{V}_S + \bar{V}_H) [\bar{V}_F \bar{V}_H - c\bar{ov}^2] - \bar{V}_S (\bar{V}_H - c\bar{ov})^2 \right\} \\ &= \left\{ (\bar{V}_S + \bar{V}_H) [\bar{V}_F \bar{V}_H - c\bar{ov}^2 - (\bar{V}_H - c\bar{ov})^2] + \bar{V}_H (\bar{V}_H - c\bar{ov})^2 \right\} \\ &= \left\{ (\bar{V}_S + \bar{V}_H) [\bar{V}_F \bar{V}_H - c\bar{ov}^2 - \bar{V}_H^2 - c\bar{ov}^2 + 2\bar{V}_H c\bar{ov}] + \bar{V}_H (\bar{V}_H - c\bar{ov})^2 \right\} \\ &= \left\{ (\bar{V}_S + \bar{V}_H) [\bar{V}_H (\bar{V}_F - \bar{V}_H) + 2c\bar{ov} (\bar{V}_H - c\bar{ov})] + \bar{V}_H (\bar{V}_H - c\bar{ov})^2 \right\} \end{aligned}$$

When $V_F > V_H > cov$, $\frac{1-2k}{k} - \frac{A_3}{A_2} > 0$.

Combing Eq. (21) with Eq. (23) and Eq. (24) :

$$\begin{aligned}
&\Rightarrow \frac{C_3}{C_2} = \frac{-1 + 2\bar{k}}{-\bar{k}} = \frac{1 - 2\bar{k}}{\bar{k}} \\
&\Rightarrow \frac{A_3}{A_2} < \frac{C_3}{C_2} \tag{38}
\end{aligned}$$

- When $V_F < cov$, sign of A_1 is negative. When $V_F < cov$, the investment decision (without prior information advantage) would be to short Home asset and to long Foreign asset. The coefficient associated with changes in V_H would be negative in both original and prior information models.
- When $V_F > cov$, V_H can be either smaller or larger than cov . If V_H is smaller than cov , then without adding any information advantage, Home investor would have already wanted to short Foreign asset. As a result there is no way to introduce prior information advantage to obtain the home bias observed in the data.

Alternative decomposition of changes in portfolio holding:

From Eq. (9) to Eq. (12), we have:

$$\begin{aligned}
B_1 + B_2 + B_3 &= \frac{1}{B} \left\{ [\bar{V}_S - (\bar{V}_S + \bar{V}_F) \bar{k}] \hat{V}_H - (\bar{V}_S + \bar{V}_H) \bar{k} \hat{V}_F + [-\bar{V}_S + 2(\bar{V}_S + cov) \bar{k}] \widehat{cov} \right\} \\
&= \frac{1}{B} \left\{ \bar{V}_S (1 - \bar{k}) \hat{V}_H - \bar{V}_F \bar{k} \hat{V}_H - \bar{V}_S \bar{k} \hat{V}_F - \bar{V}_H \bar{k} \hat{V}_F - \bar{V}_S (1 - 2\bar{k}) \widehat{cov} + 2cov \bar{k} \widehat{cov} \right\} \\
&= \frac{1}{B} \left\{ [\bar{V}_S (1 - \bar{k}) \hat{V}_H - \bar{V}_S \bar{k} \hat{V}_F - \bar{V}_S (1 - 2\bar{k}) \widehat{cov}] - \bar{V}_F \bar{k} \hat{V}_H - \bar{V}_H \bar{k} \hat{V}_F + 2cov \bar{k} \widehat{cov} \right\} \\
&= \frac{1}{B} \left\{ \begin{aligned} &[0.5\bar{V}_S (\hat{V}_H - \hat{V}_F) + \bar{V}_S (0.5 - \bar{k}) \hat{V}_H + \bar{V}_S (0.5 - \bar{k}) \hat{V}_F - \bar{V}_S (1 - 2\bar{k}) \widehat{cov}] \\ &- \bar{V}_F \bar{k} \hat{V}_H - \bar{V}_H \bar{k} \hat{V}_F + 2cov \bar{k} \widehat{cov} \end{aligned} \right\} \\
\Rightarrow k - \bar{k} &= \frac{1}{B} \left\{ 0.5\bar{V}_S (\hat{V}_H - \hat{V}_F) + \bar{V}_S (0.5 - \bar{k}) \hat{V}_{er} - \bar{k} \Delta (V_H V_F - cov^2) \right\} \tag{39}
\end{aligned}$$

C 1H-1F - with information - 2 priors - equal expected returns

Solution

I assume that the Home investor make the investment decision based on information inferred from an econometric model and his private signal.

$$\text{Private signal: } \begin{pmatrix} r_H \\ r_F \end{pmatrix} \sim N \begin{pmatrix} S_H & 1/\tau_{SH} & 0 \\ S_F & 0 & 1/\tau_{SF} \end{pmatrix} \equiv r \sim N(S, \Sigma_S)$$

$$\text{Econometric model: } \begin{pmatrix} r_H \\ r_F \end{pmatrix} \sim N \begin{pmatrix} r_{H,0} & V_H & cov \\ r_{F,0} & cov & V_F \end{pmatrix} \equiv r \sim N(r_0, \Sigma)$$

Assuming a non-informative prior about realized return, the posterior belief of the Home investor is updated using Bayesian rule:

$$\begin{pmatrix} r_H \\ r_F \end{pmatrix} \sim N \left((\Sigma^{-1} + \Sigma_S^{-1})^{-1} (\Sigma^{-1}S + \Sigma_S^{-1}r_0), (\Sigma^{-1} + \Sigma_S^{-1})^{-1} \right) \equiv r \sim N(\tilde{r}, \tilde{\Sigma})$$

In addition, assuming that both the econometric model and the prior belief suggest that expected returns are equal, the portfolio share in foreign asset is

$$k = \frac{\tilde{V}_H - c\tilde{ov}}{\tilde{V}_H + \tilde{V}_F - 2c\tilde{ov}}$$

$$\text{where } \begin{pmatrix} \tilde{V}_H & c\tilde{ov} \\ c\tilde{ov} & \tilde{V}_F \end{pmatrix} = (\Sigma^{-1} + \Sigma_S^{-1})^{-1}.$$

$$\text{Denote: } \det(\Sigma) = D^{-1}; \det(\tilde{\Sigma}) = \hat{D}^{-1}$$

$$\Rightarrow \tilde{\Sigma} = (\Sigma^{-1} + \Sigma_S^{-1})^{-1} = \begin{bmatrix} DV_F + \tau_{SH} & -Dcov \\ -Dcov & DV_H + \tau_{SF} \end{bmatrix}^{-1} = \frac{\begin{bmatrix} DV_H + \tau_{SF} & Dcov \\ Dcov & DV_F + \tau_{SH} \end{bmatrix}}{(DV_F + \tau_{SH})(DV_H + \tau_{SF}) - D^2cov^2}$$

$$\begin{aligned}
&\Rightarrow \tilde{r} = (\Sigma^{-1} + \Sigma_S^{-1})^{-1} (\Sigma^{-1}r_0 + \Sigma_S^{-1}r) \\
&= \frac{\begin{bmatrix} DV_H + \tau_{SF} & Dcov \\ Dcov & DV_F + \tau_{SH} \end{bmatrix}}{(DV_F + \tau_{SH})(DV_H + \tau_{SF}) - D^2cov^2} \left\{ \begin{bmatrix} DV_F & -Dcov \\ -Dcov & DV_H \end{bmatrix} \begin{bmatrix} r_{H,0} \\ r_{F,0} \end{bmatrix} + \begin{bmatrix} \tau_{SH} & \\ 0 & \tau_{SF} \end{bmatrix} \begin{bmatrix} r_H \\ r_F \end{bmatrix} \right\} \\
&= \frac{\begin{bmatrix} DV_H + \tau_{SF} & Dcov \\ Dcov & DV_F + \tau_{SH} \end{bmatrix}}{(DV_F + \tau_{SH})(DV_H + \tau_{SF}) - D^2cov^2} \begin{bmatrix} DV_F r_{H,0} - Dcov r_{F,0} + \tau_{SH} r_H \\ -Dcov r_{H,0} + DV_H r_{F,0} + \tau_{SF} r_F \end{bmatrix}
\end{aligned}$$

Assuming that both the econometric model and the prior belief suggest that expected returns are equal, the portfolio share in the foreign asset is:

$$\begin{aligned}
k &= \frac{\text{var}(R_H) - \text{cov}(R_H R_F)}{\text{var}(R_H) + \text{var}(R_F) - 2\text{cov}(R_H R_F)} = \frac{DV_H + \tau_{SF} - Dcov}{DV_F + \tau_{SH} + DV_H + \tau_{SF} - 2Dcov} \quad (40) \\
&= \frac{\frac{V_H}{V_H V_F - cov^2} + \tau_{SH} - \frac{cov}{V_H V_F - cov^2}}{\frac{V_H}{V_H V_F - cov^2} + \tau_{SH} + \frac{V_F}{V_H V_F - cov^2} + \tau_{SF} - 2\frac{cov}{V_H V_F - cov^2}}
\end{aligned}$$

$$\Rightarrow k = \frac{V_H + \tau_{SF}(V_H V_F - cov^2) - cov}{V_H + V_F + (\tau_{SF} + \tau_{SH})(V_H V_F - cov^2) - 2cov} = \frac{\bar{A}}{\bar{B}} \quad (41)$$

$$\begin{aligned}
&\Rightarrow k [DV_F + \tau_{SH} + DV_H + \tau_{SF} - 2Dcov] = DV_H + \tau_{SF} - Dcov \\
&\Rightarrow \tau_{SF}(1 - k) = k [DV_F + \tau_{SH} + DV_H - 2Dcov] - DV_H + Dcov \\
&\Rightarrow \tau_{SF} = \frac{k [DV_F + \tau_{SH} + DV_H - 2Dcov] - DV_H + Dcov}{1 - k}
\end{aligned}$$

Taylor expansion

$$\begin{aligned}\frac{\partial k}{\partial V_H} \Big|_{\bar{b}ar} &= \frac{(1 + \tau_{SF} \bar{V}_F) \bar{B} - [1 + (\tau_{SF} + \tau_{SH}) \bar{V}_F] \bar{A}}{\bar{B}^2} \Big|_{\bar{b}ar} \\ &= \frac{1}{\bar{B}} [1 + \tau_{SF} \bar{V}_F - (1 + (\tau_{SF} + \tau_{SH}) \bar{V}_F) \bar{k}] = \bar{A}_1\end{aligned}\quad (42)$$

$$\begin{aligned}\frac{\partial k}{\partial V_F} \Big|_{\bar{b}ar} &= \frac{\tau_{SF} \bar{V}_H \bar{B} - [1 + (\tau_{SF} + \tau_{SH}) \bar{V}_H] \bar{A}}{\bar{B}^2} \Big|_{\bar{b}ar} \\ &= \frac{1}{\bar{B}} [\tau_{SF} \bar{V}_H - (1 + (\tau_{SF} + \tau_{SH}) \bar{V}_H) \bar{k}] = \bar{A}_2\end{aligned}\quad (43)$$

$$\begin{aligned}\frac{\partial k}{\partial cov} \Big|_{\bar{b}ar} &= \frac{(-2c\bar{ov}\tau_{SF} - 1) \bar{B} - [-2c\bar{ov}(\tau_{SF} + \tau_{SH}) - 2] \bar{A}}{\bar{B}^2} \Big|_{\bar{b}ar} \\ &= \frac{1}{\bar{B}} [-(2c\bar{ov}\tau_{SF} + 1) + 2(c\bar{ov}(\tau_{SF} + \tau_{SH}) + 1) \bar{k}] = A_3\end{aligned}\quad (44)$$

Alternative decomposition of changes in portfolio holding

Combining above three equations, we have:

$$\begin{aligned}\Rightarrow k - \bar{k} &= \frac{1}{\bar{B}} \left\{ \begin{aligned} &[1 + \tau_{SF} \bar{V}_F - (1 + (\tau_{SF} + \tau_{SH}) \bar{V}_F) \bar{k}] \hat{V}_H + [\tau_{SF} \bar{V}_H - (1 + (\tau_{SF} + \tau_{SH}) \bar{V}_H) \bar{k}] \hat{V}_F \\ &+ [-(2c\bar{ov}\tau_{SF} + 1) + 2(c\bar{ov}(\tau_{SF} + \tau_{SH}) + 1) \bar{k}] \hat{cov} \end{aligned} \right\} \\ &= \frac{1}{\bar{B}} \left\{ \begin{aligned} &[(c\bar{ov}\tau_{SF} + 0.5) - (c\bar{ov}(\tau_{SF} + \tau_{SH}) + 1) \bar{k}] [\hat{V}_H + \hat{V}_F - 2 * \hat{cov}] \\ &+ [0.5 - c\bar{ov}\tau_{SF} + c\bar{ov}(\tau_{SF} + \tau_{SH}) \bar{k} + \tau_{SF} \bar{V}_F - (\tau_{SF} + \tau_{SH}) \bar{V}_F \bar{k}] \hat{V}_H \\ &+ [-0.5 - c\bar{ov}\tau_{SF} + c\bar{ov}(\tau_{SF} + \tau_{SH}) \bar{k} + \tau_{SF} \bar{V}_H - (\tau_{SF} + \tau_{SH}) \bar{V}_H \bar{k}] \hat{V}_F \end{aligned} \right\}\end{aligned}$$

D 1H-NF Model - without information

General solution for the case of non-zero expected returns

1 Home country: $r_H \sim N(\alpha, \sigma_H^2)$

N Foreign countries: $r_i = \beta_i r_F + \epsilon_i; r_F \sim N(\beta, \sigma_F^2)$, where $\epsilon_i \sim N(0, \sigma_i^2)$; ϵ_i are uncorrelated to each other and to r_H and r_F .

$$V(r_i) = \beta_i^2 V_F + V_i : V_i = V_j = \tilde{V}, \forall i, j$$

$$\text{Notation: } \beta = \sum_i \beta_i; \beta^2 = \beta \cdot \beta; k = \sum_i k_i; \tilde{\beta}k = \sum_i \beta_i k_i; \beta^2 = \sum_i \beta_i^2;$$

Home investor maximizes the following:

$$\begin{aligned} E(W) - 0.5\gamma \text{var}(W) &= (1 - \sum_i k_i) E(R_H) + (\sum_i \beta_i k_i) E(R_F) \\ -0.5\gamma \left[(1 - \sum_i k_i)^2 V_H + (\sum_i \beta_i k_i)^2 V_F + 2(1 - \sum_i k_i) (\sum_i \beta_i k_i) \text{cov} + (\sum_i k_i^2) V_i \right] \end{aligned} \quad (45)$$

$$\begin{aligned} \Rightarrow \frac{\partial U}{\partial k_i} &= \beta_i E(R_F) - E(R_H) \\ -\gamma \left[\left(\sum_i k_i - 1 \right) V_H + \beta_i \left(\sum_i \beta_i k_i \right) V_F + \left(\beta_i - \sum_i \beta_i k_i - \beta_i \sum_i k_i \right) \text{cov} + k_i V_i \right] &= \end{aligned}$$

$$\Rightarrow \beta_i E(R_F) - E(R_H) = \gamma \left[(1 - k) (\beta_i \text{cov} - V_H) + \tilde{\beta}k (\beta_i V_F - \text{cov}) + k_i V_i \right] \quad (46)$$

- Sum across i of Eq. (46), we have:

$$\beta E(R_F) - nE(R_H) - \gamma \left[(1 - k) (\beta \text{cov} - nV_H) + \tilde{\beta}k (\beta V_F - n\text{cov}) + k\tilde{V} \right] = 0$$

$$\Rightarrow \beta E(R_F) - nE(R_H) - \gamma \left[(1 - k) (\beta \text{cov} - nV_H) + k\tilde{V} \right] = \gamma \tilde{\beta}k (\beta V_F - n\text{cov})$$

$$\Rightarrow \tilde{\beta}k = \left(\sum_i \beta_i k_i \right) = \frac{\beta E(R_F) - nE(R_H) - \gamma \left[(1 - k) (\beta \text{cov} - nV_H) + k\tilde{V} \right]}{\gamma (\beta V_F - n\text{cov})} \quad (47)$$

- Combining Eq. (46) with Eq. (47), we have:

$$\begin{aligned}
&\Rightarrow \beta_i E(R_F) - E(R_H) - \gamma \left[(1-k)(\beta_i \text{cov} - V_H) + k_i \tilde{V} \right] \\
&= \gamma(\beta_i V_F - \text{cov}) \frac{\beta E(R_F) - E(R_H) - \gamma \left[(1-k)(\beta \text{cov} - nV_H) + k\tilde{V} \right]}{\gamma(\beta V_F - n\text{cov})}
\end{aligned}$$

$$\begin{aligned}
k_i &= \frac{\beta_i E(R_F) - E(R_H) - \gamma \left[(1-k)(\beta_i \text{cov} - V_H) \right]}{\gamma \tilde{V}} \\
&\frac{[\beta E(R_F) - nE(R_H)] - \gamma \left[(1-k)(\beta \text{cov} - nV_H) + k\tilde{V} \right]}{\gamma \tilde{V}} \frac{(\beta_i V_F - \text{cov})}{(\beta V_F - n\text{cov})}
\end{aligned} \tag{48}$$

- Multiply both sides of Eq. (46) with β_i , we have:

$$\beta_i^2 E(R_F) - \beta_i E(R_H) - \gamma \left[(1-k)(\beta_i^2 \text{cov} - \beta_i V_H) + \beta_i k_i \tilde{V} \right] = \gamma(\beta_i^2 V_F - \beta_i \text{cov}) \tilde{\beta} k$$

Sum across i , we have:

$$\begin{aligned}
&\tilde{\beta}^2 E(R_F) - \beta E(R_H) - \gamma \left[(1-k) \left(\tilde{\beta}^2 \text{cov} - \beta V_H \right) + \tilde{\beta} k \tilde{V} \right] = \gamma \left(\tilde{\beta}^2 V_F - \beta \text{cov} \right) \tilde{\beta} k \\
&\Rightarrow \tilde{\beta}^2 E(R_F) - \beta E(R_H) - \gamma \left[\beta(k-1)V_H + \tilde{\beta}^2(1-k)\text{cov} \right] = \gamma \left(\tilde{\beta}^2 V_F - \beta \text{cov} + \tilde{V} \right) \tilde{\beta} k
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \tilde{\beta}^2 E(R_F) - \beta E(R_H) - \gamma \left[\beta(k-1)V_H + \tilde{\beta}^2(1-k)\text{cov} \right] \\
&= \gamma \left(\tilde{\beta}^2 V_F - \beta \text{cov} + \tilde{V} \right) \frac{\beta E(R_F) - nE(R_H) - \gamma \left[(1-k)(\beta \text{cov} - nV_H) + k\tilde{V} \right]}{\gamma(\beta V_F - n\text{cov})}
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \left[\tilde{\beta}^2 E(R_F) - \beta E(R_H) \right] (\beta V_F - n\text{cov}) - \gamma \left[\beta(k-1)V_H + \tilde{\beta}^2(1-k)\text{cov} \right] (\beta V_F - n\text{cov}) \\
&= \left(\tilde{\beta}^2 V_F - \beta \text{cov} + \tilde{V} \right) \left\{ \beta E(R_F) - nE(R_H) - \gamma \left[(1-k)(\beta \text{cov} - nV_H) + k\tilde{V} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow -\widetilde{\beta}^2 E(R_F) ncov - \beta^2 E(R_H) V_F - \gamma\beta^2 (k-1) V_H V_F + \gamma\widetilde{\beta}^2 (1-k) ncov^2 \\
&= -\beta^2 E(R_F) cov - n\widetilde{\beta}^2 E(R_H) V_F - \gamma n\widetilde{\beta}^2 (k-1) V_H V_F + \gamma\beta^2 (1-k) cov^2 \\
&+ [\beta E(R_F) - nE(R_H)] \widetilde{V} - \gamma k \widetilde{V} \left(\widetilde{\beta}^2 V_F - \beta cov + \widetilde{V} \right) - \gamma [n(k-1) V_H + \beta(1-k) cov] \widetilde{V}
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow [E(R_F) cov - E(R_H) V_F - \gamma(k-1) V_H V_F - \gamma(1-k) cov^2] \left(\beta^2 - n\widetilde{\beta}^2 \right) \\
&= [\beta E(R_F) - nE(R_H)] \widetilde{V} - \gamma \widetilde{V} \left[k\widetilde{\beta}^2 V_F + k\widetilde{V} + n(k-1) V_H + \beta(1-2k) cov \right]
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow [E(R_F) cov - E(R_H) V_F + \gamma V_H V_F - \gamma cov^2] \left(\beta^2 - n\widetilde{\beta}^2 \right) + \gamma k (cov^2 - V_H V_F) \left(\beta^2 - n\widetilde{\beta}^2 \right) \\
&= [\beta E(R_F) - nE(R_H)] \widetilde{V} - \gamma \widetilde{V} k \left[\widetilde{\beta}^2 V_F + \widetilde{V} + nV_H - 2\beta cov \right] + \gamma \widetilde{V} (nV_H - \beta cov)
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \gamma k \left[(cov^2 - V_H V_F) \left(\beta^2 - n\widetilde{\beta}^2 \right) + \widetilde{V} \left(\widetilde{\beta}^2 V_F + \widetilde{V} + nV_H - 2\beta cov \right) \right] = [\beta E(R_F) - nE(R_H)] \widetilde{V} \\
&\quad + \gamma \widetilde{V} (nV_H - \beta cov) - [E(R_F) cov - E(R_H) V_F + \gamma V_H V_F - \gamma cov^2] \left(\beta^2 - n\widetilde{\beta}^2 \right) \quad (49)
\end{aligned}$$

Solution for the case of zero expected returns

If $E(R_H) = E(R_F) = 0$

- Substituting $E(R_H) = E(R_F) = 0$ into Eq. (48)

$$\Rightarrow k_i = \frac{-(1-k)(\beta_i cov - V_H)}{\widetilde{V}} + \frac{(1-k)(\beta cov - nV_H) + k\widetilde{V}(\beta_i V_F - cov)}{\widetilde{V}(\beta V_F - ncov)} \quad (50)$$

- Substituting $E(R_H) = E(R_F) = 0$ into Eq. (49)

$$\begin{aligned}
&\Rightarrow k \left[(cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2) + \widetilde{V} (\widetilde{\beta}^2 V_F + \widetilde{V} + nV_H - 2\beta cov) \right] \\
&= \widetilde{V} (nV_H - \beta cov) - [V_H V_F - cov^2] (\beta^2 - n\widetilde{\beta}^2)
\end{aligned}$$

The gross share of foreign assets is:

$$\Rightarrow k = \frac{\widetilde{V} (nV_H - \beta cov) + (cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2)}{(cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2) + \widetilde{V} (\widetilde{\beta}^2 V_F + \widetilde{V} + nV_H - 2\beta cov)} = \frac{M}{N} \quad (51)$$

$$\Rightarrow 1 - k = \frac{\widetilde{V} (\widetilde{\beta}^2 V_F + \widetilde{V} - \beta cov)}{(cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2) + \widetilde{V} (\widetilde{\beta}^2 V_F + \widetilde{V} + nV_H - 2\beta cov)} \quad (52)$$

Combining Eq. (50) and Eq. (51), we have:

$$\begin{aligned}
k_i &= - \frac{(\widetilde{\beta}^2 V_F + \widetilde{V} - \beta cov) (\beta_i cov - V_H)}{(cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2) + \widetilde{V} (\widetilde{\beta}^2 V_F + \widetilde{V} + nV_H - 2\beta cov)} \\
&+ \frac{(\widetilde{\beta}^2 V_F + \widetilde{V} - \beta cov) (\beta cov - nV_H) + \widetilde{V} (nV_H - \beta cov) + (cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2)}{(cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2) + \widetilde{V} (\widetilde{\beta}^2 V_F + \widetilde{V} + nV_H - 2\beta cov)} \frac{(\beta_i V_F - cov)}{(\beta V_F - ncov)} \\
&= - \frac{(\widetilde{\beta}^2 V_F + \widetilde{V} - \beta cov) (\beta_i cov - V_H)}{Y} + \frac{X (\beta_i V_F - cov)}{Y (\beta V_F - ncov)}
\end{aligned}$$

$$X = (\widetilde{\beta}^2 V_F + \widetilde{V} - \beta cov) (\beta cov - nV_H) + \widetilde{V} (nV_H - \beta cov) + (cov^2 - V_H V_F) (\beta^2 - n\widetilde{\beta}^2)$$

$$\begin{aligned}
\Rightarrow X &= \widetilde{\beta}^2 V_F \beta cov + \widetilde{V} \beta cov - \beta cov \beta cov - \widetilde{\beta}^2 V_F nV_H - \widetilde{V} nV_H + \beta cov nV_H \\
&+ n\widetilde{V} V_H - \widetilde{V} \beta cov + cov^2 \beta^2 - V_H V_F \beta^2 - cov^2 n\widetilde{\beta}^2 + V_H V_F n\widetilde{\beta}^2
\end{aligned}$$

$$\Rightarrow X = \widetilde{\beta}^2 V_F \beta cov + \beta cov nV_H - cov^2 n\widetilde{\beta}^2 - V_H V_F \beta^2 = (V_F \beta - ncov) (\widetilde{\beta}^2 cov - V_H \beta) \quad (53)$$

$$\begin{aligned}
\Rightarrow k_i &= -\frac{\left(\widetilde{\beta}^2 V_F + \widetilde{V} - \beta \text{cov}\right) (\beta_i \text{cov} - V_H)}{\left(\text{cov}^2 - V_H V_F\right) \left(\beta^2 - n \widetilde{\beta}^2\right) + \widetilde{V} \left(\widetilde{\beta}^2 V_F + \widetilde{V} + n V_H - 2 \beta \text{cov}\right)} \\
&+ \frac{\left(V_F \beta - n \text{cov}\right) \left(\widetilde{\beta}^2 \text{cov} - V_H \beta\right)}{\left(\text{cov}^2 - V_H V_F\right) \left(\beta^2 - n \widetilde{\beta}^2\right) + \widetilde{V} \left(\widetilde{\beta}^2 V_F + \widetilde{V} + n V_H - 2 \beta \text{cov}\right)} \frac{\left(\beta_i V_F - \text{cov}\right)}{\left(\beta V_F - n \text{cov}\right)} \\
k_i &= \frac{-\left(\widetilde{\beta}^2 V_F + \widetilde{V} - \beta \text{cov}\right) (\beta_i \text{cov} - V_H) + \left(\widetilde{\beta}^2 \text{cov} - V_H \beta\right) (\beta_i V_F - \text{cov})}{\left(\text{cov}^2 - V_H V_F\right) \left(\beta^2 - n \widetilde{\beta}^2\right) + \widetilde{V} \left(\widetilde{\beta}^2 V_F + \widetilde{V} + n V_H - 2 \beta \text{cov}\right)}
\end{aligned}$$

$$\begin{aligned}
\text{Numerator} &= -\widetilde{\beta}^2 V_F \beta_i \text{cov} - \widetilde{V} \beta_i \text{cov} + \beta \text{cov} \beta_i \text{cov} + \widetilde{\beta}^2 V_F V_H + \widetilde{V} V_H - \beta \text{cov} V_H \\
&+ \widetilde{\beta}^2 \text{cov} \beta_i V_F - V_H \beta \beta_i V_F - \widetilde{\beta}^2 \text{cov} \text{cov} + V_H \beta \text{cov} \\
&= \widetilde{\beta}^2 V_F V_H - \widetilde{V} \beta_i \text{cov} + \widetilde{V} V_H + \beta \text{cov} \beta_i \text{cov} - \widetilde{\beta}^2 \text{cov}^2 - V_H \beta \beta_i V_F \\
&= \widetilde{\beta}^2 (V_F V_H - \text{cov}^2) + \widetilde{V} V_H - \beta_i \left(\widetilde{V} \text{cov} - \beta \text{cov}^2 + \beta V_F V_H\right) \\
&= \left(\widetilde{\beta}^2 - \beta \beta_i\right) (V_F V_H - \text{cov}^2) + \widetilde{V} (V_H - \beta_i \text{cov})
\end{aligned}$$

$$\Rightarrow k_i = \frac{\left(\widetilde{\beta}^2 - \beta \beta_i\right) (V_F V_H - \text{cov}^2) + \widetilde{V} (V_H - \beta_i \text{cov})}{\left(\text{cov}^2 - V_H V_F\right) \left(\beta^2 - n \widetilde{\beta}^2\right) + \widetilde{V} \left(\widetilde{\beta}^2 V_F + \widetilde{V} + n V_H - 2 \beta \text{cov}\right)} = \frac{\bar{M}}{N} \quad (54)$$

The sign of N

$$N = (V_H V_F - \text{cov}^2) \left(n \widetilde{\beta}^2 - \beta^2\right) + \widetilde{V} \left(\widetilde{\beta}^2 V_F + \widetilde{V} + n V_H - 2 \beta \text{cov}\right) > 0, \text{ because:}$$

$$(V_H V_F - \text{cov}^2) = \det(\Sigma) > 0$$

$$\left(n \widetilde{\beta}^2 - \beta^2\right) = n \sum_{i=1}^n \beta_i^2 - \left(\sum_{i=1}^n \beta_i\right)^2 = (n-1) \sum_{i=1}^n \beta_i^2 - 2 \sum_{i \neq j} \beta_i \beta_j = \sum_{i \neq j} (\beta_i - \beta_j)^2 > 0$$

$$\left(\widetilde{\beta}^2 V_F + \widetilde{V} + n V_H - 2 \beta \text{cov}\right) = \widetilde{V} + \sum_{i=1}^n V (r_H - r_{F_i}) > 0$$

Taylor expansion

Taking Taylor expansion around trend value of Eq. (54), we have:

$$\begin{aligned}
\frac{\partial k_i}{\partial V_H} \Big|_{\bar{b}ar} &= \frac{[\tilde{V} + (\widetilde{\beta^2} - \beta\beta_i) \bar{V}_F] N - [-(\beta^2 - n\widetilde{\beta^2}) \bar{V}_F + \tilde{V}n] \bar{M}}{N^2} \\
&= \frac{1}{N} [\tilde{V} + (\widetilde{\beta^2} - \beta\beta_i) \bar{V}_F - [-(\beta^2 - n\widetilde{\beta^2}) \bar{V}_F + \tilde{V}n] \bar{k}_i] \quad (55)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k_i}{\partial V_F} \Big|_{\bar{b}ar} &= \frac{(\widetilde{\beta^2} - \beta\beta_i) \bar{V}_H N - [-\bar{V}_H (\beta^2 - n\widetilde{\beta^2}) \bar{V}_F + \tilde{V} \widetilde{\beta^2}] \bar{M}}{N^2} \\
&= \frac{1}{N} [(\widetilde{\beta^2} - \beta\beta_i) \bar{V}_H + [\bar{V}_H (\beta^2 - n\widetilde{\beta^2}) \bar{V}_F - \tilde{V} \widetilde{\beta^2}] \bar{k}_i] \quad (56)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k_i}{\partial cov} \Big|_{\bar{b}ar} &= \frac{[-2c\bar{ov} \widetilde{\beta^2} - \beta_i (\tilde{V} - 2\beta c\bar{ov})] N - [2c\bar{ov} (\beta^2 - n\widetilde{\beta^2}) - 2\beta \tilde{V}] \bar{M}}{N^2} \\
&= \frac{1}{N} [2c\bar{ov} (\beta\beta_i - \widetilde{\beta^2}) - \tilde{V} \beta_i - [2c\bar{ov} (\beta^2 - n\widetilde{\beta^2}) - \beta \tilde{V}] \bar{k}_i] \quad (57)
\end{aligned}$$

Taking Taylor expansion around trend value of Eq. (51), we have:

$$\begin{aligned}
\frac{\partial k}{\partial V_H} \Big|_{\bar{b}ar} &= \frac{[\tilde{V}n - (\beta^2 - n\widetilde{\beta^2}) \bar{V}_F] N - [\tilde{V}n - (\beta^2 - n\widetilde{\beta^2}) \bar{V}_F] M}{N^2} \\
&= \frac{1}{N} [\tilde{V}n - (\beta^2 - n\widetilde{\beta^2}) \bar{V}_F] (1 - \bar{k}) \quad (58)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k}{\partial V_F} \Big|_{\bar{b}ar} &= \frac{[-(\beta^2 - n\widetilde{\beta^2}) \bar{V}_H] N - [-\bar{V}_H (\beta^2 - n\widetilde{\beta^2}) + \tilde{V} \widetilde{\beta^2}] M}{N^2} \\
&= \frac{1}{N} [-(\beta^2 - n\widetilde{\beta^2}) \bar{V}_H + [\bar{V}_H (\beta^2 - n\widetilde{\beta^2}) - \tilde{V} \widetilde{\beta^2}] \bar{k}] \quad (59)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k}{\partial cov} \Big|_{\bar{b}ar} &= \frac{[-\beta \tilde{V} + 2c\bar{ov} (\beta^2 - n\widetilde{\beta^2})] N - [2c\bar{ov} (\beta^2 - n\widetilde{\beta^2}) - 2\beta \tilde{V}] M}{N^2} \\
&= \frac{1}{N} [2c\bar{ov} (\beta^2 - n\widetilde{\beta^2}) (1 - \bar{k}) - \beta \tilde{V} (1 - 2\bar{k})] \quad (60)
\end{aligned}$$

E 1H-NF Model - with information - 1 prior

Solution for the individual foreign asset holding

Recall from Eq. (54) that

$$k_i = \frac{\left(\widetilde{\beta}^2 - \beta\beta_i\right) (V_F V_H - cov^2) + \tilde{V} (V_H - \beta_i cov)}{(cov^2 - V_H V_F) \left(\beta^2 - n\widetilde{\beta}^2\right) + \tilde{V} \left(\widetilde{\beta}^2 V_F + \tilde{V} + nV_H - 2\beta cov\right)}$$

Combining this with the conditional moments defined in Eq. (25) we have:

$$k_i = \frac{\left(\beta\beta_i - \widetilde{\beta}^2\right) \{cov^2 V_S^2 - V_S V_H [V_F (V_S + V_H) - cov^2]\} + \tilde{V} V_S (V_H - \beta_i cov) (V_S + V_H)}{\left(\beta^2 - n\widetilde{\beta}^2\right) \{cov^2 V_S^2 - V_S V_H [V_F (V_S + V_H) - cov^2]\} + \tilde{V} (V_S + V_H) \left\{\widetilde{\beta}^2 [V_F (V_S + V_H) - cov^2] + \tilde{V} (V_S + V_H) + nV_H V_S - 2\beta cov V_S\right\}}$$

$$k_i = \frac{\left(\beta\beta_i - \widetilde{\beta}^2\right) [cov^2 V_S - V_S V_H V_F] - \tilde{V} V_S (\beta_i cov - V_H)}{\left(\beta^2 - n\widetilde{\beta}^2\right) (cov^2 V_S - V_S V_H V_F) + \tilde{V} \left\{\widetilde{\beta}^2 [V_F (V_S + V_H) - cov^2] + \tilde{V} (V_S + V_H) + nV_H V_S - 2\beta cov V_S\right\}} \quad (61)$$

$$\frac{\partial k_i}{\partial V_H} \Big|_{\bar{a}} = \frac{\left[\tilde{V} V_S - \left(\beta\beta_i - \widetilde{\beta}^2\right) \bar{V}_S \bar{V}_F\right] \bar{Q} - \left[-\bar{V}_S \bar{V}_F \left(\beta^2 - n\widetilde{\beta}^2\right) + \tilde{V} \left(\widetilde{\beta}^2 \bar{V}_F + n\bar{V}_S + \tilde{V}\right)\right] \bar{P}}{D^2}$$

$$= \frac{\tilde{V} \bar{V}_S - \left(\beta\beta_i - \widetilde{\beta}^2\right) \bar{V}_F \bar{V}_S + \left[\bar{V}_S \bar{V}_F \left(\beta^2 - n\widetilde{\beta}^2\right) - \tilde{V} \left(\widetilde{\beta}^2 \bar{V}_F + n\bar{V}_S + \tilde{V}\right)\right] \bar{k}_i}{Q} \quad (62)$$

$$\frac{\partial k_i}{\partial V_F} \Big|_{\bar{a}} = \frac{-\left(\beta\beta_i - \widetilde{\beta}^2\right) \bar{V}_S \bar{V}_H \bar{Q} - \left[-\bar{V}_S \bar{V}_H \left(\beta^2 - n\widetilde{\beta}^2\right) + \tilde{V} \widetilde{\beta}^2 (\bar{V}_H + \bar{V}_S)\right] \bar{P}}{D^2}$$

$$= \frac{-\left(\beta\beta_i - \widetilde{\beta}^2\right) \bar{V}_S \bar{V}_H + \left[\bar{V}_S \bar{V}_H \left(\beta^2 - n\widetilde{\beta}^2\right) - \tilde{V} \widetilde{\beta}^2 (\bar{V}_H + \bar{V}_S)\right] \bar{k}_i}{Q} \quad (63)$$

$$\begin{aligned}
\frac{\partial k_i}{\partial cov} \Big|_{\bar{a}r} &= \frac{\left[2 \left(\beta \beta_i - \widetilde{\beta}^2 \right) cov \bar{V}_S - \tilde{V} \beta_i \bar{V}_S \right] Q - \left[2 cov \bar{V}_S \left(\beta^2 - n \widetilde{\beta}^2 \right) - 2 \tilde{V} \left(\widetilde{\beta}^2 cov + \beta \bar{V}_S \right) \right] \bar{P}}{Q^2} \\
&= \frac{2 \left(\beta \beta_i - \widetilde{\beta}^2 \right) cov \bar{V}_S - \tilde{V} \beta_i \bar{V}_S - 2 \left[cov \bar{V}_S \left(\beta^2 - n \widetilde{\beta}^2 \right) - \tilde{V} \left(\widetilde{\beta}^2 cov + \beta \bar{V}_S \right) \right] \bar{k}_i}{Q} \quad (64)
\end{aligned}$$

Solution for the gross foreign holding

Recall from Eq. (51) that:

$$k = \frac{\tilde{V} (nV_H - \beta cov) + (cov^2 - V_H V_F) \left(\beta^2 - n \widetilde{\beta}^2 \right)}{(cov^2 - V_H V_F) \left(\beta^2 - n \widetilde{\beta}^2 \right) + \tilde{V} \left(\widetilde{\beta}^2 V_F + \tilde{V} + nV_H - 2\beta cov \right)}$$

Combining this with the conditional moments defined in Eq. (25) we have:

$$\begin{aligned}
k &= \frac{V_S \left[\tilde{V} (nV_H - \beta cov) + (cov^2 - V_H V_F) \left(\beta^2 - n \widetilde{\beta}^2 \right) \right]}{V_S (cov^2 - V_H V_F) \left(\beta^2 - n \widetilde{\beta}^2 \right) + \tilde{V} \left[\widetilde{\beta}^2 (V_F (V_S + V_H) - cov^2) + \tilde{V} (V_S + V_H) + nV_S V_H - 2\beta cov V_S \right]} \\
&= \frac{P}{\bar{Q}} \quad (65)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k}{\partial V_H} \Big|_{\bar{a}r} &= \frac{\left[\bar{V}_S \tilde{V} n - \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_S \bar{V}_F \right] Q - \left[\tilde{V} \left(\widetilde{\beta}^2 \bar{V}_F + n \bar{V}_S + \tilde{V} \right) - \bar{V}_S \bar{V}_F \left(\beta^2 - n \widetilde{\beta}^2 \right) \right] P}{Q^2} \\
&= \frac{\left[\bar{V}_S \tilde{V} n - \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_S \bar{V}_F \right] - \left[\tilde{V} \left(\widetilde{\beta}^2 \bar{V}_F + n \bar{V}_S + \tilde{V} \right) - \bar{V}_S \bar{V}_F \left(\beta^2 - n \widetilde{\beta}^2 \right) \right] \bar{k}}{Q} \quad (66)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k}{\partial V_F} \Big|_{\bar{a}r} &= \frac{- \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_H \bar{V}_S Q - \left[- \bar{V}_S \bar{V}_H \left(\beta^2 - n \widetilde{\beta}^2 \right) + \tilde{V} \widetilde{\beta}^2 \left(\bar{V}_S + \bar{V}_H \right) \right] P}{Q^2} \\
&= \frac{- \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_H \bar{V}_S + \left[\bar{V}_S \bar{V}_H \left(\beta^2 - n \widetilde{\beta}^2 \right) - \tilde{V} \widetilde{\beta}^2 \left(\bar{V}_S + \bar{V}_H \right) \right] \bar{k}}{Q} \quad (67)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k}{\partial cov} \Big|_{\bar{a}r} &= \frac{2 cov \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_S Q - \tilde{V} \beta \bar{V}_S Q - 2 \left[cov \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_S - \tilde{V} \left(\widetilde{\beta}^2 cov + \beta \bar{V}_S \right) \right] P}{Q^2} \\
&= \frac{2 cov \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_S - \tilde{V} \beta \bar{V}_S - 2 \left[cov \left(\beta^2 - n \widetilde{\beta}^2 \right) \bar{V}_S - \tilde{V} \left(\widetilde{\beta}^2 cov + \beta \bar{V}_S \right) \right] \bar{k}}{Q} \quad (68)
\end{aligned}$$

From Eq. (65), we have:

$$\begin{aligned}
& V_S \left[\tilde{V} (nV_H - \beta cov) + (cov^2 - V_H V_F) (\beta^2 - n\tilde{\beta}^2) \right] \\
= & kV_S \left\{ (cov^2 - V_H V_F) (\beta^2 - n\tilde{\beta}^2) + \tilde{V} \left[\tilde{\beta}^2 V_F + \tilde{V} + nV_H - 2\beta cov \right] \right\} + k\tilde{V} \left[\tilde{\beta}^2 (V_F V_H - cov^2 + \tilde{V} V_H) \right] \\
\Rightarrow V_S = & \frac{k\tilde{V} \left[\tilde{\beta}^2 (V_F V_H - cov^2) + \tilde{V} V_H \right]}{\left\{ (cov^2 - V_H V_F) (\beta^2 - n\tilde{\beta}^2) + \tilde{V} (nV_H - \beta cov) \right\} (1 - k) - k\tilde{V} (\tilde{\beta}^2 V_F + \tilde{V} - \beta cov)} \quad (69)
\end{aligned}$$

The trend value of Foreign position

The average Foreign position is higher under 1H-NF than that under 1H-1F model. Algebraically, this can be understood as follows.

The estimated values of $V_H=.0051$, $V_F=.0058$, $cov=.0038$, and estimated betas are usually in the range of .95 to 1.1. Given these, when one more Foreign asset is included in the portfolio, both M and N increase because generally $V_H > \beta_i cov$ and $\tilde{\beta}^2 V_F > \beta cov$ ($\beta_i^2 V_F > \beta_i cov$). Recall from Eq. (51) :

$$k = \frac{\tilde{V} (nV_H - \beta cov) + (cov^2 - V_H V_F) (\beta^2 - n\tilde{\beta}^2)}{(cov^2 - V_H V_F) (\beta^2 - n\tilde{\beta}^2) + \tilde{V} (\tilde{\beta}^2 V_F + \tilde{V} + nV_H - 2\beta cov)} = \frac{M_{1,n} + \tilde{V} M_{2,n}}{M_{1,n} + \tilde{V} M_{2,n} + \tilde{V} M_{3,n} + \tilde{V}^2}$$

As n increases, $M_{i,n}$ & $N_{j,n}$ ($\forall i, j$) increase. However, because $\beta_i < 1$, $M_{2,n}$ will dominate $M_{3,n}$:

$$M_{2,k+1} = M_{2,k} + V_H - \beta_{k+1} cov$$

$$M_{3,k+1} = M_{3,k} + V_H - \beta_{k+1} cov$$

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