Substitutability among Federal Income Tax Deductions:

Implications for Optimal Tax Policy

Tianying He

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Abstract. This paper studies households making joint decisions about two tax-deductible activities: charitable giving and paying interest on home equity lines of credit (HELOC). By allowing for interactions, either substitutability or complementarity, between deductions, I provide a fuller understanding of the elasticity of taxable income (ETI) and the tax-price elasticity of charitable giving. The tax-price of paying HELOC interest depends on the tax schedule as well as current interest rates. Therefore, the two activities have different tax-prices and the identification of their interaction is possible. Using the Survey of Consumer Finances, I find that the level of each activity falls with its own tax-price and rises with the other activity’s tax-price. I show that in theory the ETI is a weighted sum of such own and cross tax-price effects between activities. I then apply this theory and my estimates to illustrate the trade-offs of three base-broadening tax policies. A conventional policy which removes a subset of deductions while keeping other deductions intact cannot guarantee either more tax revenue or greater efficiency. In contrast, an optimal policy would lower all deductions by differing degrees. However, designing an optimal policy requires knowing the values of all own and cross tax-price elasticities between deductions. Instead, I recommend a second-best policy of “uniform partial deductibility” which lowers all deductions to the same degree; it can guarantee improvement without prior knowledge about elasticities between deductions.

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I. Introduction

The elasticity of taxable income (ETI), which shows how taxable income changes in response to the marginal tax rate, is a key parameter for tax analysis for two reasons. First, the ETI indicates whether cutting tax rates will decrease or increase tax revenue. This relationship is sometimes depicted through the well-known “Laffer Curve”. Second, the ETI characterizes the dead-weight loss of taxation; under certain assumptions, the ETI is a sufficient statistic to estimate the efficiency costs of taxation (Feldstein 1999; Saez, Slemrod and Giertz 2012) and plays an important role in optimal tax theory (Diamond and Saez 2011).

Researchers have estimated the ETI, defined with respect to the net of tax rate $1-\tau$, with $\tau$ being the marginal income tax rate, to be significantly above zero and perhaps even above one (Feldstein 1999; Gruber and Saez 2002). Kopczuk (2005) argues that the ETI is not a constant, immutable parameter, however, meaning that it varies with individuals’ access to tax-deductible items and can be altered by tax policies such as policies on deductions.

On the one hand, the ETI literature examines the overall responsiveness of taxable income to tax rate changes. On the other hand, many studies have considered isolated components of taxable income, for example, by estimating the tax-price elasticity of charitable giving or the labor supply elasticity with respect to the tax rate. Arguably, however, there are important interactions between the different components of taxable income. For example, if the home mortgage interest deduction becomes more generous, a household may take out a larger mortgage to finance a more expensive house. As a result, their property tax deduction will also become larger.\footnote{Normally itemized deductions reduce taxable income dollar for dollar (for households who itemize). The “Pease Limitation” complicates this, but does not change the way of interaction between different itemized deductions per se. This rule limits the amount of itemized deductions for households with Adjusted Gross Income over a threshold (e.g. $128,950 in 2000 for married households); for more details about the rule see, for example, 2000 Tax Form 1040 Schedule A’s Line 28 and the accompanied Form 1040 Instructions. This rule alters “tax-prices”, but not the way how “tax-prices” affect deduction amounts. I account for this rule in my empirical work.}

In this paper I consider the implication of allowing the decisions on different deductions to interact. That is, I am interested in not only own tax-price elasticities but also the cross price elasticities of deductions with respect to the tax-price of other deductions. This allows for either substitutability or complementarity between deductions and reveals whether households make
Joint decisions on different tax expenditures. In addition, examining the cross price elasticities between deductions is important for two other reasons.

First, it deepens our understanding of the ETI. In Section VI and Appendix 1 I derive the mathematical relationship between the ETI and the underlying cross price elasticities. As these cross price elasticities correspond to different components of taxable income and their interactions, tax systems that define taxable income differently will generally have different ETIs. This reasoning is similar to Kopczuk (2005). Additionally, these elasticities can cause bias in estimating the ETI for one particular tax system using observed changes in taxable income caused by a tax reform. As Slemrod (1998) explains, if a tax reform removes deductions while changing rates, the ETI estimate could bias from the true values of both the pre-reform ETI and the after reform ETI. This is because, during a tax reform that lowers tax rates and removes deductions at the same time, households change their taxable incomes responding to both the rate change and the deduction change. Thus, the rate change’s effect on taxable income (i.e. the ETI) will be confounded by the deduction change, if the latter is not accounted for. In Appendix 2 I quantify this bias in terms of cross price elasticities and also graphically illustrate possible directions of the bias.

Second, it has important implications for the design of tax reform. Broadening the tax base, by removing all deductions for example, reduces the deadweight loss per dollar of collected revenue (Kopczuk 2005). However, in light of the cross price elasticities between deductions, not all base broadening methods are equal. For example, ceteris paribus, removing the deductibility of a consumption item that is a complement of (i.e. having negative cross price elasticities with) other deductible items will broaden the tax base more than if it is a substitute. The reasoning is as follows. Both tax reform options will broaden the tax base directly by removing a deduction, but indirectly the former will cause the remaining deductions to shrink (adding to the direct broadening effect) while the latter will cause the remaining deductions to expand (offsetting the

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2 I view the substitutability or complementarity as arising mainly from the fact that different deductions encourage different types of consumptions that enter the utility function, for example charitable giving and housing. In addition, certain deductions may have very obvious substitutability from an accounting perspective. For example, the deductibility for personal interest is gradually phased out following TRA 86. In this process, households may shift away from personal loans to home equity loans, with the latter’s interest always deductible.

3 Kopczuk (2005) provides a theoretical relationship between the so called “broad income” (sum of before-tax labor income and many other types of incomes) elasticity and deductions, but not a theoretical relationship between the ETI and deductions. In his empirical work, he examines both the broad income elasticity and the ETI.
direct broadening effect). If cross price elasticities are unknown, I demonstrate that a second best policy of lowering all deductions together to the same extent – which will maintain their relative tax-prices and avoid triggering any substitution between them – broadens the base as well as reduces dead weight loss. I discuss this in Section VI.

In addition to examining the theoretical relationships, I empirically demonstrate the importance of the cross price elasticities between deductions. To do so, I use data from the Survey of Consumer Finances to examine the cross price elasticities of two tax-deductible activities, charitable giving and paying interest on home equity lines of credit (HELOC). To my knowledge, this is the first empirical paper that considers interactions across tax deductible items.

I pick these two components of taxable income for the following reasons. First, out of the many components of taxable income, I study two deductions rather than two types of income because I suspect that the responsiveness of taxable income may arise largely from deductions. In fact, the literature finds that overall labor supply responds little to tax incentives (Saez, Slemrod and Giertz 2012; Gruber and Saez 2002) while the ETI is typically estimated to be much higher than labor supply elasticities (Feldstein 1999; Gruber and Saez 2002). In contrast, many studies have found that charitable giving is responsive to tax rates (Peloza and Steel 2005). The dollar amount of deductions is also large, so their responsiveness to tax rate changes influences the overall responsiveness of taxable income. In 2011 the total amount of itemized deductions was over $1.2 trillion. The sum of itemized deductions and standard deductions amounted to 34.6% of taxable income. Second, home collateral borrowing and charitable giving are important deductions. In 2012, total giving by individuals in the US was $316.23 billion, with $199.27 billion claimed as itemized deductions (the amount subtracted from taxable income), and the total amount of the home mortgage interest claimed was $332.61 billion (Giving USA 2013; Internal Revenue Service 2014).

Third, among forms of borrowing on home collateral, HELOC repayments are flexible: after entering a HELOC agreement which determines the credit limit (the maximum amount allowed to be borrowed), the borrower is often able to borrow up to the credit limit whenever he or she

4 Other large tax expenditures are state and local taxes ($251.66 billion) and real estate taxes ($167.78 billion).
wants (FRB 2012) and faces a flexible payback schedule. Thus, I treat the payback decision as annual. This makes static modeling feasible.

Fourth, for these two deductions the identification of cross-price elasticities is possible. Many tax-deductible items face the same tax-price, \(1 - \tau\). In order to separately identify the own-price effect and the cross-price effect of the change in \(1 - \tau\), I rely on the fact that the price of a home equity loan depends not only on \(1 - \tau\) but also on \(r\), the interest rate, which for HELOCs is often variable. The cross-price effect of the home equity line price on giving is captured when \(r\) changes but \(1 - \tau\) remains unchanged. Conversely, the cross-price effect of the giving price on home equity debt is captured by comparing households with the same price of home equity line but different net-of-tax rates. As will be elaborated in Sections III and V, the HELOC tax-price will be endogenous to the amount of repayment, so I will employ an instrument and run 3SLS regressions.

I use data from the Surveys of Consumer Finances (SCF). The SCF has detailed information on demographics, income, wealth, and education, and covers a long time period with several federal tax changes (see Appendix 3). I find evidence that each of the two activities, giving to charities and repaying HELOCs, responds to their own and to cross tax-prices. I examine two specifications, one with marginal tax-prices and the other with average tax-prices. I do the latter because Liebman and Zeckhause (2004) develops theory about and finds evidence that people may respond to average prices rather than marginal prices. My two specifications produce qualitatively similar results although the average price results are more precise. I find that each activity is reduced by an increase in its own price or a decrease in the other activity’s price. Under the average tax-price specification, the estimated own price elasticity and cross price elasticity of giving are, respectively, -2.83 and 3.90, with p-values of 15.2\% and 8.9\%. The own and cross price elasticities for HELOC are -5.38 and 1.49, with p-values of 0.0\% and 4.6\%. Not all these estimates are significant at conventional levels. It is difficult to find a pair of deductions

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5 The Survey of Consumer Finances data that I use confirm this flexibility. While the survey usually asks for whether there is a fixed repayment amount per month on other types of loans, for HELOCs the survey simply does not ask this question (instead, only “typical payment” and frequency is asked).
6 The SCF surveys ask households’ ages, years of education, and the amounts of different kinds of assets and liabilities. It is not possible to obtain these information from administrative tax data. The main drawback with SCF is not observing the marginal tax rate and itemized deductions without error. I discuss how I impute them in Section IV.
7 The Slutsky symmetry indicates that the two cross price elasticities should have the same sign, as they are here.
with different but measurable tax-prices. Studying those with HELOC deduction leaves me with a relatively small sample and results in relatively large standard errors.

The rest of this paper is divided into six sections. Section II reviews the literature. Section III constructs the tax-prices. Section IV describes the data. Section V gives the estimation results. In Section VI, I apply my estimates to the theory of ETI, develop new theoretical results, and discuss policy implications. Section VII concludes.

II. Literature Review

I examine the responsiveness of charitable giving and HELOC payments to their own and each other’s tax-prices. There is no relevant literature on the sensitivity of HELOC payments to tax rates, but a large literature on the tax-price elasticity of charitable giving including Bakija and Heim (2011), Auten, Sieg and Clotfelter (2002), Reece and Zieschang (1985), and Friedberg and He (2015). These studies mostly find the tax-price elasticity of charitable giving to be around -1. This literature shares the following four characteristics, which I adapt to my research purpose.

First, the tax-price of giving is defined as the amount of personal income foregone for each dollar given to charity, or 1 minus the marginal tax rate applied to taxable income at zero giving. In this paper, I similarly define the tax-price of giving, while the tax-price for HELOC payments will be more involved. Second, I follow most studies in using a log-linear specification that regresses Log (giving amount) on Log (tax-price) together with other covariates. The coefficient for Log (tax-price) is thus the tax-price elasticity. Third, most studies except Friedberg and He (2015) restrict the sample to “exogenous itemizers”, defined as taxpayers who have more non-charity deductions than their standard deduction, so that any giving on top of other deductions reduces taxable income and forgoes only $(1-\tau)$ of after-tax consumption for each additional dollar of giving. I similarly restrict my analysis to taxpayers who have more non-charity and non-HELOC-interest-payments deductions than their standard deduction.

Lastly, the literature on charitable deductions has treated all non-charity itemized deductions as exogenous. I move a step away from this by focusing on joint decisions about charity and HELOC repayment, and treat non-charity, non-HELOC deductions as exogenous. As in the charitable giving literature, I assume that the amount of giving and amount of HELOC
repayment are not otherwise correlated with their tax-prices because of unobserved preferences. I discuss this in more details later.

My research is also relevant to the ETI literature. Gruber and Saez (2002) find the overall ETI, defined as the elasticity of taxable income with respect to the net of tax rate \( 1-\tau \), to be 0.4, implying a revenue maximizing tax rate of 71%. Kopczuk (2005) finds that the ETI increases with individuals' access to deductions. For example, a tax system that allows more deductions will yield a higher ETI; in the cross-section, a household spending a larger proportion of income on tax-deductible commodities has a higher ETI. In Section VI and Appendix 2 I derive the precise relation between ETI and deductions, taking into account interactions among deductions. It shows that the ETI decreases in the tax-price elasticity of the deductible item and increases in the proportions of income spent on the deductions. Using the formula, in Section VI I show that, due to the possible substitutability between deductions, removing a subset of deductions does not necessarily increases revenue.

Appendices 1 and 2 provide a theoretical analysis of the ETI and review the literature on the ETI, labor supply elasticity, charitable giving elasticity, and “broad income” elasticity. “Broad income” is the sum of certain kinds of incomes and defined differently in different studies. The labor supply elasticity and broad income elasticity are not directly related to my research, but nonetheless in Appendix 2 I discuss them from the perspective of the ETI analysis.

**III. The Economic Model and Tax-Prices**

In this paper I estimate the own and cross tax-price elasticities of two tax-deductible activities. The critical empirical issue in choosing deductible activities to focus on is to find two that have different tax-prices. For this reason, I focus on giving to charities and making interest repayments on home equity lines of credit (HELOC). In this section I will develop a model to determine the two tax-prices. Before writing down the model, I discuss sources of substitutability between deductions and endogeneity issues.

**III.1 Background**

Different tax-deductible activities could be substitutes or complements for two reasons. First, people who need to borrow because they lack assets may cut down on other spending, and various forms of borrowing, such as borrowing on home collateral, and spending, such as
charitable giving, are tax-preferred. Second, tax planning under the progressive tax system could also generate substitutability. For example, if a taxpayer wants to reduce taxable income by $x$ dollars, then allocating the $x$ dollars between interest payments and charitable giving makes the two activities substitutes.

Unobservable heterogeneity is a potential concern -- some types of people might give more to charities and also pay down their debt more slowly. At first glance, this “type” issue, as it creates a negative correlation between giving and repayment in the cross section and will lead one to find a spurious substitutability between the two activities. However, this will not be an empirical concern as long as the “type” does not correlate with the tax-prices (which are functions of the marginal tax rate and the interest rate). I make a similar assumption to the rest of the literature estimating tax-price elasticities (for example Gruber and Saez 2002), that unobserved heterogeneity does not generate a correlation among tax expenditures and tax-prices, conditional on controls for income and wealth; however, I move beyond the literature in investigating interactions among two of these tax expenditures instead of assuming that a single tax expenditure that is studied is uncorrelated with all others.

III.2 The Economic Model and the Tax-prices

In this section I model the tax-prices for giving, $G$, and HELOC repayments, $Q$. Notice that $Q$ includes both interest and principal payments. I model both rather than just interest payments since the marginal tax rate affects principal payments too -- in a given year, paying back the principal in this year rather than later reduces future interest payments, which are tax-deductible.

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8 Vice versa, there could be some types who like paying down their own debt faster and may not give much to charity.
9 One can perhaps come up with stories of correlation between “type” and the marginal tax rate or the interest rate, but these are likely to be rare scenarios that do not make a case for a reasonable identification threat. For example, suppose that among households with similar income, wealth and other demographics which are observable, people who are much more generous than others in donating may have been so enthusiastic in giving that they were occasionally overstretched and missed debt payments due, worsening their credit record. As a result, they have to pay high interest rate for borrowing on home equity. This would cause a positive correlation between interest rates and charitable giving. However, this may happen only rarely.
Based on the static model from Auten and Joulfaian (1996), I construct my model where future consumption enter the household’s utility function (whereas Auten and Joulfaian let children’s income enter an individual’s utility function).\(^\text{10}\) \(D\) denotes the amount of itemized deductions other than charitable giving \(G\) and HELOC interest payments. Other consumption this year, which is \(t=0\), including saving, is denoted by \(C_0\). Let \(\{C_1, C_2 \ldots C_n \ldots\}\) denote the set of future yearly after-tax consumption levels. Let \(B\) denote the HELOC balance and \(r\) the interest rate. Under this notation, the first \(B \cdot r\) dollars of repayment \(Q\) is tax-deductible interest. A household solves the following problem (given other itemized deductions \(D\) and income)

\[
\text{Max}_{\{G, Q\}} U(C_0, G, D, \{C_1, C_2 \ldots C_n \ldots\})
\]

s.t. the budget constraint

Now consider the budget constraint. Let \(I_0\) denote the household’s total pre-tax income this year and \(AGI\) the adjusted gross income.\(^\text{11}\) Then, the household’s taxable income is \(AGI – D – G – \text{Min} (Q, B \cdot r)\); here \(Q\) enters the calculation because a household may pay less than the interest due, in which case the deduction amount they can take will be \(Q = \text{Min} (Q, B \cdot r) < B \cdot r\).\(^\text{12}\) Let \(T(.)\) denote the tax function applied to taxable income, so the tax paid by the household is then \(T(AGI – D – G – \text{Min} (Q, B \cdot r))\). Then, the budget constraint for the current period’s spending is:

\[
D + C_0 + G + Q = I_0 - T(AGI – D – G – \text{Min} (Q, B \cdot r)) \tag{1}
\]

Now, I rearrange Equation (1) to let the tax-prices appear.\(^\text{13}\) First, I decompose the taxes on the right hand side of (1) into two parts as

\[
T(AGI – D – G – \text{Min} (Q, B \cdot r)) = T(AGI – D) – Ts. \tag{2}
\]

\(^\text{10}\) In Auten and Joulfaian (1996), utility is determined by own consumption, life-time charitable contributions, charitable bequests, and child heirs’ own income and gifts/bequests from the individual. In this paper I do not consider offspring or bequests.

\(^\text{11}\) I omit the personal exemption \(E\) to simplify notation.

\(^\text{12}\) For some but not all HELOCs, this underpayment may lead to a HELOC freeze (no more drawing of funds allowed) or a reduction (reduced credit limit), or even put the home at risk of foreclosure (Federal Reserve Board 2012h; Federal Trade Commission 2012; Citizens Financial Group, Inc 2015).

\(^\text{13}\) Only interest on the first \$100,000 of home equity debt balance is tax deductible. The effect on (1) and the subsequent equations is minimal: for households exceeding the \$100,000 limit on balance (about 10% of my sample), simply replace \(B \cdot r\) with \$100,000*\(r\) (or a smaller amount if there is a second mortgage; see Appendix 4). My empirical results take this into account.
On the right hand side of (2), $T(AGI - D)$ is the tax liability that would apply if the household gives zero to charities, repays zero on their HELOC and thus has a taxable income of $AGI - D$. Suppose that such taxable income puts the household in the tax bracket that has a marginal tax rate of $\tau$. Then, a dollar of giving or HELOC repayment will reduce taxable income from $AGI - D$ and reduce the tax liability by $\tau$ this year. Therefore $Ts$ on the right hand side of (2) is the amount of taxes reduced in this year from giving and repaying HELOC interest and I write it out as $Ts = [G + \text{Min}(Q, B\cdot r)] \cdot \tau$. With this and (2), I transform (1) to

$$C_0 + (1-\tau)G + (1-\tau)\text{Min}(Q, B\cdot r) + (Q - B\cdot r)^+ = I_0 - T(AGI - D) - D, \quad \ldots \quad (3)$$

where $(Q - B\cdot r)^+$ is the part of $Q$ exceeding $B\cdot r$, or principal payments. The coefficient $(1-\tau)$ on $G$ is the tax-price of charitable giving, $p_g$. The coefficients preceding $\text{Min}(Q, B\cdot r)$ and $(Q - B\cdot r)^+$, i.e. $(1-\tau)$ and 1, are not yet the tax-prices of, respectively, interest payments and principal payments. A dollar repaid now on average reduces after-tax consumption by $1-\tau$ (if it is a dollar of interest payment) or 1 (if it is a dollar of principal payment). Meanwhile, if it is repaid next year, it will have grown to $1+r$, with 1 regarded as principal and $r$ as interest and reduce after-tax consumption by $1+r(1-\tau)$ then. In general, repaying a dollar in $n$ years means forgoing $(1+r)^n-1+(1+r)^n r(1-\tau)$ of after-tax consumption then. So, the price of a dollar of HELOC repayment should be defined as current consumption forgone, which is $1-\tau$ for interest payments or 1 for principal payments, minus the benefit of saving future interest that would otherwise accrue on this dollar. Define the future benefit as the after-tax interest saved compared to repaying next year, or $r(1-\tau)$, so the final expressions of the tax-prices are $p_{in}=1-\tau-r(1-\tau)$ for interest payments and $p_{ir}=1-r(1-\tau)$ for principal payments. It follows that the marginal tax-price at $q=Q$ is

$$p_q = \begin{cases} p_{in} = 1 - \tau - r(1 - \tau), & Q < B \cdot r \\ p_{pr} = 1 - r(1 - \tau), & Q \geq B \cdot r \end{cases} \quad \ldots \quad (4)$$

In my empirical analysis, I will examine the effect of the marginal prices, $p_g$ and $p_q$, on $G$ and $Q$. From Expression (4) it is clear that $p_q$ depends on $Q$, so later when I regress the log of $Q$ on the

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14 I am aware that $G+\text{Min}(Q, B\cdot r)$ may become larger enough and taxable income may drop below the $\tau$ bracket. Above this amount, taxes reduced from an additional dollar spent on $G+\text{Min}(Q, B\cdot r)$ will be smaller than $\tau$. Here, I make the simplification of only considering the marginal tax rate associated with the first dollar of $G+\text{Min}(Q, B\cdot r)$. This is similar to some charitable giving studies such as Barrett (1991).
log of $p_q$, I will instrument for the log of $p_q$ with the log of $p_{in}$ (Expression (4), or the first line of Expression (4)). It is a valid instrument as it correlates with $p_q$ but does not depend on $Q$.

III.3 Average tax-prices

In light of the “schmeduling theory” by Liebman and Zeckhauser (2004), I develop another specification for HELOCs’ tax-price. People may respond to the average tax-price (instead of the marginal tax price), in the way of the “ironing” approach that Liebman and Zeckhauser find evidence of in their analysis of schmeduling. They formally test for ironing behavior in households’ responses to the introduction of child credit, and estimate that 54 percent of taxpayers are ironers.

Therefore, I will also consider the effect of average tax-prices, $p_h = \frac{\sum_{q} p_q q}{Q}$ (with $p_q$ being the marginal tax-price defined in the previous subsection) i.e.\(^{(5)}\)

$$p_h = \begin{cases} 1 - \tau - r(1 - \tau), & Q < B \cdot r \\ 1 - \frac{B \cdot r \tau}{Q} - r(1 - \tau), & Q \geq B \cdot r \end{cases} \quad (5)$$

In my study, an ironer would misperceive the tax benefit falling on interest payment dollars as spreading over all payment dollars. As shown in Figure 1, the actual tax-price schedule for HELOC payment $Q$, the “marginal price” line (defined in Equation (4)), would be misperceived as the “average price” line (defined in Equation (5)). The lines show that the average price is always equal to or smaller than the marginal price, implying that an ironer pays off the line of credit faster.

\(^{(5)}\) Due to the $100,000$ on the amount of equity debt that is tax deductible, about 4% of the households have a tax-price that differs from (5). See Appendix 4.
In this paper I do not formally test for ironing behavior. Rather, I estimate models using both average and marginal prices, noting that the average price model is justified by the ironing theory.

IV. The Data

IV.1 The Data Source and Sample Selection

I use data from the Survey of Consumer Finances (SCF), a repeated cross-section conducted every three years between 1989-2007 with detailed financial data for approximately 4,000 households each year. I exclude the 1983, 1986 and 2010 SCF because they lack necessary information.\textsuperscript{16} These surveys give me a total of 29,031 observations. The surveys oversampled high income individuals so as to obtain reasonable sample sizes of the wealthy, and I use survey weights to make sample statistics and regression results nationally representative. As complement for results in the main text (Section V), I present the unweighted regression results

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure1.png}
\caption{marginal and average tax-prices of HELOC payment}
\end{figure}

\textsuperscript{16} The 1986 and 2010 SCF do not report Adjusted Gross Income, which makes the computation of the marginal tax rate less accurate. The 1983 SCF does not report charitable giving.
The weighted and unweighted results are broadly similar; in particular, the price elasticity estimates are somewhat smaller but have the same signs with the weighted results.

Following the literature that I described earlier, I restrict my sample to “exogenous itemizers”, though my definition is broader than for payers that focus on a single deduction. Exogenous itemizers are defined as taxpayers who have more non-charity and non-HELOC-interest-payments than their standard deduction. It is under this restriction that any giving and repayment on top of other deductions change tax liabilities. Further, I restrict my sample to exogenous itemizers who have a HELOC with positive outstanding balance, so that they face the choice on how much to give and how much to repay on HELOCs. This means that the analysis is restricted to people who have elected to borrow.\footnote{Perhaps someone who borrows to finance current consumption may not be very interested in donating to charity to help others. However, summary statistics in Section V suggest that households in my sample have a similar distribution of donation amounts with the broader group of all exogenous itemizers. In addition to this concern of the selected sample being unrepresentative of the broader population in terms of the willingness of giving (external validity), one may also worry about sample selection bias; perhaps estimated price elasticities are not unbiased estimates of the effects of exogenous changes in tax-prices even for the selected population of HELOC borrowers (internal validity) because an unobservable factor that affects the choice of whether to borrow also correlates with both the tax-prices and donation/HELOC repayment amounts. However, it is difficult to imagine such factors that threaten the internal validity. In Appendix 5 I try to construct a story for such a factor; nonetheless, I conclude that it is at most a minor issue.}

The sample selection leaves 1,103 observations. Out of the 29,031 observations in the unrestricted sample, about 16\% of did not file a tax return in the year surveyed, and 47\% filed tax returns but are not exogenous itemizers. Of the remaining 36\%, or 10,501 observations, only about one tenth had a HELOC in the year surveyed.

\textbf{IV.2 The Variables}

In my model, the left-hand side variables are charitable giving and payments on the HELOC balance. The key right-hand side variables are the tax-prices \(1-\tau, p_q\) and \(p_h\) specified in equations (4) and (5). My other right-hand side variables, which I include for reasons discussed below, control for wealth, income, age, marital status and years of education. I construct the variables as follows.

My left-hand side variables are \textit{charity} and \textit{payment}. Following studies on charitable giving such as Bakija and Heim (2011), I add $10 to each giving amount to get \textit{charity}. I do this in
order to take logs even for zero donations.\textsuperscript{18} I also try estimating a tobit model later, though it does not work particularly well. The total payment to the HELOC, \textit{payment}, includes both principal and interest payments. For the 5 observations with zero payment, I take logs after treating them as repaying $10.

The key right-hand side variables are the tax-prices of giving and HELOC repayment. \( p_g \) is the tax-price of giving. It is the after-tax cost of giving to charity, defined as \( 1 - \tau \), with \( \tau \) being the household’s marginal tax rate (MTR). As discussed in the economic model in Section III.3, the \( \tau \) here is the marginal tax rate applied to taxable income at zero giving and zero interest payment to HELOC. I do not observe a household’s exact MTR or taxable income prior to giving and paying HELOC interests, so I calculate them using the equation

\[
\text{taxable income prior to giving and paying HELOC interests} = \text{AGI} - \text{exemptions} - \text{itemized deductions except giving and HELOC interests}
\]

I observe \text{AGI} in the data. \textit{Exemptions} and standard deduction depend on the filing status and the number of dependents.\textsuperscript{19} I observe whether the filing status is married filing jointly (the majority) or not. Number of people in the Primary Economic Unit (PEU), marital status, whether a respondent lives with a spouse/partner, and whether the spouse/partner is included in the PEU allow me to tell which one of the other three filing status the respondent falls in, and allow me to calculate the number of dependents.\textsuperscript{20}

I observe some of the most important itemized deductions in addition to the ones I consider, but not all. Studies that use survey data to estimate the tax-price of charitable giving also have to impute itemized deductions (see footnote), and the SCF offers much more concrete information

\textsuperscript{18} A robustness check by Bakija and Heim (2011) shows that this specification works well. They analyzed the sensitivity of estimates to the size of the constant added to charity by varying the value of this constant and then run regressions. The values tried include $1, $100, and $1000.

\textsuperscript{19} Although standard deduction is not in the taxable income formula, I nevertheless need to use it here. This is because, as mentioned earlier, I restrict my sample to “exogenous itemizers”, i.e. people whose \textit{itemized deductions except giving and HELOC interests} are larger than their standard deduction.

\textsuperscript{20} In SCF, the PEU consists of an economically dominant single individual or couple (married or living as partners) in a household and all other individuals who are financially dependent on that individual or couple. Therefore, the number of people in PEU combined with the information on whether the respondent has a spouse or partner and whether the spouse or partner is included in PEU allows me to calculate the number of dependents and thus whether a respondent is married filing separately, single or head of household. There are also tens of observations of widow(er)s who may qualify to be in a fifth status, “qualifying widow(er)s with dependent child”. I simply exclude these observations for simplicity.
for doing so than many other types of surveys.\footnote{Among other surveys, the Consumer Expenditure Survey provides relatively detailed records for such imputation, while other surveys only allow for assigning itemized deduction amounts proportional to income. Examples of the former are Reece (1979) and Reece and Zieschang (1985) who add up all interest payments, state and local taxes paid, and medical deductions (not considering real estate and vehicle property taxes). Examples of the latter are Tiehen (2001) using the Independent Sector Surveys on Giving and Volunteering and Boskin and Feldstein (1977) using the National Survey of Philanthropy. Meanwhile, studies that use administrative data cannot consider the possible itemization behavior of non-itemizers at all.} Therefore, I impute \textit{itemized deductions except giving and HELOC interests} as follows. They are the sum of the mortgage interest deduction except that on HELOC, state income tax deduction, real estate tax deduction and vehicle property tax deduction.\footnote{The itemized deductions I am missing are medical and dental expenses in excess of 7.5% of AGI, home mortgage deductible points, investment interest, casualty and theft losses, job expenses and other miscellaneous deductions. According to IRS statistics, for 2010, the non-exclusive percentages of taxpayers that took these 6 types of deductions were, respectively, 7.3%, 2.0%, 1.1%, 0.07% and 9.07%. In addition, I also miss non-vehicle personal property tax deduction, and the IRS statistics does not report categories of personal property tax deductions. However, the impact of missing this deduction should be very small, since vehicles are the major component of personal properties (which is defined not to include real properties).} I observe the amount of real estate tax. I compute the mortgage interest deduction from information on the loan balances at the time of the survey, the annual interest rates and total mortgage payments per period.\footnote{Specifically, I compute the interest payment in the relevant tax year by first calculating the balance at the beginning of the tax year and then multiply the balance by the annual interest rate.} The state income tax rate varies by state and the vehicle property tax rate varies by county, but I do not observe the respondents’ states or counties. Therefore, I set the state income tax rate based the respondent’s total income and based on Davis et al. (2009) which reports the national averages of all states’ income tax rates for different income groups. About 20 states have vehicle property taxes. I extensively surveyed these states and their counties’ websites online, and set the national average at 0.44%, applied to the value of vehicles reported in the data.

The tax-price of HELOC repayment is specified in Equations (4) and (5) in its marginal and average price forms. Their expressions involve the marginal tax rate $\tau$, the HELOC interest rate $r$, and payments $Q$. The SCF reports $r$ and $Q$.

I define and control for the covariates $balance$, $wealth$, $disposable\ income$, $u$, $edu$, $married$, $age$ and its dummies. $balance$ is the balance of the HELOC at the beginning of the relevant tax year. $wealth$ is calculated as the sum of all assets less the sum of all liabilities. The SCF surveys different types of assets and liabilities thoroughly. $disposable\ income$ is AGI less tax liabilities at zero giving and payment to HELOC and other deductions. I control for wealth and income in
order to avoid omitted variable bias, as they could affect both the amount one gives/pays on debt and the interest rate one gets which enters HELOC repayment’s tax-price. Besides, income affects the marginal tax rate, and controlling for income separately helps isolate exogenous variation in the tax schedule arising from the arbitrary location of tax kinks and from tax reforms (Gruber and Saez 2002).\footnote{A potential concern is that unobservable heterogeneity may determine both income and giving. For example, someone with a strong sense of social responsibility may both work hard and be selfless, and as a result, earns more income and also gives more. However, the literature suggests that it is more important to deal with omitted variable bias by including income controls than to worry about unobservable heterogeneity.}

\(u\) is the annual national unemployment rate obtained from the website of Bureau of Labor Statistics. I control for this to avoid omitted variable bias. When the economy is bad and the unemployment rate is high, people might give less or repay less because they have less income. Meanwhile, the Fed may target a lower interest rate when the unemployment rate is higher (Nechio 2011). This means that the unemployment rate has an effect on \(p_g\) and \(p_h\), which contain interest rates. I define \textit{age} as the household head’s age. I also define two age dummies, one for households with \textit{age} between 40 (included) and 60, and the other one for \textit{age} at or above 60. \textit{edu} is a household head’s years of education. \textit{married} is a dummy variable, equal to 1 for married households and 0 for others.

For the variables \textit{charity}, \textit{payment}, \textit{distance}, \textit{wealth}, and \textit{disposable income}, all values are in 2011 dollars.

\textbf{IV.3 Summary Statistics}

I report summary statistics for my SCF sample of 1,103 exogenous itemizers with HELOC balances in Table 1. As mentioned in an earlier footnote, I discuss sample selection in Appendix 5. In Table 1, \(r\) is the HELOC interest rate. \(p_g\) is the tax-price of giving, equal to 1-\(\tau\). The marginal tax rate \(\tau\) has 10 different values falling between 0 and 39.6%. The marginal tax rate reaches the maximum of 28%, 31%, 33%, 35%, or 39.6%, depending on the year, for 6.0% of the households in the sample.\footnote{As mentioned earlier, the surveys oversampled high income individuals and provide survey weights to inform the degree of oversampling. The 6% is the proportion computed using the survey weights; in other words, it is a nationally representative statistic. Without weighting, the proportion is 18.7%; in other words, 1,103\times18.7\%\,=206 households in my sample have a top marginal tax rate. For other proportions in the rest of this subsection, the weighted and unweighted figures are similar and I only present the weighted proportions.} \(p_{in}, p_q,\) and \(p_h\) are, respectively, the tax-price of interest payment,
the marginal tax-price of all payment, and the average tax-price of all payment, defined by equations (4), and (5).

Table 1  Summary Statistics

<table>
<thead>
<tr>
<th>Label</th>
<th>Mean</th>
<th>Median</th>
<th>P75%</th>
<th>P90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charity</td>
<td>3,564</td>
<td>1,303</td>
<td>3,339</td>
<td>7,439</td>
</tr>
<tr>
<td>Payment</td>
<td>7,677</td>
<td>5,342</td>
<td>9,069</td>
<td>15,174</td>
</tr>
<tr>
<td>(\tau)</td>
<td>23%</td>
<td>25%</td>
<td>28%</td>
<td>31%</td>
</tr>
<tr>
<td>(p_g)</td>
<td>0.77</td>
<td>0.75</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>(r)</td>
<td>7.9%</td>
<td>8.0%</td>
<td>9.5%</td>
<td>11.0%</td>
</tr>
<tr>
<td>(p_m)</td>
<td>0.71</td>
<td>0.69</td>
<td>0.77</td>
<td>0.80</td>
</tr>
<tr>
<td>(p_q)</td>
<td>0.91</td>
<td>0.94</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td>(p_h)</td>
<td>0.82</td>
<td>0.83</td>
<td>0.88</td>
<td>0.93</td>
</tr>
<tr>
<td>balance</td>
<td>51,500</td>
<td>31,822</td>
<td>58,683</td>
<td>107,232</td>
</tr>
<tr>
<td>wealth</td>
<td>990,888</td>
<td>442,409</td>
<td>927,725</td>
<td>2,054,249</td>
</tr>
<tr>
<td>income</td>
<td>120,847</td>
<td>90,653</td>
<td>136,477</td>
<td>206,292</td>
</tr>
<tr>
<td>Age</td>
<td>48</td>
<td>47</td>
<td>55</td>
<td>62</td>
</tr>
<tr>
<td>edu</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>(u)</td>
<td>5.3</td>
<td>5.5</td>
<td>6</td>
<td>6.1</td>
</tr>
</tbody>
</table>

This table reports the summary statistics for a subsample of the Survey of Consumer Finances between 1989 and 2007 that I use for regressions. This subsample consists of 1,103 households with positive income and wealth whose largest line of credit is secured by home equity with positive balance and whose tax filing status is not married filing separately. The means and medians are weighted with the survey weights (variable X42001 in the SCF datasets.) All monetary values are in 2011 dollars.

The 10th and 90th percentiles of the HELOC interest rate \(r\) are 4.25% and 11%. The ratio of HELOC payment to balance has a median of 0.16. The distribution of charitable giving is right-skewed. The median giving level is $1,303 and 29.6% of the sample give zero; but at the 75%, 90%, and 99% percentiles, the giving levels are, respectively, $3,339, $7,439 and $33,389. 80.3% of the households are married, 14.7% are single and 5.0% have a head of household filing status. In Appendix 6 I compare statistics in this table with their counterparts in the broader

\(26\) The sample covers 7 years, and the years of 2003 and 2006 contribute about 47% of all observations, reflecting increased use of HELOCs in the 2000s. However, this increase does not mean that lenders began to give out
sample that is not restricted to HELOC borrowers but that are still exogenous itemizers with positive income and wealth. Overall, my smaller sample is a little wealthier, while the samples have similar distributions of charitable giving.

Table 2 lists the means of giving and repayment by low, medium, and high levels of the average tax-prices of giving and HELOC, \( p_g \) and \( p_h \). It displays how (charity, HELOC payment) vary with \((p_g, p_h)\), without controlling for other covariates from Table 1. Holding the level of \( p_h \) constant, both charity and payment fall with \( p_g \). Holding the level of \( p_g \) constant, payment rises with \( p_h \) while charity does not systematically vary with \( p_h \).

The brackets for \( p_h \) are chosen as follows: 0.7907 is the 33th percentile and 0.8614 is the 66th percentile. \( p_g \) is a discrete variable and does not allow for brackets with 33% probability mass. In each pair of numbers separated by ||, the first number is the mean of charity and the second number is the mean of payment.

### Table 2  The means of charity and payment by tax-price brackets

<table>
<thead>
<tr>
<th></th>
<th>( p_h \leq 0.7907 )</th>
<th>( 0.7907 &lt; p_h \leq 0.8614 )</th>
<th>( p_h &gt; 0.8614 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>charity</td>
<td>12,616</td>
<td>7,779</td>
<td>13,987</td>
</tr>
<tr>
<td>payment</td>
<td>9,587</td>
<td>13,398</td>
<td>23,653</td>
</tr>
</tbody>
</table>

The brackets for \( p_h \) are chosen as follows: 0.7907 is the 33th percentile and 0.8614 is the 66th percentile. \( p_g \) is a discrete variable and does not allow for brackets with 33% probability mass. In each pair of numbers separated by ||, the first number is the mean of charity and the second number is the mean of payment.

### V. Empirical Specification and Results

#### V.1 Log-linear regression specification with marginal tax-prices

I jointly regress giving \( G \) and HELOC payment \( Q \) on their own and each other’s marginal tax-prices. In other words:

\[
\log(G) = \beta_0 + \beta_1 \log(p_g) + \beta_2 \log(p_q) + X'B + \varepsilon_1 \quad \ldots (6)
\]

\[
\log(Q) = \gamma_0 + \gamma_1 \log(p_q) + \gamma_2 \log(p_g) + X'C + \varepsilon_2 \quad \ldots (7)
\]

\( X \) includes the log of wealth, the log of income, age, dummies for age groups, years of education, unemployment rate, and the log of balance, where balance is the beginning balance in HELOCs to many households with lower ability to repay. In fact, in my sample households from these two years have higher real wealth and income than those from earlier years.
a year. The error terms $\varepsilon_1$ and $\varepsilon_2$ are allowed to be correlated, in order to account for the correlation between giving and HELOC payment that results from unobserved factors such as unexpected medical expenses, say, or preferences for giving or paying debt. $p_g$ is the tax-price of charitable giving, $1-\tau$. $Log(p_q)$ is endogenous because $p_q$ depends on the amount of the HELOC payment $Q$, as revealed in Equation (4). Therefore I instrument for $Log(p_q)$. The instrument is $Log(p_{in})$, with $p_{in}$ being the price of interest payment specified in Equation (4). $Log(p_{in})$ correlates with $Log(p_q)$ and is exogenous. In particular, it does not depend on $Q$.

I ran a 3SLS regression. This method has three steps. In the first step, it regresses $Log(p_q)$ on its instrument, $Log(p_{in})$, $Log(p_g)$ and other control variables, and obtains the predicted values $Log(\hat{p}_q)$. Then in the second and the third steps, instead of just running OLS separately for (6) and (7) as the second stage of a 2SLS regression does, it uses seemingly unrelated regression. This allows for a correlation between $\varepsilon_1$ and $\varepsilon_2$. Specifically, in the second step, it estimates (6) and (7) separately using OLS to obtain residuals, and then uses the residuals to estimate $\hat{\Sigma}$, the $2*2$ variance covariance matrix of ($\varepsilon_1, \varepsilon_2$). In the third step, it estimates the coefficients in (6) and (7) using $\hat{\Sigma}$ with the Feasible GLS method.

V.2 Log-linear regression specification with average tax-prices

The specification with average prices is the same with that in Section V.1, except that I replace the marginal tax-price of HELOC repayment, $p_q$, with the average tax-price, $p_h$. They are defined in Equations (4) and (5). This regression uses the 3SLS method as well and has the same instrument, $Log(p_{in})$.

V.3 Regression results from both specifications

Table 3 shows the regression results for the marginal price specification (Section V.1) and for the average price specification (Section V.2). The first stage results show that the instruments are strong (see Appendix 8).

Under the marginal price specification, the two own-price elasticities, -1.21 (not statistically significant, p-value 44.2%) and -9.24 (significant, p-value 0.0%), and the cross effect of the

---

27 In contrast, $p_g$ is not a function of $G$ or $Q$, so it is not endogenous.
28 As the two equations (15) and (16) have the same set of regressors and the same instrument, the 3SLS method produces the same results with the 2SLS method.
HELOC repayment’s price on giving, 6.65 (p-value 8.5%), have the expected signs, while the cross-price effect of giving’s price $p_g$ on HELOC repayment is negative but insignificant, -0.76 (p-value 34.0%).

Under the average price specification, all 4 price parameter estimates have the expected signs and p-values are similar or smaller: the own-price elasticities, $\frac{\partial \ln(\text{charity})}{\partial \ln(p_g)} = -2.83$ and $\frac{\partial \ln(\text{payment})}{\partial \ln(p_h)} = -5.38$ are negative (p-values 15.2% and 0.0%), and the cross-price effects, $\frac{\partial \ln(\text{charity})}{\partial \ln(p_h)} = 3.90$ and $\frac{\partial \ln(\text{payment})}{\partial \ln(p_g)} = 1.49$ are positive (p-values 8.9% and 4.6%). The narrower confidence intervals suggest that the average price model fits the data better than does the marginal price model.

In various respects, the average price specification works better than the marginal price specification. The marginal price specification and the average price specification produce similar estimates for three of the four key parameters. Under either specification, the two own price elasticities are negative and the cross price elasticity of charitable giving with respect to HELOC repayment’s tax-price is positive, with two of the three elasticities significant at the 10% level. The own and cross price elasticities with respect to $p_q$ (marginal price), -9.24 and 6.65, are larger in absolute value than those with respect to $p_h$ (average price), -5.38 and 3.90.

The two specifications produce different estimates for the cross price elasticity of HELOC repayment with respect to giving’s tax-price. The marginal price specification produces an insignificant negative estimate of -0.76 (standard error=0.80, p-value=34.0%), while the average price specification produces a significant positive estimate of 1.49 (standard error = 0.75, p-value=4.6%). They are statistically different at the 5% level. A positive sign for this cross elasticity, together with the positive sign of the cross price elasticity of giving with respect to HELOC’s tax-price (under either specification), suggests that giving and HELOC repayment are substitutes. A negative sign is difficult to interpret as it differs from the positive sign of the other cross elasticity. In this paper I do not formally test whether households more often respond to marginal prices (rational) or average prices (ironing). However, in the policy implication

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29 Roughly, their difference is $1.49 - (-0.76) = 2.25$ with a standard error of $\sqrt{0.80^2 + 0.75^2} = 1.10$ and a t-statistic of 2.05. As $p_g$ and $p_h$ are positively correlated by construction, the covariance between the two estimates should be positive. This will only make the difference’s standard error even smaller and the difference more significant.

30 Although I do not formally test for ironing, I would like to emphasize that I do not observe obvious bunching at paying exactly the amount of interest due (Appendix 9). This can be regarded as evidence of ironing. In their child
section (Section VI), I will use the average price results rather than the marginal price results to demonstrate the advantage of my recommended policies when deductions are substitutes. Although I do not assert which set of results is right or wrong, the average price results seem more sensible. The marginal price results do not satisfy the Slutsky symmetry that requires symmetric cross price elasticities to have the same sign. Moreover, in Section VI I will introduce a type of policy that is especially appropriate when the correct values of cross price elasticities are uncertain.

Table 3  Regression Results

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Marginal Price</th>
<th></th>
<th>Average Price</th>
<th></th>
<th>Regular Giving Elasticity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>p-value</td>
<td>Estimate</td>
<td>p-value</td>
<td>Estimate</td>
<td>p-value</td>
</tr>
<tr>
<td></td>
<td>(Standard Error)</td>
<td></td>
<td>(Standard Error)</td>
<td></td>
<td>(Standard Error)</td>
<td></td>
</tr>
<tr>
<td>The Ln(charit) equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-12.93 (2.25)***</td>
<td>0.0%</td>
<td>-12.26 (2.24)***</td>
<td>0.0%</td>
<td>-11.12 (2.38)***</td>
<td>0.0%</td>
</tr>
<tr>
<td>Ln(p_3)</td>
<td>-1.21 (1.57)</td>
<td>44.2%</td>
<td>-2.83 (1.97)</td>
<td>15.2%</td>
<td>-2.08 (1.62)</td>
<td>20.1%</td>
</tr>
<tr>
<td>Ln(p_4)</td>
<td>6.65 (3.86)*</td>
<td>8.5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.90 (2.29)*</td>
<td>8.9%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(wealth)</td>
<td>0.28 (0.08)***</td>
<td>0.1%</td>
<td>0.27 (0.08)***</td>
<td>0.1%</td>
<td>0.34 (0.08)***</td>
<td>0.0%</td>
</tr>
<tr>
<td>Ln(income)</td>
<td>0.68 (0.26)***</td>
<td>0.8%</td>
<td>0.60 (0.25)**</td>
<td>1.7%</td>
<td>0.45 (0.27)</td>
<td>10.1%</td>
</tr>
<tr>
<td>Dummy for middle aged</td>
<td>0.36 (0.28)</td>
<td>20.5%</td>
<td>0.43 (0.26)</td>
<td>10.2%</td>
<td>0.45 (0.26)*</td>
<td>9.1%</td>
</tr>
<tr>
<td>Dummy for the elder</td>
<td>0.82 (0.51)</td>
<td>10.9%</td>
<td>0.85 (0.50)*</td>
<td>8.9%</td>
<td>0.83 (0.50)*</td>
<td>9.6%</td>
</tr>
<tr>
<td>Age</td>
<td>0.01 (0.02)</td>
<td>37.5%</td>
<td>0.01 (0.02)</td>
<td>46.9%</td>
<td>0.01 (0.01)</td>
<td>39.2%</td>
</tr>
<tr>
<td>Years of education</td>
<td>0.21 (0.04)***</td>
<td>0.0%</td>
<td>0.20 (0.04)***</td>
<td>0.0%</td>
<td>0.20 (0.09)**</td>
<td>4.6%</td>
</tr>
<tr>
<td>Married</td>
<td>0.61 (0.22)***</td>
<td>0.7%</td>
<td>0.64 (0.22)***</td>
<td>0.4%</td>
<td>0.75 (0.22)***</td>
<td>0.1%</td>
</tr>
<tr>
<td>Ln(balance)</td>
<td>0.29 (0.13)**</td>
<td>2.4%</td>
<td>0.32 (0.14)**</td>
<td>2.0%</td>
<td>0.18 (0.09)**</td>
<td>4.6%</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.09 (0.09)</td>
<td>33.3%</td>
<td>0.10 (0.09)</td>
<td>28.4%</td>
<td>0.15 (0.08)**</td>
<td>6.9%</td>
</tr>
</tbody>
</table>

The Ln(payment) equation

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Estimate (Standard Error)</th>
<th>p-value</th>
<th>Estimate (Standard Error)</th>
<th>p-value</th>
<th>Estimate (Standard Error)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>2.64 (1.19)**</td>
<td>2.7%</td>
<td>1.91 (1.03)*</td>
<td>6.3%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

credit analysis, Liebman and Zeckhauser (2004) point out that the rational model would predict bunching at $25,000 while the ironing model would predict no bunching. When using marginal prices, in addition to a linear regression, I also estimate a maximum likelihood model similar to and extending to my higher-dimensional case Friedberg (2000) and Moffit (1986), but the results are odd and difficult to interpret, and also not robust across the Survey of Consumer Finance’s five implicate datasets.
The interpretations for the average price results are as follows. When the tax-price of charitable giving increases by 1% (while holding the HELOC price constant), giving decreases by 2.83% and HELOC payments increase by 1.49%; when the tax-price of HELOC payment increases by 1% (while holding the tax-price of giving constant), HELOC payments decrease by 5.38% and charitable giving increases by 3.90%. Meanwhile, when the net-of-tax rate $1 - \tau$ increases by 1%, which affects both tax-prices, giving decreases by $2.83\% - 3.90\% \times 0.36 = 1.41\%$ and HELOC repayments decrease by $5.38\% \times 0.36 - 1.49\% = 0.46\%$ for a household with median levels of balance, payment, $r$, and $\tau$.\(^{31}\) For a household repaying less than interest due on their

\(^{31}\) By (5), when $Q$ is larger than $B \cdot r$, a 1% increase in $1-\tau$ does not lead to a 1% increase in $p_h$ (it does when $Q$ is smaller than $B \cdot r$). Under $Q \geq B \cdot r$, we have $p_h = 1 - \frac{B \cdot r}{Q} - r(1 - \tau) = 1 + \frac{B \cdot r - Q}{Q} - r(1 - \tau) = 1 + \frac{B \cdot (1-\tau) - B \cdot r}{Q} - r(1 - \tau) = 1 - \frac{B \cdot r}{Q} + \frac{B \cdot (1-\tau)}{Q} - r(1 - \tau) = \left(1 - \frac{B \cdot r}{Q}\right) + \frac{B \cdot (1-\tau)}{Q} - r(1 - \tau)$. As $(1 - \tau)$ increases by 1%, $p_h$ increases by $\left(\frac{B}{Q} - 1\right) r (1-\tau) \times 1\% / \left(1 - \frac{B \cdot r}{Q}\right) + \frac{B \cdot (1-\tau)}{Q} - r(1 - \tau)$, which is 0.36% at $B=31822$, $r=8.0\%$, $Q=5342$ ($>31822\times 8.0\%=2546$) and $\tau=25\%$. 

\[\text{Ln}(p_q) \quad -9.24 (1.94)*** \quad 0.0\% \]
\[\text{Ln}(p_h) \quad -5.38 (0.88)*** \quad 0.0\% \]
\[\text{Ln}(p_d) \quad -0.76 (0.80) \quad 34.0\% \quad 1.49 (0.75)** \quad 4.6\% \]
\[\text{Ln(wealth)} \quad 0.12 (0.05)** \quad 1.8\% \quad 0.14 (0.04)*** \quad 0.0\% \]
\[\text{Ln(income)} \quad -0.22 (0.14) \quad 11.0\% \quad -0.10 (0.11) \quad 36.7\% \]
\[\text{Dummy for middle aged} \quad 0.26 (0.16)* \quad 9.6\% \quad 0.16 (0.12) \quad 19.7\% \]
\[\text{Dummy for the elder} \quad 0.06 (0.29) \quad 83.2\% \quad 0.02 (0.22) \quad 92.6\% \]
\[\text{Age} \quad 0.00 (0.01) \quad 79.2\% \quad 0.01 (0.01) \quad 39.1\% \]
\[\text{Years of education} \quad -0.01 (0.02) \quad 63.1\% \quad -0.00 (0.02) \quad 98.0\% \]
\[\text{Married} \quad 0.40 (0.13)*** \quad 0.2\% \quad 0.36 (0.10)*** \quad 0.0\% \]
\[\text{Ln(balance)} \quad 0.47 (0.06)*** \quad 0.0\% \quad 0.43 (0.05)*** \quad 0.0\% \]
\[\text{Unemployment rate} \quad 0.08 (0.05) \quad 14.2\% \quad 0.07 (0.04)* \quad 7.2\% \]
HELOC, giving increases by 3.90% - 2.83% = 1.08%, and HELOC payments decrease by 5.38% – 1.49% = 3.89%.

The two specifications produce similar estimates for the overall effect of changing $1-\tau$ (i.e. the effect without holding $p_q$ or $p_h$ constant) on giving; as calculated earlier, at the covariates’ medians, a 1% increase $1-\tau$ in leads to a 1.63% (the marginal price specification) or 1.41% (the average price specification) decrease in giving. The numbers -1.63 (standard error=1.65) and -1.41 (standard error = 1.52) are the estimates for the regular tax-price elasticity of charitable giving.

In this paper elasticities are computed from the two-equation coefficients. In the charitable giving literature, the tax-price elasticity of giving is mostly estimated using single-equation models that regress the log of giving on the log of $1-\tau$. In the last two columns of Table 2, I replicate the conventional single-equation method in the charitable giving literature and estimate a charitable giving tax-price elasticity of -2.08 (standard error = 1.62), similar to the -1.63 and -1.41 above. These estimates’ magnitudes are higher than a few representative estimates in the literature (mostly between -1.09 and -1.26, see Table A.2 in Appendix 2). This appears to be driven by the sample selection criteria. In particular, if my sample is expanded to non-HELOC borrowers in SCF, the estimate would be -0.74 (standard error = 0.46). These estimates perhaps suggest that the HELOC borrowers are more sensitive to tax incentives compared with others, although their difference (2.08-0.74=1.34) is not statistically different (standard error=1.68).

An alternative way to handle 0 giving levels is Tobit regression. I ran Tobit regression for charitable giving, where the independent variables are $p_g$, $p_{in}$ (the instrument for $p_q$ and $p_h$), and other covariates. The two price coefficients, -28,181 and 46,522, have the right signs and are significant at the 1% level. They are large because they are the regressions coefficients of giving on the tax-prices, rather than those of the log of giving on the logs of the tax-prices. They imply that giving decreases in its own tax-price and increases in HELOC’s tax-price. Since there

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32 To increase the precision of the estimate, I also did the following exercise. I
33 However, in the two-stage IV tobit regression, the coefficient for the tax-price of giving is 135 (wrong sign) and not significant under the average price specification, and is 12,580 (wrong sign) and significant at the 10% level under the marginal price specification.
are only 5 observations of zero HELOC repayments, I did not run a Tobit regression for HELOCs.

VI. Links with the ETI and Optimal Tax Policy

In this section I demonstrate the implications of my results for understanding estimates of the elasticity of taxable income (ETI) that appear in the literature. I also compare different base broadening tax reform approaches, highlighting the role of cross price elasticities among deductions. I prove theoretically that, when we do not know exactly the signs and magnitudes of cross price elasticities among deductions, reducing all deductions rather than removing a subset of deductions can guarantee improved efficiency and revenue.

VI.1 Towards a Fuller Understanding of the ETI

As discussed in my introduction and literature review, in order to have a full picture of the ETI, it is necessary to decompose it into parts that correspond to the various components of taxable income. In Appendix 1 I have derived a decomposition built on Kopczuk (2005) and Varian (1992). This decomposition indicates that, when taxable income has \( N \) components (i.e. \( N \) different types of incomes and deductions), the ETI is the sum of \( N \times N \) terms. These terms are additive, and in this section I isolate and discuss 2 \( \times \) 2 of them (Expression (8) below). In other words, I am essentially looking at a “sub-ETI”, or the charitable giving-HELOC portion of the ETI when other deductions are exogenous (but the basic intuition and implications implied in my analysis of this sub-ETI applies to a full ETI):

\[
\text{sub-ETI} = -\left( \frac{\partial G}{\partial p_g} \cdot \frac{dp_g}{pg} \cdot \frac{G}{\text{taxable income}} \right) \cdot (a) - \left( \frac{\partial G}{\partial p_h} \cdot \frac{dp_h}{ph} \cdot \frac{G}{\text{taxable income}} \right) \cdot (b) + \left( \frac{\partial G}{\partial G} \cdot \frac{dp_g}{pg} \cdot \frac{G}{\text{taxable income}} \right) \cdot (c) - \left( \frac{\partial G}{\partial G} \cdot \frac{dp_h}{ph} \cdot \frac{G}{\text{taxable income}} \right) \cdot (d) + \left( \frac{\partial h}{\partial p_g} \cdot \frac{dp_g}{pg} \cdot \frac{h}{\text{taxable income}} \right) \cdot (e) - \left( \frac{\partial h}{\partial p_h} \cdot \frac{dp_h}{ph} \cdot \frac{h}{\text{taxable income}} \right) \cdot (f) + \left( \frac{\partial h}{\partial p_g} \cdot \frac{dp_g}{pg} \cdot \frac{h}{\text{taxable income}} \right) \cdot (g) - \left( \frac{\partial h}{\partial p_h} \cdot \frac{dp_h}{ph} \cdot \frac{h}{\text{taxable income}} \right) \cdot (h)
\]
This expression shows that the ETI is not an invariant parameter, as it is the sum of all four terms. Terms 1 to 4 measure the effect of changes in the net-of-tax rate 1-\(\tau\) on the 2 deductions (1 and 2 for \(G\); 3 and 4 for \(h\)) through changing the 2 tax-prices, therefore there are 2*2=4 of these terms. It depends, for instance, on whether \(G\) is deductible. If \(G\) were no longer deductible, \(p_g\) would equal 1 and \(dp_g\) would equal 0. As a result, Terms 1 and 4 will become 0, altering the value of the ETI. I estimated the terms (a), (b), (c) and (d) in the above Equation (8). The other terms’ values depend on the tax system in place and the households’ expenditure allocation, which I will explain further in Subsections VI.2 and VI.3 with examples.

**VI.2 Conventional policy vs. optimal policy**

In this and the next subsections I compare different base broadening policies and show how this comparison depends on the ETI and sub ETI that I have derived. I will supply both theoretical results and simulations. I evaluate to what extent each policy can achieve the desired outcomes of (1) increasing tax revenue, and (2) reducing the deadweight loss per dollar of tax base. This boils down to evaluating whether each policy can reduce the ETI, because a smaller ETI means both higher revenue and less deadweight loss per dollar of tax base (Feldstein 1999; Saez, Slemrod and Giertz 2012).

I begin with the conventional base broadening policy of eliminating some deductions and keeping others (for example the “blank-slate” approach proposed by the Senate Finance Committee in June 2013). In theory, this type of policy cannot guarantee reducing the ETI. To show this, I continue with Equation (8). If legislation eliminates the HELOC deduction, then terms 3 and 4, which represent the change in HELOC deduction with respect to changes in 1-\(\tau\), will disappear, helping to reduce the ETI. But term 2, the effect of changes in 1- \(\tau\) on \(G\) through changing \(p_h\), will also disappear, working against the reduction. The sub-ETI in (8) (and hence the ETI, which depends additively on the sub-ETI) will only fall if the two deductions are not extremely substitutable. In the extreme case, if they are sufficiently substitutable, then eliminating one deduction will lead to so much more of the other deduction that tax revenue could fall.

Applying my estimates, I simulate outcomes for two conventional policy reforms, eliminating either the deduction for charitable giving or for HELOC interest. Suppose, for example, that a
household’s pretax income is $106. When the tax rate is zero, suppose that the amounts of giving $G$, HELOC interest payment $h$, and other ordinary consumption $C$ are, respectively, $2$, $4$, and $100$. Actual $G$ and $h$ depend on the tax rate.

Then, under the current system, with both giving and HELOC payment deductible, the inputs for percentage terms (g) and (h) in for Equation (8) are $\frac{G}{\text{taxable income}} = \frac{G}{C} = \frac{2}{100}$ and $\frac{h}{\text{taxable income}} = \frac{h}{C} = \frac{4}{100}$. Further, plugging in my price elasticity estimates into Equation (8) too, I have a sub-ETI value of $-2.83 \times 1 \times \frac{2}{100} - 3.90 \times 1 \times \frac{2}{100} - (-5.38) \times 1 \times \frac{4}{100} - 1.49 \times 1 \times \frac{4}{100} = 0.134$. Figure 2 draws the “Laffer curve”, labelled “before reform”, that depicts the relation between the tax revenue and the tax rate under this sub-ETI of 0.134. I also record this result for the sub-ETI in Table 4’s “before reform” row.

The first conventional Policy, named Policy G1, eliminates the HELOC interest deduction, and thus increases the tax base from $100 to $104 and leaves a sub-ETI of $-2.83 \times 1 \times \frac{2}{104} = 0.054$. Figure 2 plots the resulting Laffer curve, labelled “G1”. It lies above the “before reform” curve, because it starts with a larger tax base at zero tax rate, and also has a smaller ETI.

The second conventional Policy, named Policy h1, eliminates the charitable giving deduction, thus increasing the tax base from $100 to $102 and leaving a sub-ETI of $-5.38 \times 1 \times \frac{4}{102} = 0.211$, even larger than the before reform ETI. Figure 2 draws the corresponding Laffer curve, labelled “h1”. Notice that Policy h1 raises slightly more revenue than the “before reform” system at small tax rates, because it closes off the charitable giving deduction and has a larger tax base when the tax rate is zero. But the tax base is much more elastic. Once the tax rate goes above a

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34 Here I plug in 2 and 4, the values when the tax rate of 0, because I evaluate the ETI at tax rate equal 0, following Feldstein (1999). For simplicity, I do not evaluate ETI at other tax rates and also treat the ETI as constant when plotting Laffer curves later on. Evaluating the ETI at other tax rates will not change the policy implications.

35 To see where the numbers come from, notice that I estimated terms (a), (b), (c), and (d), or the average tax-price elasticities of $G$ and $h$ with respect to $p_g$ and $p_h$. As for the other partial derivative terms (e) and (f), $p_g=1-\tau$ and $p_h=1-\tau-\tau(1-\tau)1-\tau=1$. The equation for the curve is $\text{Revenue}=100(1-\tau)^{0.134}\tau$. It is derived as follows. First, I have $\text{Taxable Income}=100*(1-\tau)^{0.134}$, pinned down by two conditions, $\text{Taxable Income}(\tau=0)=100$ and $\text{ETI} = \frac{\partial \ln(\text{Taxable Income})}{\partial \ln(1-\tau)} = 0.134$. Then, I have $\text{Revenue}=\text{Taxable Income} \cdot \tau = 100(1-\tau)^{0.134}\tau$. The equations for other policies will be the same except that I replace 0.134 with their respective ETIs.
threshold level (22.7%), Policy h1 raises less revenue. Table 4’s “conventional” panel records G1’s and h1’s key results.

Figure 2  the Laffer Curves

Now, I discuss optimal base broadening policies that preserve both the tax rate and the size of the tax base under conventional policies while maximizing tax revenue (and simultaneously minimizing deadweight loss, since they are interdependent). While a conventional policy stipulates that a deduction is either 100% deductible or 0% deductible, for an optimal policy I consider a “deductible proportion” between 0 and 100% for each deduction. For example, if we stipulate that charitable giving is 80% deductible, then each dollar of charitable giving will reduce taxable income by 80 cents rather than by 1 dollar, and, under a tax rate of 10%, decreases tax liability by 8 cents rather than by 10 cents. To be prepared for the ensuing simulations, notice that reducing deductible proportions from 100% to x% will increase the tax-price of giving from 1- τ to 1- x% · τ (affecting Terms ① and ④ in Expression (8)), and increase the tax-price of HELOC interest payment from 1- τ·r(1- τ) to 1- x% · τ·r(1- x% · τ)) (affecting Terms ② and ③ in Expression (8)).

37 In a base broadening reform, given the tax authority’s chosen tax base, an optimal policy chooses a set of optimal deductible proportions that minimizes the after reform ETI.

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37 (1) To see the calculation, notice that under the usual system we currently have (x%=100%), each dollar of giving reduces tax liability by τ dollar, while if only x% is deductible, then each dollar of giving reduces tax liability by x% · τ dollar. For more rigorous proof, please see Appendix 10. (2) Feldstein (1999) shows that, in a tax system with fully tax deductible items, the dead weight loss is proportional to the ETI. Here my proposed tax system is different: each tax deductible item is partially (x%) instead of 100% deductible. However, the result that the dead weight loss is proportional to the ETI still holds approximately. Yet, although the approximation error is small, it can be shown that my proposed system has smaller deadweight loss than the traditional tax system. I provide a proof in Appendix 11.
The conventional Policy G1 has a tax base of $104 and sets the deductible proportions for $G$ and $h$ to be, respectively, 100% and 0%. In contrast, given my cross price elasticity estimates, the optimal policy with the same tax base ($104), which I name Policy G2, sets the deductible proportions to be 49% for $G$ and 25% for $h$ (see Appendix 12 for calculations). The ETI is 
\[-(−2.83) \times 49\% \times \frac{49\% + 2}{104} - 3.90 \times 25\% \times \frac{25\% + 2}{104} - (−5.38) \times 25\% \times \frac{25\% + 4}{104} - 1.49 \times 49\% \times \frac{49\% + 4}{104} = 0.010\] (see Appendix 12). The optimal Policy G2 produces a lower ETI, or in other words higher revenue and lower dead weight loss per dollar of tax base, than both the conventional Policy G1 or the before reform system.

The conventional Policy h1 has a tax base of $102 and sets the deductible proportions for $G$ and $h$ to be, respectively, 0% and 100%. In contrast, given my cross price elasticity estimates, the optimal policy with the same tax base, named h2, sets the deductible proportions to be 99% for $G$ and 51% for $h$ (see Appendix 12 for calculations). The ETI is 
\[-(−2.83) \times 99\% \times \frac{99\% + 2}{102} - 3.90 \times 51\% \times \frac{51\% + 2}{102} - (−5.38) \times 51\% \times \frac{51\% + 4}{102} - 1.49 \times 99\% \times \frac{99\% + 4}{102} = 0.041\] (see Appendix 12). The optimal Policy h2 does better in ETI, revenue, and efficiency than the conventional Policy h1 or before reform. Table 4’s “optimal” panel records G2’s and h2’s key elements.

In sum, in base broadening attempts, the optimal policies that are based on the own and cross tax-price elasticity values outperform the conventional policies. In particular, an optimal policy can guarantee reducing the ETI. This is not only true in the above two deduction case, but also true in the general situation where taxable income has $N$ components (theoretically proved in Appendix 12). Further, by definition an optimal policy reduces the ETI to the minimum subject to a constraint on the tax base. In contrast, a conventional policy cannot guarantee reducing the ETI (e.g. Policy h1 does not).

**VI.3 Uniform partial deductibility: the second best policy under uncertainty**

The previous subsection illustrates the advantage of optimal policy design in reducing the ETI (to the minimum). However, it requires knowing the correct values of all own and cross tax-price elasticities of taxable income’s components (see Appendix 12). Designing policy based on estimates fail to reduce the ETI. This subsection shows that implementing a “uniform partial
deductibility’’ policy can guarantee a reduction in the ETI even when the own and cross price elasticities are unknown.

As my naming suggests, a uniform partial deductibility policy sets one same deductible proportion for all deductions, e.g. legislating that only x% of h and x% of G are deductible in the previous subsection’s example. Theoretically this policy can guarantee reducing the ETI, and this is not only true for my two deduction case but also true in general under an N-component taxable income definition (see Appendix 12 for my proof).

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Policy simulation results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Policy</td>
<td>Description</td>
</tr>
<tr>
<td>Before reform</td>
<td>G deductible</td>
</tr>
<tr>
<td></td>
<td>h deductible</td>
</tr>
<tr>
<td></td>
<td>tax base=100</td>
</tr>
<tr>
<td>G1</td>
<td>G deductible</td>
</tr>
<tr>
<td></td>
<td>h not deductible</td>
</tr>
<tr>
<td></td>
<td>tax base=104</td>
</tr>
<tr>
<td>Conventional</td>
<td>G deductible</td>
</tr>
<tr>
<td>G2</td>
<td>G 49% deductible</td>
</tr>
<tr>
<td></td>
<td>h 25% deductible</td>
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<tr>
<td></td>
<td>tax base=104</td>
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<tr>
<td></td>
<td>G 99% deductible</td>
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<tr>
<td></td>
<td>h 51% deductible</td>
</tr>
<tr>
<td></td>
<td>tax base=102</td>
</tr>
<tr>
<td>Optimal</td>
<td>G deductible</td>
</tr>
<tr>
<td>h3</td>
<td>G 67% deductible</td>
</tr>
<tr>
<td></td>
<td>h 67% deductible</td>
</tr>
<tr>
<td></td>
<td>tax base=102</td>
</tr>
<tr>
<td>Uniform partial deductibility</td>
<td>G deductible</td>
</tr>
<tr>
<td>G3</td>
<td>G 33% deductible</td>
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<tr>
<td></td>
<td>h 33% deductible</td>
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<tr>
<td></td>
<td>tax base=104</td>
</tr>
<tr>
<td>h2</td>
<td>G 67% deductible</td>
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<tr>
<td></td>
<td>h 67% deductible</td>
</tr>
<tr>
<td></td>
<td>tax base=102</td>
</tr>
</tbody>
</table>


Continuing with previous simulations, consider a uniform partial deductibility Policy G3 that sets the tax base to be $104 as Policy G1 does. It deducts from pretax income $106 - $104 = $2 dollars, or 33% of total amounts of $G$ and $h$ at 0 tax rate (6 dollars). So Policy G3 sets all deductions to be 33% deductible and produces an ETI of $-(-2.83) * 33% * \frac{33%+2}{104} - 3.90 * 33% * \frac{33%+2}{104} - (-5.38) * 33% * \frac{33%+4}{104} - 1.49 * 33% * \frac{33%+4}{104} = 0.014$. This is not as small as the optimal Policy G2’s ETI (0.010), but it is smaller than the before reform ETI (0.134), as predicted by theory. In addition, it dominates Policy G1 (0.054).

A uniform partial deductibility Policy h3 sets the tax base to be $102 as Policy h1 does. It sets all deductions to be 67% deductible and produces an ETI of $-(-2.83) * 67% * \frac{67%+2}{104} - 3.90 * 67% * \frac{67%+2}{104} - (-5.38) * 67% * \frac{67%+4}{104} - 1.49 * 67% * \frac{67%+4}{104} = 0.058$. This is not as small as the optimal Policy h2’s ETI (0.041), but it is smaller than the before reform ETI (0.134), as predicted by theory. In addition, it is also better than Policy h1 (0.211).

Above, I show the advantages of optimal policies (in guaranteeing reducing the ETI) and the advantages of uniform partial deductibility policies (in both guaranteeing reducing the ETI and requiring less information) under the same chosen tax base.

An additional feature of uniform partial deductibility is the predictability of its effects. So far, I have demonstrated that under any desired tax base target, the optimal and uniform partial deductibility policies dominate the conventional policy of removing a single deduction (or a subset of deductions in general). The next step is to predict the revenue and efficiency outcomes under the new tax base (or, alternatively, what tax base leads to a particular revenue and efficiency target). In this regard, the uniform partial deductibility policy has a unique advantage: one only needs to know the before reform ETI to predict the effect on revenue and efficiency. In contrast, under a conventional policy or an optimal policy, one needs to know the own and cross tax-price elasticities (since these policies change the relative prices of deductions). The intuitive reason is that a uniform partial deductibility policy maintains the deductions’ relative prices. In fact, a simple relationship exists between the ETI under the uniform partial policy and the before reform ETI, which is $uniform partial ETI = (x')^2 before reform tax base new (broader) tax base before reform ETI$, with $x'$ being the uniform deductible proportion. This formula holds not
only for the two deduction case but also for multiple deductions (see Appendix 12 for proof). This relation does not depend on the own and cross price elasticity values. Since the ETI is a sufficient statistic for computing the deadweight loss (Feldstein 1999) as well as for forecasting revenue (by definition), this relation means the following: in order to predict the revenue and deadweight loss under a uniform partial deductibility policy, knowing the before reform ETI is enough.

While estimates of a before reform ETI could be biased due to the confounding effect of deduction changes in a reform (elaborated in Section I and Appendix 2), estimating an overall ETI is a less formidable task than measuring all own and cross elasticities. Moreover, it is relatively straightforward to undertake an informative sensitivity analysis based on a range of plausible values for the ETI. For example, suppose that the tax authority believes that the true before reform ETI value today is somewhere between 0.4 and 0.8 (taken from existing estimates in Appendix 2’s Table A.1 that are smaller than 1). Also, suppose that the total amount of itemized deductions is 40% of the current taxable income and the current tax rate is 20%. Then, if the tax authority aims to increase revenue by 10% by broadening the tax base, then the correct uniform deductible proportion $x'$ will be somewhere between 78.6% and 81.4% (see Figure 3 for the correct $x'$ as a function of ETI; see calculations in Appendix 12’s Section A12.5). This range is narrow enough for it to provide meaningful guidance.

![Figure 3](image)

Lastly, even if no narrow range of $x'$ can be determined as in Figure 3 due to the high degree of uncertainty about the true ETI value, there is still a reasonable approach to base broadening. For example, suppose that the true ETI is 0.58 and thus the correct $x'$ is 80%, but the tax
authority knows neither number. Nonetheless, it can gradually phase in a uniform partial deductibility policy over the course of several years – e.g. start from the existing full deductibility of $x' = 100\%$, and then bring it down by, say 2\% every year, until it reaches $x' = 80\%$ when they find that the revenue goal is reached. Along the way, it can accelerate the pace (just like the acceleration of the Bush tax cut) if it finds that the 2\% steps increase revenue too slowly.

VII. Conclusions

This paper finds evidence of households making joint decisions on different deductions, using the Survey of Consumer Finances (SCF). The SCF has detailed information on demographics such as income, wealth, age, and education. The key to identify the interaction between deductions is to find a pair of two deductions with different tax-prices. For this purpose I have studied the charitable giving deduction and the HELOC interest payment deduction. I find negative own price elasticities consistent with downward sloping demand curves; and positive cross price elasticities in my preferred specification. The latter indicates substitutability between the two deductions.

By considering interactions between deductions, I move beyond the charitable giving literature that studies one deduction in isolation. I decompose the ETI as a weighted sum of taxable income’s components’ elasticities. This theoretical framework allows me to discuss the design of base broadening policies. Illustrated with simulations, I show that a conventional base broadening policy cannot guarantee increasing revenue (and improving efficiency) due to substitutability between deductions. In contrast, under a tax base target, an optimal policy involves reducing different deductions to optimal degrees that maximize the revenue raising capacity and efficiency. Calculating these optimal degrees requires knowing the correct values of all own and cross tax-price elasticities of taxable income’s components, such as those I estimated in this paper. Contrary to the optimal policy, we can implement a “uniform partial deductibility” policy that reduces all deductions to the same degree, even when the own and cross price elasticities are unknown. This policy can guarantee improved outcomes.

This paper extends the static modelling methods employed in the charitable giving literature and the ETI literature to considering the interaction between a pair of deductions. The results can
thus be compared with the charitable giving tax price elasticity and the ETI. Future research could attempt to model interactions between longer term decisions about deductions, such as the home purchase mortgage interest deduction and charitable giving literature. This kind of research will give us more insight about the structure of the long run ETI.

Appendix 1 Decomposing the Elasticity of Taxable Income (ETI)

Taxable income consists of more basic components. As a result, the responsiveness of taxable income to tax rate can be decomposed into more basic parts too. In this appendix I provide a formal decomposition, built on Kopczuk (2005) and Varian (1992). I will present that the elasticity of taxable income (ETI) is the summation of more basic elasticities, including several “own-price” elasticities and several “cross-price” elasticities.

Suppose that taxable income has $N$ components, i.e.

$$\text{Taxable Income} = I_1 + I_2 + \cdots + I_j - D_{j+1} - D_{j+2} - \cdots - D_N,$$

where $I_j$ ($1 \leq j \leq J$) is the amount of a type of income, e.g. labor income of $5,000, and $D_j$ ($J + 1 \leq j \leq N$) is the amount of a type of tax-deductible consumption, e.g. $1,000 given to charities.

Let $p_1, p_2, \ldots, p_N$ denote the tax-prices for $I_1, I_2, \ldots, D_N$, respectively. A household chooses the levels of $I_1, I_2, \ldots, D_N$ based on the tax-prices. For example, the tax-price of charitable giving is defined as the forgone after-tax money from giving 1 dollar, i.e. $1 - \tau$, with $\tau$ being the marginal tax rate. As another example, the tax-price of labor income could also be written as $1 - \tau$. This is because, choosing labor supply is equivalent to choosing the amount of leisure, and the after-tax money forgone for having 1 hour of leisure is $(1 - \tau) \times \text{hourly wage}$.\footnote{Thus one implicit assumption for $1 - \tau$ to suffice as the tax-price for labor income is that the hourly wage does not change when $\tau$ changes, so the only variable part in $(1 - \tau) \times \text{hourly wage}$ is $1 - \tau$. However, this assumption may not be true when $\tau$ changes for a large number of households, for example in a tax reform. In fact, Kubik (2004) finds that as the marginal tax rates changed differently for different industries after the 1986 tax reform, wages also changed differently in different industries. In this appendix I do not discuss how tax rates affect the tax base through changing market equilibrium prices, and instead examines only how tax rate changes alter individual households’ choices directly. This does not mean that market equilibrium price changes do not play an important part in the ETI. In a working project I examine the effect of changing tax rates on home mortgage interest rates, and discuss how much this effect increases the ETI.} In general, $p_1, p_2, \ldots, p_N$ are all functions of $1 - \tau$, but not necessarily exactly $1 - \tau$. Therefore the above formula is in fact
**Taxable Income**

\[
= l_1(p_1, p_2, \ldots, p_N) + l_2(p_1, p_2, \ldots, p_N) + \cdots + l_j(p_1, p_2, \ldots, p_N) - D_{j+1}(p_1, p_2, \ldots, p_N) \\
- D_{j+2}(p_1, p_2, \ldots, p_N) - \cdots - D_N(p_1, p_2, \ldots, p_N).
\]

Then I have

\[
\frac{d(\text{Taxable Income})}{d(1-\tau)} = \frac{d(l_1 + l_2 + \cdots + l_j - D_{j+1} - D_{j+2} - \cdots - D_N)}{d(1-\tau)} = \sum_{j=1}^j \frac{d l_j}{d(1-\tau)} - \sum_{j=j+1}^N \frac{d D_j}{d(1-\tau)}
\]

\[
= \sum_{j=1}^j \sum_{n=1}^N \frac{\partial l_j}{\partial p_n} \frac{dp_n}{d(1-\tau)} - \sum_{j=j+1}^N \sum_{n=1}^N \frac{\partial D_j}{\partial p_n} \frac{dp_n}{d(1-\tau)}
\]

The ETI is

\[
\frac{d(\text{Taxable Income})}{d(1-\tau)} \frac{1-\tau}{\text{Taxable Income}} = \left[ \sum_{j=1}^j \frac{d l_j}{d(1-\tau)} - \sum_{j=j+1}^N \frac{d D_j}{d(1-\tau)} \right] \frac{1-\tau}{\text{Taxable Income}}
\]

\[
= \sum_{j=1}^j \frac{1-\tau}{\text{Taxable Income}} \frac{l_j}{d(1-\tau)} - \sum_{j=j+1}^N \frac{1-\tau}{\text{Taxable Income}} \frac{D_j}{d(1-\tau)}
\]

\[
= \sum_{j=1}^j \frac{l_j}{\text{Taxable Income}} \times \text{the Elasticity of } l_j \text{ with respect to } (1-\tau)
\]

\[
- \sum_{j=j+1}^N \frac{D_j}{\text{Taxable Income}} \times \text{the Elasticity of } D_j \text{ with respect to } (1-\tau).
\]

This expression says that the ETI is the weighted average of its components’ elasticities, with weights being the respective components’ shares in taxable income. For example, suppose that \(l_j\) is labor income, then, assuming that the hourly wage for the taxpayer does not change when \((1-\tau)\) changes, the Elasticity of \(l_j \) with respect to \((1-\tau)\) is nothing but the labor supply elasticity. As another example, suppose that \(D_j\) is the amount of charitable giving, then the Elasticity of \(D_j \) with respect to \((1-\tau)\) is nothing but the tax-price elasticity of charitable giving talked about in the charitable giving literature.

However, these weighted component elasticities are still not the most basic building blocks of the ETI. In what follows, I further decompose the ETI. Along the way, I also show that the components’ elasticities, for example the labor supply elasticity and the charitable giving elasticity, are also not immutable parameters; in
other words, they could potentially change if the definition of taxable income changes, for example if the mortgage interests paid are no longer deductible after a tax reform.

Continuing with the above expression, I decompose each total derivative into $N$ partial derivatives corresponding to $N$ tax-prices. I have

$$\begin{align*}
ETI &= \sum_{j=1}^{J} \frac{I_j}{\text{Taxable Income}} \cdot \frac{1 - \tau}{I_j} \frac{dl_j}{d(1 - \tau)} - \sum_{j=j+1}^{N} \frac{D_j}{\text{Taxable Income}} \cdot \frac{1 - \tau}{D_j} \frac{dD_j}{d(1 - \tau)} \\
&= \sum_{j=1}^{J} \frac{I_j}{\text{Taxable Income}} \cdot \frac{1 - \tau}{I_j} \left[ \sum_{n=1}^{N} \frac{\partial I_j}{\partial p_n} \frac{dp_n}{d(1 - \tau)} \right] \\
&\quad - \sum_{j=j+1}^{N} \frac{D_j}{\text{Taxable Income}} \cdot \frac{1 - \tau}{D_j} \left[ \sum_{n=1}^{N} \frac{\partial D_j}{\partial p_n} \frac{dp_n}{d(1 - \tau)} \right]
\end{align*}$$

To make it clearer, I now extend the terms without using the summation signs to get

$$\begin{align*}
ETI &= \frac{I_1}{\text{Taxable Income}} \cdot \frac{1 - \tau}{I_1} \left[ \frac{\partial I_1}{\partial p_1} \frac{dp_1}{d(1 - \tau)} + \frac{\partial I_1}{\partial p_2} \frac{dp_2}{d(1 - \tau)} + \cdots + \frac{\partial I_1}{\partial p_N} \frac{dp_N}{d(1 - \tau)} \right] \\
&= \text{(the tax-price elasticity of } I_1) \\
&\quad + \frac{I_2}{\text{Taxable Income}} \cdot \frac{1 - \tau}{I_2} \left[ \frac{\partial I_2}{\partial p_1} \frac{dp_1}{d(1 - \tau)} + \frac{\partial I_2}{\partial p_2} \frac{dp_2}{d(1 - \tau)} + \cdots + \frac{\partial I_2}{\partial p_N} \frac{dp_N}{d(1 - \tau)} \right] \\
&= \text{(the tax-price elasticity of } I_2) \\
&\quad + \cdots \\
&\quad + \frac{I_J}{\text{Taxable Income}} \cdot \frac{1 - \tau}{I_J} \left[ \frac{\partial I_J}{\partial p_1} \frac{dp_1}{d(1 - \tau)} + \frac{\partial I_J}{\partial p_2} \frac{dp_2}{d(1 - \tau)} + \cdots + \frac{\partial I_J}{\partial p_N} \frac{dp_N}{d(1 - \tau)} \right] \\
&= \text{(the tax-price elasticity of } I_J) \\
&\quad - \frac{D_{J+1}}{\text{Taxable Income}} \cdot \frac{1 - \tau}{D_{J+1}} \left[ \frac{\partial D_{J+1}}{\partial p_1} \frac{dp_1}{d(1 - \tau)} + \frac{\partial D_{J+1}}{\partial p_2} \frac{dp_2}{d(1 - \tau)} + \cdots + \frac{\partial D_{J+1}}{\partial p_N} \frac{dp_N}{d(1 - \tau)} \right] \\
&= \text{(the tax-price elasticity of } D_{J+1})
\end{align*}$$
\[
\frac{-D_{J+2}}{\text{Taxable Income}} \left( 1 - \tau \right) \left[ \frac{\partial D_{J+2}}{\partial p_1} d \frac{1}{(1 - \tau)} + \frac{\partial D_{J+2}}{\partial p_2} \frac{dp}{d} \left( 1 - \tau \right) + \cdots + \frac{\partial D_{J+2}}{\partial p_N} \frac{dp}{d} \left( 1 - \tau \right) \right]
\]

(\text{the tax-price elasticity of } D_{J+2})

\[
\frac{-...}{\text{Taxable Income}} \left( 1 - \tau \right) \left[ \frac{\partial D_N}{\partial p_1} d \frac{1}{(1 - \tau)} + \frac{\partial D_N}{\partial p_2} \frac{dp}{d} \left( 1 - \tau \right) + \cdots + \frac{\partial D_N}{\partial p_N} \frac{dp}{d} \left( 1 - \tau \right) \right].
\]

(\text{the tax-price elasticity of } D_N)

This final expression says that the ETI is, again, the weighted average of taxable income’s components’ elasticities (the bold and shadowed terms) e.g. the labor supply elasticity and the charitable giving elasticity under the US tax system, and that each component’s elasticity consists of one own-price part (e.g. the \( \frac{\partial D_{J+1}}{\partial p_{J+1}} \frac{1}{d(1-\tau)} \frac{d}{d} \)) and \( N-1 \) cross price parts. It is thus clear that not only the value of the ETI but also taxable income’s components’ elasticities depend on tax law. For example, if the legislation makes \( D_{J+1} \) no longer deductible, then all own-price and cross-price terms with \( \frac{\partial p_{J+1}}{\partial p_{J+1}} \) will disappear from the ETI expression, changing the ETI as well as the components’ elasticities such as the charitable giving tax-price elasticity. In summary, there are \( N \times N \) channels through which \( 1-\tau \) affects taxable income: \( 1-\tau \) affects \( N \) tax-prices, and each of the \( N \) tax-prices affects each of the \( N \) taxable income components. The above expression rigorously exhibits this structure.

Appendix 2  Mathematical analysis of the relation between the ETI and other types of elasticities and review of existing estimates

In this appendix I thoroughly analyze mathematically the relation between different elasticities. In Section A2.2 I address a topic I alluded to in the introduction, namely the potential bias associated with estimating the ETI using tax reform data. The bias is caused by removal/addition of deductions which affects taxable income and correlates with tax rate change.

In what follows, I start with an ETI decomposition formula. It follows that different categories of empirical studies estimate different parameters in this formula.
A2.1 A decomposition of the ETI

The tax literature has estimated different types of elasticities with respect to 1-\(\tau\), such as the labor supply elasticity, the tax-price elasticity of charitable giving, the ETI, and the elasticity of broad income. In Appendix 1 I have provided a decomposition of the ETI for a generic tax system where taxable income has \(N\) components. Here, to keep expressions short, I present a simple version of it with \(N\) set to 4 which suffices for my purpose here: that is, to put all kinds of elasticities in one picture and clearly illustrate their relations.

Consider a simple, generic tax system defining taxable income as \(TI = L + R - D_1 - D_2\), with \(L\) and \(R\) being the amounts of two types of income and \(D_1\) and \(D_2\) being the amounts spent on two tax-deductible commodities. Let \(\tau\) denote the marginal tax rate and let \(p_l, p_R, p_1,\) and \(p_2\) denote, respectively, the tax-prices \(L, R, D_1,\) and \(D_2\). The four tax-prices may or may not be \(1 - \tau\). Based on Appendix 1, the ETI is

\[
ETI = \frac{L}{TI} \frac{d \ln(L)}{d \ln(1 - \tau)} + \frac{R}{TI} \frac{d \ln(R)}{d \ln(1 - \tau)} - \frac{D_1}{TI} \frac{d \ln(D_1)}{d \ln(1 - \tau)} - \frac{D_2}{TI} \frac{d \ln(D_2)}{d \ln(1 - \tau)}
\]

... (A.1)

Elasticity of \(D_1\)

\[
= \frac{L}{TI} \left[ \frac{\partial \ln(L)}{\partial \ln(p_L)} \frac{d \ln(p_L)}{d \ln(1 - \tau)} + \frac{\partial \ln(L)}{\partial \ln(p_R)} \frac{d \ln(p_R)}{d \ln(1 - \tau)} + \frac{\partial \ln(L)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{\partial \ln(L)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1 - \tau)} \right]
\]

\[
+ \frac{R}{TI} \left[ \frac{\partial \ln(R)}{\partial \ln(p_L)} \frac{d \ln(p_L)}{d \ln(1 - \tau)} + \frac{\partial \ln(R)}{\partial \ln(p_R)} \frac{d \ln(p_R)}{d \ln(1 - \tau)} + \frac{\partial \ln(R)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{\partial \ln(R)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1 - \tau)} \right]
\]

\[
- \frac{D_1}{TI} \left[ \frac{\partial \ln(D_1)}{\partial \ln(p_L)} \frac{d \ln(p_L)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_1)}{\partial \ln(p_R)} \frac{d \ln(p_R)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_1)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_1)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1 - \tau)} \right]
\]

\[
- \frac{D_2}{TI} \left[ \frac{\partial \ln(D_2)}{\partial \ln(p_L)} \frac{d \ln(p_L)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_2)}{\partial \ln(p_R)} \frac{d \ln(p_R)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_2)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_2)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1 - \tau)} \right]
\]

... (A.2)

\[
= \left[ \frac{L}{TI} \frac{R}{TI} - \frac{D_1}{TI} - \frac{D_2}{TI} \right] \frac{\partial (\ln(L), \ln(R), \ln(D_1), \ln(D_2))}{\partial (\ln(p_L), \ln(p_R), \ln(p_1), \ln(p_2))} \frac{d \ln(p_L)}{d \ln(1 - \tau)} + \frac{d \ln(p_R)}{d \ln(1 - \tau)} + \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{d \ln(p_2)}{d \ln(1 - \tau)}
\]

... (A.3)
Expression (A.3) is a succinct form of (A.2). I will use this form frequently for ease of writing. In (A.3),
\[
\frac{\partial (\ln(L), \ln(R), \ln(D_1), \ln(D_2))}{\partial (\ln(p_L), \ln(p_R), \ln(p_1), \ln(p_2))}
\]
equals the matrix of
\[
\begin{bmatrix}
\frac{\partial \ln(L)}{\partial \ln(p_L)} & \frac{\partial \ln(L)}{\partial \ln(p_R)} & \frac{\partial \ln(L)}{\partial \ln(p_1)} & \frac{\partial \ln(L)}{\partial \ln(p_2)} \\
\frac{\partial \ln(R)}{\partial \ln(p_L)} & \frac{\partial \ln(R)}{\partial \ln(p_R)} & \frac{\partial \ln(R)}{\partial \ln(p_1)} & \frac{\partial \ln(R)}{\partial \ln(p_2)} \\
\frac{\partial \ln(D_1)}{\partial \ln(p_L)} & \frac{\partial \ln(D_1)}{\partial \ln(p_R)} & \frac{\partial \ln(D_1)}{\partial \ln(p_1)} & \frac{\partial \ln(D_1)}{\partial \ln(p_2)} \\
\frac{\partial \ln(D_2)}{\partial \ln(p_L)} & \frac{\partial \ln(D_2)}{\partial \ln(p_R)} & \frac{\partial \ln(D_2)}{\partial \ln(p_1)} & \frac{\partial \ln(D_2)}{\partial \ln(p_2)}
\end{bmatrix}
\]

and \(\frac{d(\ln(p_L), \ln(p_R), \ln(p_1), \ln(p_2))}{d\ln(1-\tau)}\) equals the matrix of
\[
\begin{bmatrix}
\frac{d\ln(p_L)}{d\ln(1-\tau)} \\
\frac{d\ln(1-\tau)}{d\ln(p_R)} \\
\frac{d\ln(1-\tau)}{d\ln(p_1)} \\
\frac{d\ln(1-\tau)}{d\ln(p_2)}
\end{bmatrix}
\]

In (A.1) to (A.3), all quantities are evaluated at \(\tau = 0\). Expressions (A.1) says that the ETI is the weighted average of taxable income’s 4 components’ elasticities – the elasticity of \(L\), the elasticity of \(R\), the elasticity of \(D_1\), and the elasticity of \(D_2\), all with respect to \(1 - \tau\). If \(L\) is labor income, then \(\frac{d\ln(L)}{d\ln(1-\tau)}\) is the labor supply elasticity.\(^{39}\) If \(D_1\) is charitable giving, then its elasticity is the tax-price elasticity of charitable giving. The weights, \(\frac{L}{TI}, \frac{R}{TI}, \frac{-D_1}{TI}\), and \(\frac{-D_2}{TI}\), are the components’ relative sizes as proportions of taxable income, signed positively or negatively based on whether a component is an income or a deduction. The weights sum up to 1 since \(TI\) equals \(L+R-D_1-D_2\). Expression (A.2) says that each of the 4 component elasticities in (A.1) consists of one own price elasticity with respect to the component’s own tax-price and three cross price elasticities with respect to, respectively, the other three components’ tax-prices, weighted by each tax-price’s derivative with respect to \(1-\tau\). So if the tax rule regarding any one component changes, the \(ETI\) and all 4 component elasticities in (A.1) could potentially change. For example, if \(p_1\) changes from \(1-\tau\) to \(1\) as a result of the repealing of the deductibility of \(D_1\), then all derivatives in (A.2) involving \(p_1\) (in the vertical dashed box in (A.2)) would change. As a result, aside from the \(ETI\) and the elasticity of \(D_1\) with respect to \(1-\tau\), (A.1)’s elasticities of \(L\), \(R\), and \(D_2\) (all with respect to \(1-\tau\)) will change as well through the cross price effects. In this paper I estimate a pair of cross price effects in a case study of two specific deductions. In other words, the

\(^{39}\) More rigorously, it is the labor supply elasticity only if wage is not affected by marginal tax rates. Otherwise it can only be called the labor income elasticity. Kubik (2004) finds that TRA86 affected equilibrium sector wages. Please see the footnote in Appendix 1 for more details.
interests of this paper are terms like $\frac{\partial \ln (D_1)}{\partial \ln (P_2)}$ and $\frac{\partial \ln (P_2)}{\partial \ln (P_1)}$ in (A.2). In the rest of this section, I review the literature. For each category of empirical studies, I point out what parameter(s) it estimates in terms of (A.1) and (A.2). This will clarify relations of the studies, and evidence the importance of understanding ETI’s own and cross elasticity structure.

**A2.2 the ETI**

In terms of (A.1) and (A.2), empirical studies on ETI estimate the single parameter, $ETI$, on the left hand side of (A.1). Feldstein (1996) estimates it to be between 1 and 3, and the subsequent literature have lower estimates. A representative estimate is Gruber and Saez’s (2002) 0.40. Table A.1 lists existing estimates varying between -1.3 and 2.40 Saez, Slemrod and Giertz (2012) do a thorough literature review, and conclude that the best available long-run estimates range from 0.12 to 0.4.41

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</table>

Source: Gruber and Saez (2002)

---

40 This table also contains estimates of the elasticity of broad income, which I discuss later.
41 Meanwhile, they also conclude that there are not truly convincing estimates of the long-run elasticity, because it is very hard to identify the effect of tax rate changes separately from other factors changing the income distribution.
Tax reforms often redefine taxable income. Expressions (A.1) and (A.2) make it clear that the tax system before such a reform and the system after it can have different ETIs. A related estimation issue highlights the importance of the own and cross elasticity structure in (2). If the pre-reform taxable income definition differs from the post-reform taxable income definition, estimating the ETI by “naively” regressing the difference between the two differently defined taxable incomes on the change in tax rate is wrong. It will confound tax-induced changes in behavior with definitional changes (Slemrod 1998). For example, consider a hypothetical tax reform that removes the charitable giving deduction and decreases the tax rate from 20% to 10%. A hypothetical household does not respond to tax law changes at all, and always earn $100 in income and donate $20 to charities. Although their ETI is always zero, the aforementioned “naive” regression will find their taxable income increasing from $100-$20=$80 to $100 responding to the 10% tax cut. Aware of this, researchers have used either a consistent pre- or post-definition of taxable income (Slemrod 1998). However, the more hidden issue is: as long as the responsiveness to tax rates is not zero, even if we use a consistent pre- or post-definition, the ETI estimate using data before and after the reform may be neither an unbiased estimate of the pre-reform ETI nor an unbiased estimate of the post-reform ETI. In addition, it may be neither an unbiased estimate of the responsiveness of the post-reform style taxable income to tax rates under the pre-reform system, nor an unbiased estimate of the responsiveness of the pre-reform style taxable income to tax rates under the post-reform system. In short, the estimate may bias from any meaningful elasticity. I will explain why thoroughly. Gruber and Saez (2002) briefly refer to this issue pertaining to their estimation. Slemrod (1998) discusses this issue in more details with an example. In both papers, the biases have certain directions. Now I explain this issue more formally and more systematically from the perspective of expression (A.2). Along the way I will pick up Gruber and Seaz (2002)’s and Slemrod (1998)’s examples.

Consider a tax reform that broadens the base from $TI_0 = L + R - D_1 - D_2$ to $TI_1 = L + R - D_1$, and cuts the tax rate from $\tau = \tau_0$ to $\tau = \tau_1$. Call the tax system before the reform System 0 and the tax system after the reform System 1. Now we are perhaps the most interested in the elasticity of taxable income under System 1. Call it $ETI_{11}$. This is the elasticity of $TI_1 = L + R - D_1$ with respect to $1-\tau$ under System 1. In addition, we may also want to know $ETI_{00}$, the elasticity of $L + R - D_1 - D_2$ with respect to $1-\tau$ under System 0; $ETI_{10}$, the elasticity of $L + R - D_1$ with respect to $1-\tau$ under System 0; and $ETI_{01}$, the elasticity of $L + R - D_1 - D_2$ with respect to $1-\tau$ under System 0. Now I illustrate that, in light of the structure in (A.2), the reform-induced change in $\ln(L + R - D_1)$ (or the change in $\ln(L + R - D_1 - D_2)$ as well) divided by $\ln(1 - \tau_1) - \ln(1 - \tau_0)$, i.e. $\hat{e}_1 = \frac{\Delta \ln(L+R-D_1)}{\ln(1-\tau)}$ (or $\hat{e}_0 = \frac{\Delta \ln(L+R-D_1-D_2)}{\ln(1-\tau)}$) may not be an unbiased estimate of any of the four elasticities of $ETI_{11}$, $ETI_{00}$, $ETI_{10}$, and $ETI_{01}$. Without loss of generality, normalize the amount of $\ln(L + R - D_1)$ at $\tau = 0$ to 1 and the amount of $\ln(L + R - D_1 - D_2)$ at $\tau = 0$ to $\omega < 1$. By definition of the elasticities, the relations between $L + R - D_1 - D_2$ as well as $L + R - D_1$ and $1 - \tau$ satisfy the following four equations:

Under System 0: $\ln(L + R - D_1 - D_2) = w + ETL_{00} \times \ln(1 - \tau)$  \( \ldots (A.4) \)

$\ln(L + R - D_1) = 1 + ETL_{10} \times \ln(1 - \tau)$  \( \ldots (A.5) \)
Under System 1: \( \ln(L + R - D_1 - D_2) = w + ETI_{01} \times \ln(1 - \tau) \) \ldots (A.6)

\( \ln(L + R - D_1) = 1 + ETI_{11} \times \ln(1 - \tau) \) \ldots (A.7)

Now I plot all four equations on one graph. They will be four straight lines with slopes of, respectively, \( ETI_{00}, ETI_{10}, ETI_{01}, \) and \( ETI_{11} \), and with intercepts of, respectively, \( w, 1, w, \) and \( 1 \). The plot will demonstrate how \( \hat{e}_1 \) and \( \hat{e}_0 \) could be biased from \( ETI_{00}, ETI_{10}, ETI_{01}, \) and \( ETI_{11} \). Before plotting, I discuss the relative magnitudes of the four slopes. In succinct expressions like (A.3), the four slopes are:

\[ ETI_{00} = \frac{d}{d \ln(1 - \tau)} \left[ \frac{L}{TI_0} \right] = \frac{R}{TI_0} - \frac{D_1}{TI_0} - \frac{D_2}{TI_0} \frac{\partial \ln(L) + \ln(R), \ln(D_1), \ln(D_2)}{\partial \ln(p_1), \ln(p_R), \ln(p_1), \ln(p_2)} \]

\[ ETI_{10} = \frac{d}{d \ln(1 - \tau)} \left[ \frac{L}{TI_1} \right] = \frac{R}{TI_1} - \frac{D_1}{TI_1} \frac{\partial \ln(L) + \ln(R), \ln(D_1)}{\partial \ln(p_1), \ln(p_R), \ln(p_1), \ln(p_2)} \]

\[ ETI_{01} = \frac{d}{d \ln(1 - \tau)} \left[ \frac{L}{TI_0} \right] = \frac{D_1}{TI_0} \frac{\partial \ln(L) + \ln(R), \ln(D_1), \ln(D_2)}{\partial \ln(p_1), \ln(p_R), \ln(p_1), \ln(p_2)} \]

\[ ETI_{11} = \frac{d}{d \ln(1 - \tau)} \left[ \frac{L}{TI_1} \right] = \frac{D_1}{TI_1} \frac{\partial \ln(L) + \ln(R), \ln(D_1)}{\partial \ln(p_1), \ln(p_R), \ln(p_1)} \]

Their differences depend on various own and cross elasticities in (A.2). For example, \( ETI_{10} \) and \( ETI_{11} \) differ by \( \frac{d}{d \ln(1 - \tau)} \left[ \frac{L}{TI_1} \right] - \frac{d}{d \ln(1 - \tau)} \left[ \frac{L}{TI_0} \right] = \frac{D_1}{TI_1} \frac{\partial \ln(D_2)}{\partial \ln(p_2)} \frac{\ln(p_1)}{\ln(1 - \tau)} \), whose sign is uncertain. In general, depending on the values of the own and cross elasticities, any one of the four slopes can possibly be larger, smaller, or equal to another. Figure A.1 is plotted under one possible scenario of \( ETI_{11} > ETI_{10}, ETI_{00} > ETI_{10} \) and \( ETI_{00} > ETI_{01} \). The four thin lines plot (A.4) to (A.7), with slopes labelled. I claim without showing the tedious proof that, one set of sufficient conditions that could produce this ordinal relationship is: (c1) \( \frac{\partial \ln(L)}{\partial \ln(p_2)} \frac{\partial \ln(D_2)}{\partial \ln(p_1)} \), (c2) \( \frac{\partial \ln(D_1)}{\partial \ln(p_1)} \), and (c3) \( \frac{\partial \ln(D_2)}{\partial \ln(p_2)} \) are negligible; (c2) \( \frac{\partial \ln(D_2)}{\partial \ln(p_1)} < 0 \), (c3) \( \frac{\partial \ln(D_2)}{\partial \ln(p_2)} > 0 \); and (c3) \( \frac{\partial \ln(D_2)}{\partial \ln(p_1)} + \frac{\partial \ln(D_2)}{\partial \ln(p_2)} \frac{\ln(p_1)}{\ln(1 - \tau)} < 0 \). Condition (c1) says that incomes do not respond to the tax-price of \( D_2 \) and vice versa. Condition (c2) says that \( D_2 \) decreases in its own tax-price and that \( D_1 \) and \( D_2 \) are substitutes. An example of substitutable deductions are consumer interest and home equity loan interest, both deductible before TRA 86 (Slemrod 1998). Condition (c3) says that the absolute value of the own price elasticity of \( D_2 \) is larger than the cross elasticities between \( D_1 \) and \( D_2 \).

In Figure A.1, Point A marks the pre reform level of \( \ln(L + R - D_1) \) under \( \tau = \tau_0 \). Point B marks the post reform level of \( \ln(L + R - D_1) \) under \( \tau = \tau_1 \). Therefore the slope of line AB is \( \hat{e}_1 = \frac{\Delta \ln(L + R - D_1)}{\Delta \ln(1 - \tau)} \). The graph shows that \( \hat{e}_1 \) can be different from \( ETI_{11} \), and deviate from the other three meaningful taxable income elasticities as well. Similarly, Point C and Point D mark the pre- and post-reform levels of \( \ln(L + R - D_1 - D_2) \), and the slope of line CD is \( \hat{e}_0 = \frac{\Delta \ln(L + R - D_1 - D_2)}{\Delta \ln(1 - \tau)} \). As in Figure A.1, \( \hat{e}_0 \) deviates from \( ETI_{00} \) and \( ETI_{01} \), and
is not guaranteed to be close to $ETI_{10}$ or $ETI_{11}$ either. Despite all the lengthy discussion above, the underlying intuition is simple. Both $\hat{e}_0$ and $\hat{e}_1$ embody the effects of both the tax rate change and the taxable income definition change, while all the taxable elasticities we are interested in, especially $ETI_{11}$ and $ETI_{00}$, purely measure the effect of tax rate changes. So the biases of $\hat{e}_0$ and $\hat{e}_1$ from $ETI_{11}$ and $ETI_{00}$ are essentially omitted variable bias. The directions of the biases depend on values of terms in (A.2).

Figure A.1

Conditions (c1) to (c3) are chosen such that Figure A.1 displays the same bias directions with special cases in Gruber and Saez (2002) and Slemrod (1998). Gruber and Saez (2002) say that “we potentially understate the responsiveness of taxable income to taxation, even from the perspective of 1990 [p. 8]”. What causes it is the substitutability between different “avoidance avenues”. This resembles that $ETI_{10}$ understates $ETI_{11}$ due to the substitutability between $D_1$ and $D_2$. In Slemrod (1998) [p. 784], the actual response of the broader base (this response is measured as $\hat{e}_1$ in Figure A.1) understates the would-be response of the broader base with $t_0$ dropping to $t_1$ but without changing the definition of taxable income (this response is $ETI_{10}$ in Figure A.1; “understate” means $\hat{e}_1 < ETI_{10}$). And, the actual response of the narrower base (this response is $\hat{e}_0$ in Figure A.1) overstates the would-be response of the broader base with $t_0$ dropping to $t_1$ but without changing the definition of taxable income (this response is $ETI_{10}$ in Figure A.1; “overstate” means $\hat{e}_0 > ETI_{10}$).

Figure A.1 only displays a baseline scenario of the biases in $\hat{e}_0$ and $\hat{e}_1$. Without restrictions like (c1) to (c3), magnitudes and directions of the biases can be arbitrary, depending on values of the own and cross price elasticities.

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42 They also mention that this understatement is offset by overstatement due to income shifting from individual to corporate income tax base.
A2.3 the labor supply elasticity and the tax-price elasticity of charitable giving

Both elasticities are for a taxable income component with respect to $1 - \tau$. As I mentioned earlier, in my simple hypothetical example with $TI = L + R - D_1 - D_2$, if $L$ represents labor income and $D_1$ represents charitable giving amount, $\frac{d \ln(L)}{d \ln(1 - \tau)}$ and $\frac{d \ln(D_1)}{d \ln(1 - \tau)}$ in (A.1) will be, respectively, the labor supply elasticity with respect to $1 - \tau$ and the charitable giving tax-price elasticity. However, notice that what the tax-price elasticity of charitable giving literature attempts to estimate is not the same with $\frac{d \ln(D_1)}{d \ln(1 - \tau)}$. Based on the literature’s assumptions of other deductions are exogenous and that $\tau$ is constructed to be the marginal tax rate calculated using zero giving but the full amounts of other deductions, what supposedly estimated in the literature is $\frac{d \ln(D_1)}{d \ln(1 - \tau)}$. Before the focus on the ETI, researchers analyzing the income tax primarily focused on estimating the elasticity of labor supply with respect to $1 - \tau$; existing estimates suggest that it is fairly small overall and even close to zero for prime-age males, although it is higher for secondary earners (Saez, Slemrod and Giertz 2012; Gruber and Saez 2002). In contrast, existing studies find significant tax-price elasticities of charitable giving. Table A.2 lists a few representative estimates, all around -1. Table A.3 reproduces the summary of findings in Peloza and Steel (2005)’s meta analysis. These estimates range between -7.07 and 0.12.

### Table A.2 Estimates on the Tax-price Elasticities of Charitable Giving

<table>
<thead>
<tr>
<th>Study</th>
<th>Price Elasticity Estimate (Standard Error)</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakija and Heim (2011)</td>
<td>-1.10 (0.45) a</td>
<td>1979 – 2006 tax returns, panel</td>
</tr>
<tr>
<td>Auten, Sieg, and Clotfelter (2002)</td>
<td>-1.26 (0.04); -0.46 b</td>
<td>1979 – 1993 tax returns, panel</td>
</tr>
<tr>
<td>Barrett (1991)</td>
<td>-1.09 (0.11)</td>
<td>1979 – 1986 tax returns, panel</td>
</tr>
<tr>
<td>Friedberg and He (2015)</td>
<td>-1.23 (0.55)</td>
<td>1989 – 2007 Survey of Consumer Finances</td>
</tr>
<tr>
<td>Tiehen (2001)</td>
<td>-1.15 (0.68)</td>
<td>1987 – 1995 Independent Sector Surveys on Giving and Volunteering</td>
</tr>
<tr>
<td>Reece (1979)</td>
<td>-1.19 (0.29)</td>
<td>1972 – 73 Consumer Expenditure Survey c</td>
</tr>
<tr>
<td>Reece and Zieschang (1985)</td>
<td>-0.85</td>
<td>1972 – 73 Consumer Expenditure Survey c</td>
</tr>
</tbody>
</table>

a: They estimated the elasticities for “persistent” price”, “future” price, and “transitory” price. -1.10 is the estimate for the persistent price elasticity.

b: -1.26, is their core estimate under certain econometric assumptions for the change of permanent and transitory income and price; -0.46, which only appears in a footnote, is from a pooled regression model using fixed effects and therefore more comparable to other estimates in the literature.

c: these two studies used the same data, but Reece (1979) does a reduced form regression while Reece and Zieschang (1985) uses the Hausman method.

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43 Again, $\frac{d \ln(L)}{d \ln(1 - \tau)}$ is labor supply elasticity assuming away wage change. See an earlier footnote.

44 Blundell and Macurdy (1999) and McClelland and Mok (2012) provide through reviews of the literature on labor supply elasticity.
A2.4 The broad income elasticity

The literature frequently estimate the elasticity of a measure of income broader than taxable income (see Table A.1 earlier). In my illustrative example with $TI = L + R - D_1 - D_2$, a “broad income” could be defined as $B = L + R$, and a “broad income elasticity” refers to the elasticity of $B$ with respect to $1 - \tau$. In forms like Expressions (A.1) and (A.2), this means rewriting taxable income as $TI = B - D_1 - D_2$ and rewriting Expressions (A.1) and (A.2) as

$$ETI = \frac{B}{TI} \frac{d \ln(B)}{d \ln(1 - \tau)} - \frac{D_1}{TI} \frac{d \ln(D_1)}{d \ln(1 - \tau)} - \frac{D_2}{TI} \frac{d \ln(D_2)}{d \ln(1 - \tau)}$$

$$= \frac{B}{TI} \left[ \frac{\partial \ln(B)}{\partial \ln(p_B)} \frac{d \ln(p_B)}{d \ln(1 - \tau)} + \frac{\partial \ln(B)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{\partial \ln(B)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1 - \tau)} \right]$$

$$\frac{D_1}{TI} \left[ \frac{\partial \ln(D_1)}{\partial \ln(p_B)} \frac{d \ln(p_B)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_1)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_1)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1 - \tau)} \right]$$

$$\frac{D_2}{TI} \left[ \frac{\partial \ln(D_2)}{\partial \ln(p_B)} \frac{d \ln(p_B)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_2)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{\partial \ln(D_2)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1 - \tau)} \right]$$

$$\ldots (A.1')$$

$$= \left[ \frac{B}{TI} - \frac{D_1}{TI} - \frac{D_2}{TI} \right] \frac{\partial (\ln(B), \ln(D_1), \ln(D_2))}{\partial (\ln(p_B), \ln(p_1), \ln(p_2))} \frac{d \ln(p_B)}{d \ln(1 - \tau)} + \frac{d \ln(p_1)}{d \ln(1 - \tau)} + \frac{d \ln(p_2)}{d \ln(1 - \tau)}$$

$$\ldots (A.2')$$

The broad income elasticity would be $\frac{d \ln(B)}{d \ln(1 - \tau)}$ in (A.1’), equal to the three-term sum in (A.2’’s first bracket. So broad income elasticity characterizes the “average” responsiveness of $L$ and $R$ to $1 - \tau$. In this view the broad income elasticity is more like a component elasticity such as the charitable giving elasticity than the ETI in nature.

Gruber and Saez (2002) find a large difference between the broad income elasticity (0.120) and the ETI (0.400). In terms of (A.1’), it is the difference between $\frac{d \ln(B)}{d \ln(1 - \tau)}$ and $ETI$. They mention two sources of the difference, one “mechanical” (see footnote) and one “behavioral” (i.e. the itemized deductions being responsive). To understand their meaning of “mechanical”, consider a household who respond to a tax rate change by increasing broad income by $x$ dollars. As broad income is larger than taxable income (by Gruber and Saez (2002)’s definition, taxable income equals broad income less deductions and exemptions), the resulted percentage change in

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45 To understand their meaning of “mechanical”, consider a household who respond to a tax rate change by increasing broad income by $x$ dollars. As broad income is larger than taxable income (by Gruber and Saez (2002)’s definition, taxable income equals broad income less deductions and exemptions), the resulted percentage change in
the right hand side and thus makes crystal clear what exactly the two sources of difference are: the “mechanical” source is \( \frac{B}{T} \) and the “behavioral” source is \( -\frac{D_1}{T} \frac{d \ln(D_1)}{d \ln(1-\tau)} - \frac{D_2}{T} \frac{d \ln(D_2)}{d \ln(1-\tau)} \). From (A.1’) we also know that these two sources of difference Gruber and Saez (2002) give are exhaustive, i.e. there are no other sources of difference.

**A2.5 Cross price effect portions of the broad income elasticity and the ETI**

Kopczuk (2005) estimates the broad income elasticity as a function of the proportion of spending on tax deductible commodities. Similar to previous sections, I describe what he estimates in terms of notations in my simple example.

The broad income part of the decomposition from (A.1’) to (A.2’) is

\[
\frac{d \ln(B)}{d \ln(1-\tau)} = \frac{\partial \ln(B)}{\partial \ln(p_B)} \frac{d \ln(p_B)}{d \ln(1-\tau)} + \frac{\partial \ln(B)}{\partial \ln(p_1)} \frac{d \ln(p_1)}{d \ln(1-\tau)} + \frac{\partial \ln(B)}{\partial \ln(p_2)} \frac{d \ln(p_2)}{d \ln(1-\tau)}.
\]

In terms of this decomposition, what Kopczuk (2005) examines is the role of a certain average of the last two terms. I illustrate his process below.

I condense the last two terms on the right hand side. In Kopczuk (2005), all prices equal \( 1 - \tau \). In my example, this means \( p_B = p_1 = p_2 = 1 - \tau \) and \( \frac{d \ln(p_B)}{d \ln(1-\tau)} = \frac{d \ln(p_1)}{d \ln(1-\tau)} = \frac{d \ln(p_2)}{d \ln(1-\tau)} = 1 \). It follows that

\[
\frac{d \ln(B)}{d \ln(1-\tau)} = \frac{\partial \ln(B)}{\partial \ln(p_B)} + \frac{\partial \ln(B)}{\partial \ln(p_1)} + \frac{\partial \ln(B)}{\partial \ln(p_2)} + \left[ \frac{\partial \ln(B)}{\partial \ln(p_1)} + \frac{\partial \ln(B)}{\partial \ln(p_2)} \right].
\]

Using the symmetry of the Slutsky matrix gives

\[
\frac{d \ln(B)}{d \ln(1-\tau)} = \frac{\partial \ln(B)}{\partial \ln(p_B)} + \left[ \frac{D_1}{B} \frac{\partial \ln(D_1)}{\partial \ln(p_B)} + \frac{D_2}{B} \frac{\partial \ln(D_2)}{\partial \ln(p_B)} \right].
\]

Introducing the “tax base” measure of \( \gamma = \frac{D_1 + D_2}{B} \), or the total of deductions as a proportion of broad income, the above equation becomes

\[
\frac{d \ln(B)}{d \ln(1-\tau)} = \frac{\partial \ln(B)}{\partial \ln(p_B)} + \left[ \frac{D_1}{D_1 + D_2} \frac{\partial \ln(D_1)}{\partial \ln(p_B)} + \frac{D_2}{D_1 + D_2} \frac{\partial \ln(D_2)}{\partial \ln(p_B)} \right] \frac{D_1 + D_2}{B}.
\]

---

46 This symmetry only holds for compensated elasticities. Therefore, for all the math steps onwards to hold, all elasticities should be treated as compensated elasticities. Empirically though, the difference (which is the income effect of tax rate changes) between uncompensated and compensated elasticity of broad income or taxable income is not important. Gruber and Saez (2002) find the income effect of tax rate changes on taxable income as well as broad income quite small and highly insignificant, and believe it safe to assume that the compensated and uncompensated elasticity are identical. The review of the ETI literature by Saez, Slemrod and Giertz (2012) says that compelling evidence about significant income effects is absent. Hendren (2015) points out that causal effects, which can be uncompensated elasticity and not necessarily compensated elasticity, are meaningful welfare measurement as well.
Let $\epsilon$ denote $\frac{\partial \ln(B)}{\partial \ln(p_B)}$ and let $\beta$ denote everything in the bracket in the previous line. Then the above equation becomes

$$\frac{d \ln(B)}{d \ln(1-\tau)} = \epsilon + \beta \gamma.$$ 

So in terms of my simplified 4-component example, what Kopczuk (2005) does is to condense the decomposed broad income elasticity in (A.2')’s first bracket into one own price elasticity, $\epsilon$, and an average of all cross price elasticities, $\beta$. It says that the broad income elasticity varies in the proportion of deductions $\gamma$. The corresponding form of regression equation with panel data is then

$$\Delta \ln(B) = \epsilon \Delta \ln(1-\tau) + \beta \gamma \Delta \ln(1-\tau) + \text{other controls}.$$ 

Further, Kopczuk considers the effect of changes in $\gamma$, denoted as $\Delta \gamma$, from introductions of new deductions. For example, if the tax law adds a new deduction $\text{-}D_3$ to $TI$’s definition, $\gamma$ will increase from $\frac{D_1 + D_2}{B}$ to $\frac{D_1 + D_2 + D_3}{B}$. The differential relation that incorporates $\Delta \gamma$ can be written as

$$\Delta \ln(B) = \epsilon \Delta \ln(1-\tau) + \beta \Delta [\gamma \ln(1-\tau)] + \text{other controls},$$ 

under an assumption. The assumption is that the cross elasticities of newly added deductions with respect to $p_B$, e.g. $\frac{\partial \ln(D_3)}{\partial \ln(p_B)}$, are equal to $\beta = \frac{D_1}{D_1 + D_2} \frac{\partial \ln(D_1)}{\partial \ln(p_B)} + \frac{D_2}{D_1 + D_2} \frac{\partial \ln(D_2)}{\partial \ln(p_B)}$, i.e. the average of existing deductions’ cross elasticities with respect to $p_B$. In this paper I examine the heterogeneity among different cross elasticities.

Kopczuk (2005) also estimates a similar equation for taxable income, which is $\Delta \ln(TI) = \epsilon' \Delta \ln(1-\tau) + \beta' \Delta [\gamma \ln(1-\tau)] + \text{other controls}$. He finds that both the broad income elasticity and the ETI increases in the proportion of deductions $\gamma$. This implies that broadening the tax base, by removing deductions for example, reduces the deadweight loss per dollar of collected revenue. However, not all base broadening methods are equal. Depending on the values of cross elasticities such as $\frac{\partial \ln(D_1)}{\partial \ln(p_2)}$ and $\frac{\partial \ln(D_2)}{\partial \ln(p_1)}$ in (A.2), removing certain deductions will reduce the deadweight loss more effectively than removing others. If we are agnostic on the cross elasticity values, lowering all deductions can more safely reduce the deadweight loss and increase revenue than removing a subset of deductions.

Kopczuk (2005) concludes that models in previous ETI studies neglecting $\gamma$ are misspecified. In addition, he concludes that ETI is not an immutable parameter - meaning that it can be to some extent controlled by policy makers – as it varies with individual’s access to tax-deductible items.
Appendix 4  Accounting for the limit on home equity debt deduction

By the tax code (see IRS publication 936), interest payment on home equity debt above $100,000 are not tax deductible unless it is paid for investment activities (observed in the SCF). Suppose that a household’s non-HELOC home equity loans’ balance is $b$, then the limit left for their HELOC is $l = $100,000-$b$. For households with balance $B$ larger than the limit $l$ (and not borrowing to finance investment activities for which interest is deductible), the first $l \cdot r$ dollars of interest payment is deductible while the following $(B - l) \cdot r$ is not different from principal payment for tax purposes.

(1) For households (about 6% of the sample) with $B > l$ and $Q \leq l$ (why $Q \leq l$ is included will be clear in (2)) (and not borrowing to finance investment activities for which interest is deductible), expression (5) is trivially different from other households. Simply replace $B \cdot r$ with $l \cdot r$. In other words, we have:

$$p_h = \begin{cases} 
1 - \tau - r(1 - \tau), & Q < l \cdot r \\
1 - \frac{r \cdot r}{Q} - r(1 - \tau), & Q \geq l \cdot r
\end{cases} \quad \ldots \text{(A4.1)}$$

(2) For households (about 4% of the sample) with $B > l$ and $Q > l$ (and not borrowing to finance investment activities for which interest is deductible), their future interest saving benefit is different from the case discussed in the main text. Specifically, out of the total repayment of $Q$, if a household postpones the last $l$ dollars to the next year, these $l$ dollars will generate deductible interest. However, if a household postpones the last $l+1$ dollars to the next year, the $(l+1)$th dollar postponed will generate non-deductible interest. In sum, the benefit of saving future interest for future after tax consumption is $r$ for each of the first $Q - l$ dollars and $r(1 - \tau)$ for the last $l$ dollars. For the current year after tax consumption, each of the first $l \cdot r$ dollars of
repayment forgoes $1 - \tau$ dollar of after tax consumption and each of the last $Q - l \cdot r$ dollars of repayment forgoes 1 dollar of after tax consumption. So, if $Q - l \geq l \cdot r$, then the average price at $Q$ is

$$p_h = \frac{l \cdot r \cdot (1 - \tau - r) + (Q - l - l \cdot r)(1 - r) + l \cdot [1 - r(1 - \tau)]}{Q} = 1 - r.$$  

If $Q - l < l \cdot r$, then the average price at $Q$ is

$$p_h = \frac{(Q - l)(1 - \tau - r) + (l + l \cdot r - Q)[1 - \tau - r(1 - \tau)] + (Q - l \cdot r)[1 - r(1 - \tau)]}{Q} = 1 - r.$$  

**Appendix 5  A discussion of sample selection bias**

As described in the literature review, I restrict my sample to “exogenous itemizers”, defined as taxpayers who have more non-charity and non-HELOC-interest-payments than their standard deduction. Further, I restrict my sample to exogenous itemizers who have a HELOC with positive outstanding balance, so that they face the choice on how much to give and how much to repay on HELOCs. This could result in sample selection bias if an unobservable factor affects both the choice of whether to borrow and spending on donations and debt payments. The degree of one’s impatience could be such a factor. For example, a household may face a high interest rate but, due to a high level of impatience, still decides to borrow money to buy a new piece of furniture, despite the fact that a high interest rate hurts future consumption. This impatience may also induce the household to donate instead of paying down debt (and being able to donate more in the future). Under this scenario, households in the selected sample that begin to borrowing under high interest rates are the more impatient ones and not representative. That they donate more and pay less on debt than an average household does will lead to an underestimate of the cross-price effect of interest rate on giving. However, concerns about this issue may be moderated by two reasons. First, HELOC interest rates are variable over time. Therefore, observing a high HELOC rate for a household does not necessarily imply high impatience. Second, impatient people do not always have much say in the choice of borrowing. In situations like replacing an old piece of furniture with a new one, more patient households may wait until a low interest year to borrow. But, in other situations such as repairing one’s roof or borrowing for children’s college education, one may not have the option to wait.
### Appendix 6  Comparing my HELOC borrower sample’s statistics with a broader sample

<table>
<thead>
<tr>
<th>Label</th>
<th>Mean</th>
<th>Median</th>
<th>P75%</th>
<th>P90%</th>
</tr>
</thead>
<tbody>
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<td>Charity</td>
<td>3,564</td>
<td>1,303</td>
<td>3,339</td>
<td>7,439</td>
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<tr>
<td></td>
<td>4,356</td>
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<td>$\tau$</td>
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<td>28%</td>
<td>31%</td>
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<tr>
<td></td>
<td>21%</td>
<td>25%</td>
<td>28%</td>
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<td>118,852</td>
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<td>5.5</td>
<td>6.0</td>
<td>6.8</td>
</tr>
</tbody>
</table>

This table compares the summary statistics of my sample ($N=1,103$) and the larger sample of all exogenous itemizers ($N=12,209$). To the right of each variable name, I report my sample’s statistics in the first row and the larger sample’s in the second row.
## Appendix 7 Unweighted regression results

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Marginal Price</th>
<th></th>
<th></th>
<th>Average Price</th>
<th></th>
<th></th>
<th>Regular Giving Elasticity</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>(Standard Error)</td>
<td>p-value</td>
<td>Estimate</td>
<td>(Standard Error)</td>
<td>p-value</td>
<td>Estimate</td>
<td>(Standard Error)</td>
<td>p-value</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td></td>
<td></td>
<td>p-value</td>
<td></td>
<td></td>
<td>p-value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Ln(charit) equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-13.78 (1.39)**</td>
<td>0.00%</td>
<td></td>
<td>-13.60 (1.34)**</td>
<td>0.0%</td>
<td></td>
<td>-12.81 (1.38)**</td>
<td>0.00%</td>
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</tr>
<tr>
<td>Ln(p_g)</td>
<td>-0.64 (1.02)</td>
<td>53.1%</td>
<td></td>
<td>-1.23 (1.24)</td>
<td>32.2%</td>
<td></td>
<td>-1.12 (1.02)</td>
<td>27.4%</td>
<td></td>
</tr>
<tr>
<td>Ln(p_h)</td>
<td>2.84 (2.59)</td>
<td>27.3%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(p_k)</td>
<td></td>
<td></td>
<td></td>
<td>1.85 (1.68)</td>
<td>27.2%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(wealth)</td>
<td>0.36 (0.07)**</td>
<td>0.0%</td>
<td></td>
<td>0.35 (0.07)**</td>
<td>0.0%</td>
<td></td>
<td>0.38 (0.07)**</td>
<td>0.0%</td>
<td></td>
</tr>
<tr>
<td>Ln(disposable income)</td>
<td>0.74 (0.15)**</td>
<td>0.8%</td>
<td></td>
<td>0.74 (0.15)**</td>
<td>1.5%</td>
<td></td>
<td>0.65 (0.16)**</td>
<td>0.0%</td>
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</tr>
<tr>
<td>Dummy for middle aged</td>
<td>0.47 (0.27)*</td>
<td>8.1%</td>
<td></td>
<td>0.54 (0.26)**</td>
<td>3.6%</td>
<td></td>
<td>0.54 (0.26)**</td>
<td>3.7%</td>
<td></td>
</tr>
<tr>
<td>Dummy for the elder</td>
<td>0.74 (0.47)</td>
<td>11.6%</td>
<td></td>
<td>0.87 (0.45)*</td>
<td>5.5%</td>
<td></td>
<td>0.84 (0.45)*</td>
<td>6.2%</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>0.01 (0.01)</td>
<td>41.2%</td>
<td></td>
<td>0.01 (0.01)</td>
<td>55.6%</td>
<td></td>
<td>0.01 (0.01)</td>
<td>55.1%</td>
<td></td>
</tr>
<tr>
<td>Years of education</td>
<td>0.20 (0.04)**</td>
<td>0.0%</td>
<td></td>
<td>0.20 (0.04)**</td>
<td>0.0%</td>
<td></td>
<td>0.20 (0.04)**</td>
<td>0.0%</td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>0.70 (0.21)**</td>
<td>0.1%</td>
<td></td>
<td>0.70 (0.21)**</td>
<td>0.1%</td>
<td></td>
<td>0.75 (0.20)**</td>
<td>0.0%</td>
<td></td>
</tr>
<tr>
<td>Ln(balance)</td>
<td>0.22 (0.09)**</td>
<td>1.5%</td>
<td></td>
<td>0.24 (0.10)**</td>
<td>1.8%</td>
<td></td>
<td>0.17 (0.07)**</td>
<td>2.0%</td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.01 (0.08)</td>
<td>92.4%</td>
<td></td>
<td>0.00 (0.08)</td>
<td>95.9%</td>
<td></td>
<td>0.36 (0.08)**</td>
<td>6.9%</td>
<td></td>
</tr>
<tr>
<td>The Ln(payment) equation</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>2.07 (0.87)**</td>
<td>1.7%</td>
<td></td>
<td>1.66 (0.69)**</td>
<td>1.6%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(p_g)</td>
<td>-6.38 (1.64)**</td>
<td>0.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(p_h)</td>
<td></td>
<td></td>
<td></td>
<td>-4.14 (0.88)**</td>
<td>0.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(p_k)</td>
<td>-0.39 (0.62)</td>
<td>53.5%</td>
<td></td>
<td>0.94 (0.64)</td>
<td>13.9%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(wealth)</td>
<td>0.05 (0.04)</td>
<td>22.7%</td>
<td></td>
<td>0.08 (0.04)**</td>
<td>2.8%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(disposable income)</td>
<td>-0.10 (0.09)</td>
<td>25.0%</td>
<td></td>
<td>-0.09 (0.07)</td>
<td>32.3%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy for middle aged</td>
<td>0.26 (0.17)</td>
<td>12.2%</td>
<td></td>
<td>0.11 (0.14)</td>
<td>41.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy for the elder</td>
<td>0.22 (0.30)</td>
<td>45.0%</td>
<td></td>
<td>-0.06 (0.23)</td>
<td>79.5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.01 (0.01)</td>
<td>47.6%</td>
<td></td>
<td>0.00 (0.01)</td>
<td>80.1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years of education</td>
<td>0.00 (0.02)</td>
<td>95.7%</td>
<td></td>
<td>0.01 (0.02)</td>
<td>66.5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>0.38 (0.13)**</td>
<td>0.3%</td>
<td></td>
<td>0.38 (0.11)**</td>
<td>0.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln(balance)</td>
<td>0.54 (0.06)**</td>
<td>0.0%</td>
<td></td>
<td>0.50 (0.05)**</td>
<td>0.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.08 (0.05)</td>
<td>11.6%</td>
<td></td>
<td>0.09 (0.04)**</td>
<td>3.3%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This table presents results for the two-equation regressions of \( \ln(\text{charity}+10) \) and \( \ln(\text{payment}) \), defined in the text, on a group of covariates. The weights for the regression are used to make the results nationally representative. The sample consists of households who are “exogenous itemizers” (defined in the text) and whose largest line of credit is a home equity line of credit with a positive balance. The observations are drawn from the Surveys of Consumer Finances between 1989 and 2007.

### Appendix 8  First stage results

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Marginal Price (( \ln(p_q) ))</th>
<th>Average Price (( \ln(p_h) ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (Standard Error)</td>
<td>p-value</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.29 (0.09)**</td>
<td>0.18%</td>
</tr>
<tr>
<td>( \ln(p_{in}) )</td>
<td>0.84 (0.12)**</td>
<td>0.00%</td>
</tr>
<tr>
<td>( \ln(p_{e}) )</td>
<td>-0.89 (0.16)**</td>
<td>0.00%</td>
</tr>
<tr>
<td>( \ln(wealth) )</td>
<td>3.04E-4 (4.07E-3)</td>
<td>94.04%</td>
</tr>
<tr>
<td>( \ln(disposable income) )</td>
<td>-0.02 (0.01)**</td>
<td>4.93%</td>
</tr>
<tr>
<td>Dummy for middle aged</td>
<td>0.01 (0.01)</td>
<td>29.47%</td>
</tr>
<tr>
<td>Dummy for the elder</td>
<td>-2.48E-3 (0.02)</td>
<td>91.10%</td>
</tr>
<tr>
<td>Age</td>
<td>4.74E-4 (6.48E-4)</td>
<td>46.44%</td>
</tr>
<tr>
<td>Years of education</td>
<td>-1.55E-3 (1.75E-3)</td>
<td>37.41%</td>
</tr>
<tr>
<td>Married</td>
<td>0.01 (0.01)</td>
<td>16.78%</td>
</tr>
<tr>
<td>( \ln(balance) )</td>
<td>-0.02 (3.66E-3)**</td>
<td>0.00%</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.01 (4.48E-3)</td>
<td>17.58%</td>
</tr>
</tbody>
</table>

Notes: \( N = 1,023 \)  ***Significant at 1% level  **significant at 5% level  *significant at 10% level

This table presents the first stage results corresponding to the 3SLS regression results in Table 3.
Appendix 9  Kernel Density Estimation of HELOC Payments

This graph shows that there is no bunching at paying exactly the interest due. This graph plots the kernel density estimation for HELOC payment for households whose principal payments are below 10000, using the first implicate dataset of the Survey of Consumer Finances. Other implicate datasets produce highly similar graphs which are therefore not presented here. The variable “diff” labeled on the horizontal axis is defined as the difference between payment and interest due. For example, 0 means paying exactly the interest amount due; 5,000 means paying interest and then paying down principle by 5,000. I do not think that there is bunching at paying exactly interest: from both the left and the right of the peak the density curve is rising smoothly without having a sudden jump. Also, the peak is not exactly at zero, rather it is at a little to the right of zero. Therefore, even though the density below zero is very low, this looks more like households paying interest due under minimum payment requirements and then on top of that paying a little of principal, than them responding to a price kink. I also do not observe minimum payment requirement on the households and cannot examine this further.

Appendix 10  Tax-prices under partial deductibility

In this appendix I provide a proof for the tax-price under partial deductibility, and corresponding changes in Equation (4). The proof is more rigorous than the one in the main text.
Under the existing tax system where \( x\% = 100\% \), for each dollar of pretax income, if I spend it all on myself, I pay \( \tau \) dollar as tax, and spend \((1-\tau)\) on personal consumption. To check that it’s correctly calculated, I have

\[
\text{Personal Consumption + Tax Liability} = (1-\tau) + \tau = 1 = \text{Pretax Income}.
\]

If I donate it to the charity, I pay zero dollar as tax, and the charities receive \$1. To check that it’s correctly calculated, I have Charities’ receipts + Tax Liability = \( 1+ 0 = 1 = \text{Pretax Income} \). These two scenarios suggest that for every \$1 given to charities, I forgo \$\( (1-\tau) \) of personal consumption. By definition the tax-price of giving is \( 1-\tau \).

Now, suppose instead that only \( x\% \) of giving are deductible. Then for each dollar of pretax income, if I spend it all on myself, I still have \( (1-\tau) \) of personal consumption. If I do not spend any on myself and donate as much of the \$1 to charities while also meet my tax liability. Specifically, suppose that I do not spend on myself and donate as much of the \$1 to charities while also meet my tax liability.

Suppose that I end up donating \( g \) dollar to charities. My taxable income would be \( 1-x\% \cdot g \) and my tax liability would be \((1-x\% \cdot g) \cdot \tau \). Now, Charities’ receipts + Tax Liability = \( g + (1-x\% \cdot g) \cdot \tau = 1 = \text{Pretax Income} \) yields \( g = (1-\tau)/(1-x\% \cdot \tau) \). These two scenarios suggest that for every \((1-\tau) \) \((1-x\% \cdot \tau) \) dollar given to charities, I forgo \$\( (1-\tau) \) of personal consumption. Proportionally, for every dollar given to charities, I forgo \( (1-x\% \cdot \tau) \) of personal consumption. By definition the tax-price of giving is \( p_g = (1-x\% \cdot \tau) \). It decreases in \( x \). The old tax-price of \( 1-\tau \) is its special case at \( x = 100 \), or when giving is fully deductible.

Similarly, the new tax-price for HELOC interest payment is \( p_h = (1-x\% \cdot \tau) - \tau(1-x\% \cdot \tau) \).

Equation (4)’s terms (e) and (f) contains \( p_g \) and \( p_h \), and thus varies in \( x \). The exact relation is as follows.

\[
\text{Term (e)} = \frac{d p_g}{p_g} = \frac{d (1-x\% \cdot \tau)}{1-\tau} = d\left( \frac{1-x\% \cdot \tau}{1-\tau} \right) = \frac{d (1-x\% \cdot \tau)}{1-\tau} \cdot \frac{1-\tau}{1-x\% \cdot \tau} = \frac{d [x\% (1-\tau) + (1-x\%)]}{d (1-\tau)} \cdot \frac{1-\tau}{1-x\% \cdot \tau} = x\% \cdot \frac{1-\tau}{1-x\% \cdot \tau} \leq x\%
\]

\[
\text{Term (f)} = \frac{d p_h}{p_h} = \frac{d (1-x\% \cdot \tau - \tau(1-x\% \cdot \tau))}{1-\tau} = \frac{d [x\% (1-\tau) - \tau(1-x\% \cdot \tau)]}{d (1-\tau)} = \frac{d (1-x\% \cdot \tau)}{1-\tau} \cdot \frac{1-\tau}{1-x\% \cdot \tau} = \frac{d [x\% (1-\tau) - \tau(1-x\% \cdot \tau)]}{d (1-\tau)} \cdot \frac{1-\tau}{1-x\% \cdot \tau} = x\% \cdot \frac{1-\tau}{1-x\% \cdot \tau} \leq x\%
\]
Appendix 11  Dead Weight Loss and the ETI

Feldstein (1999) derives a formula for calculating the dead weight loss of a tax system:

\[ DWL = 0.5\tau^2(1-\tau)\epsilon TI, \quad (A11.1) \]

where \( \tau \) is the marginal tax rate (his paper uses the notation \( t \) but I have replaced with \( \tau \) to keep consistency with notations in my main text). He has derived this formula for a tax system where a consumption item is either fully taxable, i.e. subject to a marginal tax rate of \( t \) on the part of pre-tax income used to purchase it, or fully deductible. Below I explain that, in my proposed system which stipulates that deductible items are only partly deductible, the above result still approximately holds. Without loss of generality, I explain with the setup and notations in my main text: I consider three types of consumptions, ordinary consumption \( C \), charitable giving \( G \), and HELOC interest \( h \).

Feldstein (1999) notices that, when consumption rather than the ordinary consumption are all fully deductible, the income tax \( \tau \) is equivalent to an excise tax on ordinary consumption at rate \( \upsilon \), with

\[ 1 + \upsilon = (1 - \tau)^{-1}. \]  

(Again, these notations are different from what Feldstein(1999) uses.) Applying the Hicks-Harberger approximation, the deadweight loss in Feldstein (1999) is \( DWL = -0.5\upsilon dC \). A few steps transformed this expression into (A1), whereas the sufficient conditions underlying these steps are (1) \( 1 + \upsilon = (1 - \tau)^{-1} \); (2) when \( \tau \) is zero, \( TI = C \); and (3) the compensated change in \( C \) equals the compensated change in \( TI \), i.e. \( dC = dTI \).

Now under my system, the equivalent excise tax rates are \( \upsilon \) on \( C \), \( \Theta_1 \) on \( G \) and \( \Theta_2 \) on \( h \), with

\[ 1 + \Theta_i = [1 - (1 - x_i)\tau]^{-1} \] and \( 1 + \Theta_2 = [1 - (1 - x_2)\tau]^{-1} \), where \( x_1 \) and \( x_2 \) are the deductible proportions of, respectively, \( G \) and \( h \). We have

\[ \frac{\Theta_i}{\upsilon} = \frac{(1-x_i)\tau/[1-(1-x_i)\tau]}{\tau/(1-\tau)} = (1 - x_i) \frac{1 - \tau}{1-(1-x_i)\tau} \approx (1 - x_i), \quad (i = 1,2); \] or 

\[ \Theta_i \approx (1 - x_i)\upsilon, \quad (i = 1,2). \quad (A11.2) \]

Also applying the Hicks-Harberger approximation, the deadweight loss is

\[ DWL = -0.5(udC + \Theta_1dG + \Theta_2dh) \approx -0.5[udC + (1-x_1)udG + (1-x_2)udh] \]

\[ = -0.5\upsilon[ dC + (1-x_1)dG + (1-x_2)dh] = -0.5\upsilon d[C + (1-x_1)G + (1-x_2)h] \]
Notice that we have (1) $1 + \nu = (1 - \tau)^{1/\tau}$; (2) when \(\tau\) is zero, $TI = C + (1-x_1)G + (1-x_2)h$; and (3) the total compensated change of $C$, $(1-x_1)G$, and $(1-x_2)h$ equals the compensated change in $TI$, i.e. $d[C + (1-x_1)G + (1-x_2)h] = dTI$. Therefore, Feldstein (1999)’s Formula (A11.1) still holds. However, it only holds only approximately as (A11.2) holds approximately. But the approximation error is small, especially when \(\tau\) is small and/or \(x\) is close to 1. The direction of this approximation error is certain –Formula (A11.1) overestimates the dead weight loss under my partial deductibility system. To see why, notice that, based on the line above (A11.2), (A11.2) is in fact $\Theta_i < (1 - x_i)\nu$. Then I have $DWL = -0.5(\nu dC + \Theta_1 dG + \Theta_2 dh) < -0.5[\nu dC + (1-x_1)\nu dG + (1-x_2)\nu dh]$ $\ldots$ $= Feldstein (1999)$’s (A11.1) value.

Appendix 12  Theoretical results for optimal and uniform partial deductibility policies

In this appendix I present some theoretical results associated with the optimal policy and the partial deductibility policy. Sections A12.1 to A12.5 are results for the two deduction case. Sections A12.6 and A12.7 make extensions to the general setting with \(N\) components in taxable income.

A12.1 The general expression of the ETI with deductible proportions

As mentioned in the main text, given the tax authority’s chosen tax base, an optimal base broadening policy chooses a set of optimal deductible proportions that minimizes the after reform ETI. The (general) ETI expression involving deductible proportions (the “general expression” hereinafter) will be longer than the ETI expression in (8), although the latter expression is in fact just a special case of the former. Specifically, notice that (8) is derived under the taxable income definition of taxable income $= income - G - h$, while the general expression has taxable income $= income - x_G * G - x_h * h$, with $x_G$ and $x_h$ being the deductible proportions of, respectively, $G$ and $h$. Consequently, applying the same kind of algebra in Appendix 1, I have derived that the general expression differs from Expression (8) in two ways: (1) explicitly, the only difference is that $G$’s and $h$’s in (8)’s Terms (g) and (h) become $x_G * G$ and $x_h * h$ in the general expression; (2) implicitly, $p_g$’s and $p_h$’s that appear in Terms (a) to (f) now depend on $x_G$ and $x_h$ in the way described in Appendix 10.

A12.2 Solving for the optimal policy

With the general expression of ETI, the set of optimal proportions solves the following problem:

$$
\min_{x_G, x_h} ETI = -\varepsilon_{GG} * x_G * \frac{G_0}{T_0} - \varepsilon_{Gh} * x_h * \frac{G_0}{T_0} - \varepsilon_{hh} * x_h * \frac{h_0}{T_0} - \varepsilon_{hG} * x_G * \frac{h_0}{T_0}
$$

\begin{align*}
\text{s.t.:} \quad & Income - x_G * G_0 - x_h * h_0 = T_0 \\
& 0 \leq x_G \leq 1 \\
& 0 \leq x_h \leq 1
\end{align*}

(A12.1)
The notations are as follows. $\varepsilon_{GG}, \varepsilon_{Gh}, \varepsilon_{hG}$, and $\varepsilon_{hG}$ are, respectively, the own tax-price elasticity of $G$, the cross price elasticity of $G$ with respect to $h$’s tax-price, the own tax-price elasticity of $h$, and the cross price elasticity of $h$ with respect to $G$’s tax-price. $x_G$ and $x_h$ are the deductible proportions for, respectively, $G$ and $h$. $G_0$ and $h_0$ are the amounts of $G$ and $h$ when the tax rate is 0. $TI_0$ is the tax authority’s chosen tax base. Income is the pre-tax income. The minimization problem is a simple quadratic one, with lengthy discussion on corner solutions to ensure that both $x_G$ and $x_h$ fall on $[0,1]$:

1. Under the condition of $h_0 \varepsilon_{GG} + G_0 \varepsilon_{hh} < G_0 \varepsilon_{gh} + h_0 \varepsilon_{hG}$ (a condition likely to be met with own price elasticities (usually negative) on the left hand side and cross price elasticities (usually positive) on the right hand side), the solution is

$$x_G = \max(\frac{\text{Income} - TI_0 - h_0}{G_0}, 0)$$
$$x_h = \min(\frac{\text{Income} - TI_0}{h_0}, 1)$$

2. Under the condition of $h_0 \varepsilon_{GG} + G_0 \varepsilon_{hh} = G_0 \varepsilon_{gh} + h_0 \varepsilon_{hG}$ (unlikely)

(2.1) the solution is (A12.4) if $2G_0 \varepsilon_{hh} > G_0 \varepsilon_{gh} + h_0 \varepsilon_{hG}$.
(2.2) if we have \(2G_0 \varepsilon_{hh} = G_0 \varepsilon_{gh} + h_0 \varepsilon_{hg}, \) then the ETI is constantly \(-\frac{\varepsilon_{BB}}{B \cdot TI} \cdot (Income - TI_0)^2.\)

(2.3) the solution is (A12.3) if \(2G_0 \varepsilon_{hh} < G_0 \varepsilon_{gh} + h_0 \varepsilon_{hg}.\)

(3) Under the condition \(h_0 \varepsilon_{gg} + G_0 \varepsilon_{hh} < G_0 \varepsilon_{gh} + h_0 \varepsilon_{hg}\) (unlikely)

(3.1) the solution is (A12.4) if
\[
\frac{\varepsilon_{hh}^2 G_0 - \varepsilon_{gh} G_0 h_0 + \varepsilon_{hh} h_0 (\varepsilon_{gh} + \varepsilon_{hg})}{2G_0} 
\cdot \left(\frac{Income - TI_0}{G_0}\right) \leq \frac{1}{2} \left[ \max \left(\frac{Income - TI_0 - h_0}{G_0}, 0\right) + \min \left(\frac{Income - TI_0}{G_0}, 1\right) \right]
\]

(3.2) the solution is (A12.3) if
\[
\frac{\varepsilon_{hh}^2 G_0 - \varepsilon_{gh} G_0 h_0 + \varepsilon_{hh} h_0 (\varepsilon_{gh} + \varepsilon_{hg})}{2G_0} 
\cdot \left(\frac{Income - TI_0}{G_0}\right) > \frac{1}{2} \left[ \max \left(\frac{Income - TI_0 - h_0}{G_0}, 0\right) + \min \left(\frac{Income - TI_0}{G_0}, 1\right) \right]
\]

My estimated elasticities satisfy the condition in (1). Policy G2’s deductible proportions are (Expression (A12.2))
\[
\begin{align*}
x_g &= \frac{-5.55 \cdot \frac{2}{4} + 4.02 \cdot \frac{2}{4} + 1.58}{2 \cdot 2 \cdot \left[ -(-2.89) - \frac{2}{4} \cdot (-5.55) + \frac{2}{4} \cdot 4.02 + 1.58 \right]} (106 - 104) = 49\% \\
x_h &= \frac{(-2.89) \cdot \frac{2}{4} + 1.58 \cdot \frac{4}{2} + 4.02}{2 \cdot 4 \cdot \left[ (-5.55) - \frac{4}{2} \cdot (-2.89) + \frac{4}{2} \cdot 1.58 + 4.02 \right]} (106 - 104) = 25\%
\end{align*}
\]

Policy h2’s deductible proportions are (also Expression (A12.2))
\[
\begin{align*}
x_g &= \frac{-5.55 \cdot \frac{2}{4} + 4.02 \cdot \frac{2}{4} + 1.58}{2 \cdot 2 \cdot \left[ -(-2.89) - \frac{2}{4} \cdot (-5.55) + \frac{2}{4} \cdot 4.02 + 1.58 \right]} (106 - 102) = 99\% \\
x_h &= \frac{(-2.89) \cdot \frac{2}{4} + 1.58 \cdot \frac{4}{2} + 4.02}{2 \cdot 4 \cdot \left[ (-5.55) - \frac{4}{2} \cdot (-2.89) + \frac{4}{2} \cdot 1.58 + 4.02 \right]} (106 - 102) = 51\%
\end{align*}
\]

The ETI’s under these two policies can then be computed using (A12.1).

**A12.3 An optimal policy guarantees reducing the before reform ETI**

From (A12.1), it is straightforward to prove that an optimal policy will reduce the before reform (in its meaning defined in Section VI) ETI. Specifically, suppose the tax authority decides to broaden the tax base to \(TI_0.\) Pick a feasible set of deductible proportions that meets the tax base requirement (i.e. a set of deductible
proportions that satisfies the constraint below (A12.1) as \( x_G' = x_h' = \frac{\text{Income} - Tl_0}{G_0 + h_0} \). Since the base is broadened, there is \( 0 < \frac{\text{Income} - Tl_0}{G_0 + h_0} < \frac{\text{Income} - (\text{Income} - G_0 - h_0)}{G_0 + h_0} = 1 \). Therefore both \( x_G' \) and \( x_h' \) lie between 0 and 1. Below is the proof. Its basic idea is that the policy with \( x_G' \) and \( x_h' \) can already reduce the ETI, let alone the optimal policy:

\[
0 < \text{before reform ETI} \\
= -\varepsilon_{GG} * \frac{G_0}{\text{Income} - G_0 - h_0} - \varepsilon_{Gh} * \frac{G_0}{\text{Income} - G_0 - h_0} - \varepsilon_{hh} * \frac{h_0}{\text{Income} - G_0 - h_0} - \varepsilon_{hG} \\
* \frac{h_0}{\text{Income} - G_0 - h_0} = \frac{1}{\text{Income} - G_0 - h_0}(- \varepsilon_{GG} * G_0 - \varepsilon_{Gh} * G_0 - \varepsilon_{hh} * h_0 - \varepsilon_{hG} * h_0) \\
> \frac{1}{\text{Income} - x_G' * G_0 - x_h' * h_0}(- \varepsilon_{GG} * G_0 - \varepsilon_{Gh} * G_0 - \varepsilon_{hh} * h_0 - \varepsilon_{hG} * h_0) \\
= \frac{1}{Tl_0}(- \varepsilon_{GG} * G_0 - \varepsilon_{Gh} * G_0 - \varepsilon_{hh} * h_0 - \varepsilon_{hG} * h_0) > \left( \frac{x_G'}{Tl_0} \right)^2(- \varepsilon_{GG} * G_0 - \varepsilon_{Gh} * G_0 - \varepsilon_{hh} * h_0 - \varepsilon_{hG} * h_0) \\
= -\varepsilon_{GG} * x_G' * \frac{x_G' * G_0}{Tl_0} - \varepsilon_{Gh} * x_h' * \frac{x_h' * h_0}{Tl_0} - \varepsilon_{hh} * x_G' * \frac{x_G' * h_0}{Tl_0} - \varepsilon_{hG} * x_h' * \frac{x_h' * h_0}{Tl_0} \\
= -\varepsilon_{GG} * x_G' * \frac{x_G' * G_0}{Tl_0} - \varepsilon_{Gh} * x_h' * \frac{x_h' * h_0}{Tl_0} - \varepsilon_{hh} * x_G' * \frac{x_G' * h_0}{Tl_0} - \varepsilon_{hG} * x_h' * \frac{x_h' * h_0}{Tl_0} \\
\geq \min_{x_G' * x_h'} \left\{ -\varepsilon_{GG} * x_G' * \frac{x_G' * G_0}{Tl_0} - \varepsilon_{Gh} * x_h' * \frac{x_h' * h_0}{Tl_0} - \varepsilon_{hh} * x_G' * \frac{x_G' * h_0}{Tl_0} - \varepsilon_{hG} * x_h' * \frac{x_h' * h_0}{Tl_0} \right\} \text{ s.t. constraints} \\
= \text{the optimal policy's ETI}
\]

A12.4 A uniform partial deductibility policy guarantees reducing the before reform ETI

In the above proof, \( x_G' \) and \( x_h' \) are nothing but the deductible proportions under the uniform partial deductibility. The part of the above proof before the step of “\( \geq \min_{x_G' * x_h'} \)” proves that the uniform partial deductibility can guarantee reducing the ETI.

A12.5 The relation between the old and new ETIs before and after implementing a uniform partial deductibility policy

After the uniform partial deductibility policy is in place, the new ETI has a simple relationship with the before reform ETI that does not depend on the own and cross price elasticities, as proved below: (suppressing notations as \( x_G' = x_h' = x' \)
uniform partial ETI = \(-\varepsilon_{Gh} \cdot x' \cdot \frac{x' \cdot G_0}{Tl_0} - \varepsilon_{Gh} \cdot x' \cdot \frac{x' \cdot G_0}{Tl_0} - \varepsilon_{hh} \cdot x' \cdot \frac{x' \cdot h_0}{Tl_0} - \varepsilon_{hG} \cdot x' \cdot \frac{x' \cdot h_0}{Tl_0}\)

\[= (x')^2 \left( -\varepsilon_{Gh} \cdot \frac{G_0}{Tl_0} - \varepsilon_{Gh} \cdot \frac{G_0}{Tl_0} - \varepsilon_{hh} \cdot \frac{h_0}{Tl_0} - \varepsilon_{hG} \cdot \frac{h_0}{Tl_0} \right)\]

\[= (x')^2 \cdot income \cdot \frac{G_0}{Tl_0} \cdot \frac{h_0}{Tl_0} \cdot Income \cdot \frac{h_0}{Tl_0} \cdot \frac{Income \cdot -g_0 - h_0 - \varepsilon_{hG} \cdot Income \cdot -g_0 - h_0 - \varepsilon_{hh}}{Tl_0}\]

\[= (x')^2 \cdot income \cdot \frac{g_0 - h_0 - \varepsilon_{hG} \cdot Income \cdot -g_0 - h_0 - \varepsilon_{hh}}{Tl_0}\]

\[= (x')^2 \cdot \frac{\text{before reform tax base}}{\text{new (broader) tax base}} \cdot before reform ETI.\]

In other words, the relation is uniform partial ETI = \((x')^2 \cdot \frac{\text{before reform tax base}}{\text{new (broader) tax base}} \cdot before reform ETI.\)

Example (the same with the one in the last part of the main text’s policy implication section, but more calculation details are provided)

Suppose that currently (1) the tax rate is \(\tau = 20\%\); (2) the taxable income at the 20% tax rate is \(TI = 1\); (3) the amount of itemized deductions at 20% tax rate of is \$0.4; and (4) the ETI is somewhere between 0.4 and 0.8. Then the current revenue is \(R_C = \tau \cdot TI\). For any uniform partial deductible proportion \(x'\), the new ETI will equal \((x')^2 \cdot \frac{\text{before reform tax base}}{\text{new (broader) tax base}} \cdot before reform ETI = (x')^2 \cdot \frac{\$1/(1-\tau)^{ETI}}{\$1 + \$0.4 - x' \cdot \$1 + \$0.4 - \frac{\$1}{(1-\tau)^{ETI}}} \cdot ETI.\)

Therefore the new revenue will be \(R_N = \tau \cdot \{\$1 + \$0.4 - x' \cdot \$1 + \$0.4 - \frac{\$1}{(1-\tau)^{ETI}}\} \cdot (1 - \tau) \cdot (x')^2 \cdot \frac{\$1/(1-\tau)^{ETI}}{\$1 + \$0.4 - x' \cdot \$1 + \$0.4 - \frac{\$1}{(1-\tau)^{ETI}}} \cdot ETI.\). Solving for \(R_N = (1 + 10\%) \cdot R_C\) with ETI between 0.4 and 0.8 yields that \(x'\) is between 78.6% (for ETI = 0.4) and 81.4% (for ETI = 0.8).

A12.6 Extension to \(N\)-deduction taxable income

Sections A12.1 to A12.5 consider the situation where the only two responsive components of taxable income are deductions \(G\) and \(h\). If instead, the responsive components of taxable income are \(N\) deductions, with their amounts at tax rate of 0 denoted \(D_1, D_2, \ldots, D_N\) and deductible proportions being \(x_1, x_2, \ldots, x_N\). Results in previous sections can be extended. I discuss them one by one below. Basically, there is no longer closed form optimal policy solution but other things still holds (optimal policy solution exists; optimal and partial deductibility policies guarantee reducing the ETI).

(1) Solving for the optimal deductible proportions

To make later notations shorter, notice that the problem of minimizing (A12.1) can be rewritten as (in the spirit of the matrix form in Appendix 2’s Expression (A.3) for ETI)
\[
\begin{align*}
\min_{x_G,x_h} & \quad ETI = [x_G \quad x_h] \begin{bmatrix}
\frac{G_0}{TI_0} & \frac{G_0}{TI_0} \\
\frac{h_0}{TI_0} & \frac{h_0}{TI_0}
\end{bmatrix} [x_G \quad x_h]^T \\
\text{s.t.} & \quad Income - x_G \cdot G_0 - x_h \cdot h_0 = TI_0 \\
& \quad 0 \leq x_G \leq 1, \\
& \quad 0 \leq x_h \leq 1
\end{align*}
\]

with (A12.2) being the solution.

Writing down the problem of minimizing the N-deduction ETI can similarly invoke a matrix form (with \(\varepsilon_{ij}\) being the elasticity of \(D_i\) with respect to \(D_j\)'s tax-price):

\[
\begin{align*}
\min_{x_1,x_2,\ldots,x_N} & \quad ETI = [x_1 \quad x_2 \quad \cdots \quad x_N] \begin{bmatrix}
\frac{D_1}{TI_0} & \frac{D_1}{TI_0} & \cdots & \frac{D_1}{TI_0} \\
\frac{D_2}{TI_0} & \frac{D_2}{TI_0} & \cdots & \frac{D_2}{TI_0} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{D_N}{TI_0} & \frac{D_N}{TI_0} & \cdots & \frac{D_N}{TI_0}
\end{bmatrix} [x_1 \quad x_2 \quad \cdots \quad x_N]^T \\
\text{s.t.} & \quad Income - x_1 \cdot D_1 - x_2 \cdot D_2 - \cdots - x_N \cdot D_N = TI_0 \\
& \quad 0 \leq x_i \leq 1 \text{ for } i = 1,2,\ldots,N
\end{align*}
\]

The solution exists, since the minimized function is continuous, and the choice set of the deductible proportions is closed and bounded. However, I don’t think that there are closed form expressions. (There are neat eigenvalue and eigenvector results if the constraint is like \([x_1 \quad x_2 \quad \cdots \quad x_N]B[x_1 \quad x_2 \quad \cdots \quad x_N]^T\), with \(B\) being positive definite (see, for example, Cízek, Härdle, and Weron (2005)). But apparently my constraints are not in this form.)

**(2) An optimal policy guarantees reducing the before reform ETI**

As is similar with Section 12.3, pick a feasible set of deductible proportions that meets the tax base requirement (i.e. a set of deductible proportions that satisfies the constraint below (A12.1)) as \(x'_1 = x'_2 = \cdots = x'_N = \frac{Income-TI_0}{D_1+D_2+\cdots+D_N}\). Since the base is broadened, there is \(0 < \frac{Income-TI_0}{D_1+D_2+\cdots+D_N} < \frac{Income-(Income-D_1-D_2-\cdots-D_N)}{D_1+D_2+\cdots+D_N} = 1\).

Therefore \(x'_1, x'_2, \ldots\) and \(x'_N\) all lie between 0 and 1. Below is the proof. Again, its basic idea is that the policy with \(x'_1, x'_2, \ldots\) and \(x'_N\) can already reduce the ETI, let alone the optimal policy: \(0 < before\ reform\ ETI =

\[
\begin{bmatrix}
\frac{D_1}{Income-D_1-D_2-\cdots-D_N} & \frac{D_1}{Income-D_1-D_2-\cdots-D_N} & \cdots & \frac{D_1}{Income-D_1-D_2-\cdots-D_N} \\
\frac{D_2}{Income-D_1-D_2-\cdots-D_N} & \frac{D_2}{Income-D_1-D_2-\cdots-D_N} & \cdots & \frac{D_2}{Income-D_1-D_2-\cdots-D_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{D_N}{Income-D_1-D_2-\cdots-D_N} & \frac{D_N}{Income-D_1-D_2-\cdots-D_N} & \cdots & \frac{D_N}{Income-D_1-D_2-\cdots-D_N}
\end{bmatrix} \begin{bmatrix}1 \ 1 \ \cdots \ 1\end{bmatrix} > 0
\]
\[
\begin{align*}
\frac{1}{\text{Income}-x'_1D_1-x'_2D_2-\cdots-x'_ND_N} & \left[ -\frac{D_1}{T_1} \varepsilon_{11} & -\frac{D_1}{T_1} \varepsilon_{12} & \cdots & -\frac{D_1}{T_1} \varepsilon_{1N} \\
-\frac{D_1}{T_1} \varepsilon_{21} & -\frac{D_2}{T_2} \varepsilon_{22} & \cdots & -\frac{D_2}{T_2} \varepsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{D_N}{T_N} \varepsilon_{N1} & -\frac{D_N}{T_N} \varepsilon_{N2} & \cdots & -\frac{D_N}{T_N} \varepsilon_{NN} \right] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \\
\frac{1}{T_{I0}} \left[ -\frac{D_1}{T_1} \varepsilon_{11} & -\frac{D_1}{T_1} \varepsilon_{12} & \cdots & -\frac{D_1}{T_1} \varepsilon_{1N} \\
-\frac{D_1}{T_1} \varepsilon_{21} & -\frac{D_2}{T_2} \varepsilon_{22} & \cdots & -\frac{D_2}{T_2} \varepsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{D_N}{T_N} \varepsilon_{N1} & -\frac{D_N}{T_N} \varepsilon_{N2} & \cdots & -\frac{D_N}{T_N} \varepsilon_{NN} \right] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} > \\
\frac{1}{T_{I0}} \left[ -\frac{D_1}{T_1} \varepsilon_{11} & -\frac{D_1}{T_1} \varepsilon_{12} & \cdots & -\frac{D_1}{T_1} \varepsilon_{1N} \\
-\frac{D_1}{T_1} \varepsilon_{21} & -\frac{D_2}{T_2} \varepsilon_{22} & \cdots & -\frac{D_2}{T_2} \varepsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{D_N}{T_N} \varepsilon_{N1} & -\frac{D_N}{T_N} \varepsilon_{N2} & \cdots & -\frac{D_N}{T_N} \varepsilon_{NN} \right] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \\
(x'_1)^2 \left[ -\frac{D_1}{T_1} \varepsilon_{11} & -\frac{D_1}{T_1} \varepsilon_{12} & \cdots & -\frac{D_1}{T_1} \varepsilon_{1N} \\
-\frac{D_1}{T_1} \varepsilon_{21} & -\frac{D_2}{T_2} \varepsilon_{22} & \cdots & -\frac{D_2}{T_2} \varepsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{D_N}{T_N} \varepsilon_{N1} & -\frac{D_N}{T_N} \varepsilon_{N2} & \cdots & -\frac{D_N}{T_N} \varepsilon_{NN} \right] \begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_N \end{bmatrix} = \\
(x'_1)^2 \left[ -\frac{D_1}{T_1} \varepsilon_{11} & -\frac{D_1}{T_1} \varepsilon_{12} & \cdots & -\frac{D_1}{T_1} \varepsilon_{1N} \\
-\frac{D_1}{T_1} \varepsilon_{21} & -\frac{D_2}{T_2} \varepsilon_{22} & \cdots & -\frac{D_2}{T_2} \varepsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{D_N}{T_N} \varepsilon_{N1} & -\frac{D_N}{T_N} \varepsilon_{N2} & \cdots & -\frac{D_N}{T_N} \varepsilon_{NN} \right] \begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_N \end{bmatrix} > \begin{bmatrix} x'_{1} \\ x'_{2} \\ \vdots \\ x'_{N} \end{bmatrix}
\end{align*}
\]

\[
\min_{x_1, x_2, \ldots, x_N} \begin{bmatrix} -\frac{D_1}{T_1} \varepsilon_{11} & -\frac{D_1}{T_1} \varepsilon_{12} & \cdots & -\frac{D_1}{T_1} \varepsilon_{1N} \\
-\frac{D_1}{T_1} \varepsilon_{21} & -\frac{D_2}{T_2} \varepsilon_{22} & \cdots & -\frac{D_2}{T_2} \varepsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{D_N}{T_N} \varepsilon_{N1} & -\frac{D_N}{T_N} \varepsilon_{N2} & \cdots & -\frac{D_N}{T_N} \varepsilon_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \text{ s.t. constraints} \]
\]

the optimal policy's ETI

(3) A uniform partial deductibility policy guarantees reducing the before reform ETI
This paragraph is similar with Section A12.4. In the above proof, $x'_1, x'_2, \ldots, x'_N$ are nothing but the deductible proportions under the uniform partial deductibility. The part of the above proof before the step of \[\geq \min \{\ldots\}\] proves that the uniform partial deductibility can guarantee reducing the ETI.

(4) The relation between the old and new ETIs before and after implementing a uniform partial deductibility policy

Again, this is just rewriting Section A12.5 in the matrix form for an $N$-deduction ETI. After the uniform partial deductibility policy is in place, the new ETI has a simple relationship with the before reform ETI that does not depend on the own and cross price elasticities, as proved below: (suppressing notations as $x'_1 = x'_2 = \ldots = x'_N \equiv x'$)

$$
\text{uniform partial ETI} = [x' \; x' \; \cdots \; x'] \begin{bmatrix}
-\frac{D_1}{TI_0} \epsilon_{11} & -\frac{D_1}{TI_0} \epsilon_{12} & \cdots & -\frac{D_1}{TI_0} \epsilon_{1N} \\
-\frac{D_2}{TI_0} \epsilon_{21} & -\frac{D_2}{TI_0} \epsilon_{22} & \cdots & -\frac{D_2}{TI_0} \epsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{D_N}{TI_0} \epsilon_{N1} & -\frac{D_N}{TI_0} \epsilon_{N2} & \cdots & -\frac{D_N}{TI_0} \epsilon_{NN}
\end{bmatrix}
$$

$$
= (x')^2 \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}
\begin{bmatrix}
\frac{-D_1}{TI_0} \epsilon_{11} & -\frac{D_1}{TI_0} \epsilon_{12} & \cdots & -\frac{D_1}{TI_0} \epsilon_{1N} \\
\frac{-D_2}{TI_0} \epsilon_{21} & -\frac{D_2}{TI_0} \epsilon_{22} & \cdots & -\frac{D_2}{TI_0} \epsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{-D_N}{TI_0} \epsilon_{N1} & -\frac{D_N}{TI_0} \epsilon_{N2} & \cdots & -\frac{D_N}{TI_0} \epsilon_{NN}
\end{bmatrix}
\begin{bmatrix} 1 \\
\vdots \\
1
\end{bmatrix}
$$

$$
= (x')^2 \frac{\text{Income} - D_1 - D_2 - \cdots - D_N}{TI_0} [1 \; 1 \; \ldots \; 1]
\begin{bmatrix}
1 \\
\vdots \\
1
\end{bmatrix}
\begin{bmatrix}
\frac{-D_1}{TI_0} \epsilon_{11} & -\frac{D_1}{TI_0} \epsilon_{12} & \cdots & -\frac{D_1}{TI_0} \epsilon_{1N} \\
\frac{-D_2}{TI_0} \epsilon_{21} & -\frac{D_2}{TI_0} \epsilon_{22} & \cdots & -\frac{D_2}{TI_0} \epsilon_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{-D_N}{TI_0} \epsilon_{N1} & -\frac{D_N}{TI_0} \epsilon_{N2} & \cdots & -\frac{D_N}{TI_0} \epsilon_{NN}
\end{bmatrix}
\begin{bmatrix} 1 \\
\vdots \\
1
\end{bmatrix}
$$

In other words, the relation is \[\text{uniform partial ETI} = (x')^2 \frac{\text{before reform tax base}}{\text{new (broader) tax base}} \ast \text{before reform ETI}.\]

A12.6 Extension to $N$-component taxable income

The previous section makes extension to the case where the responsive components of taxable income are $N$ deductions. Now consider the most general case that incomes are also responsive. To extend the optimal and
uniform partial deductibility policy results to this case, conceptually I need to do something similar to textbook models on labor supply to “deductionize” incomes, i.e. to view taxing incomes as deducting “leisure”s. For example, suppose that taxable income is defined as taxable income = \( L - D_1 - D_2 - \ldots - D_{N-1} \), with \( L \) being labor income. Suppose that one’s total time is measured in such unit that the wage per unit of time is $1. Suppose that one’s total time is \( T \) and that one’s leisure time is \( l \), both measured in the aforementioned time unit. Then the taxable income is taxable income = \( T - l - D_1 - D_2 - \ldots - D_{N-1} \). It follows that, since \( T \) is not responsive to tax, the responsive components of this taxable income are the \( N \) deductions of \( l, D_1, D_2 \ldots \) and \( D_{N-1} \). Then everything is exactly the same with A12.5. The only noteworthy thing is that to implement the optimal or uniform partial deductibility policy, \( l \) is treated the same with other deductible consumptions – \( l \) is only partially deductible now i.e. households pay taxes on leisure.

This “deductionizing” concept can extend to other forms of incomes. As Saez, Slemrod and Giertz (2012) writes in their “Conceptual Framwork” section: “…individuals supply effort to earn income \( z \)”, with \( z \) being the tax base, not just labor income. In my view, whatever one forgoes when exerting more effort, be it leisure or relaxed mental state, can be regarded as deductions. This kind of thinking completes my theoretical discussion on the optimal and uniform partial deductibility policies; whether taxing leisure or relaxation is practical is a separate issue.

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Statistical Table 2.1 Returns with Itemized Deductions: Sources of Income, Adjustments, Itemized Deductions by Type, Exemptions, and Tax Items, by Size of Adjusted Gross Income, Tax Year 2009, retrieved from www.irs.gov/file_source/pub/irs-soi/09in21id.xls


Statistical Table 2.1 Returns with Itemized Deductions: Sources of Income, Adjustments, Itemized Deductions by Type, Exemptions, and Tax Items, by Size of Adjusted Gross Income, Tax Year 2012, retrieved from http://www.irs.gov/pub/irs-soi/12inalcr.pdf


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