Abstract This paper argues that the network structure of an economy, or more specifically, how economic sectors are connected through input-output links, matters to issues such as tax burden distribution, tax revenue, labor supply response to taxes, etc. I discuss the role of network links by comparing a parallel-structure economy and a chain-structure economy. In my model, if two sectors are parallel with both producing final consumption goods, the taxation of one sector will create larger utility loss for the taxed sector. In contrast, if the two sectors have a upstream-downstream chain structure, the two sectors have exactly the same utility loss. Moreover, in an n-sector-chain economy, taxing different combinations of the sectors achieves the identical same equilibrium result: labor supply in each sector, utility loss, consumption, tax revenue, GDP, etc. In addition, in a chain economy the elasticity of taxable income with respect to a uniform labor income tax is higher. Lastly, I argue that this network structure view impose reveals threats to the validity of various elasticity estimates.
I. Introduction

This paper argues that the network structure of an economy, or more specifically, how economic sectors are connected through input-output links, matters to issues such as tax burden distribution, tax revenue, labor supply response to taxes, etc. Section II sets my model up. Section III and IV examine theoretically the role of network structure in a group of tax-related economic questions. Section V and VI discuss threats raised by the network-view to the validity of empirical estimation of elasticities.

II. A Model of a Multi-sector Economy

I present below my model setup on which I base my discussions of a series of economic issues.

An economy is composed of \( n \) sectors. If one sector provides input for the production of another sector, the two sectors have a directed link. All sectors are competitive; each sector employs labor to process the input(s) from its upstream sector(s) into one unique output, which is either an input for a downstream sector or a final consumption good. Homogeneous workers in each sector consume the final consumption good(s) produced in the economy. They make labor supply decisions to maximize utility that come from consumption and leisure. In this paper I focus on parallel structures (all sectors are parallel; each sector uses labor to produce a final consumption good) and chain structures (all sectors form a “chain”; each sector has exactly one upstream sector and one downstream sector, except the most upstream one and the one which produces the final consumption good). In the short run labor is immobile across sectors; in the long run it is mobile. I computationally solve most of my models.

More specific function forms are as follows:

There are \( n \) sector, indexed as Sector 1, 2, 3, ..., \( n \). In each sector, there are workers with mass 1. Among the \( n \) sectors, \( k \) sectors produce final consumption goods; they are sectors 1, 2, ..., \( k \). A representative worker solves the following problem:

Max \( u(l_i, c_i) = \frac{(1 - l_i)^{1-b}}{1 - b} + \frac{k^{1-b}c_1^{1-b}c_2^{1-b}c_k^{1-b}}{1-b} \)

s. t. \( c_1p_1 + c_2p_2 + \cdots + c_kp_k = w_il_i \)

with \( w_i \) being the wage in Sector \( i \) and \( p_1, p_2, \ldots, p_k \) being the prices of final consumption goods produced by Sector 1, 2, ..., \( k \), respectively. The consumption part of this functional form is Cobb-Douglas; the implication of which is that the consumer always spends a fixed proportion of her income on a consumption good. The powers on \( c_i \)’s are chosen so that optimal labor supply under zero tax is always 1/2.

For Sector \( i \), if it doesn’t have an upstream sector, then the production technology is

\( y_i = f_i(l_i) \).

As a result, the after tax wage will be

\( w_i = f_i(p_iy_i(1 - t_i)) \).

If Sector \( i \) has a upstream sector, say Sector \( j \), then the production technology is to couple labor with input \( j \) with a fixed ratio to produce output \( i \) (this ratio is characterized by \( g_j \)); i.e.

\( y_i = f_i \min\{l_i \frac{y_j}{g_j}\} \).
As a result, the after tax wage will be

\[ w_i = (f_i p_i - g_j p_j)(1 - t_i) \]

III. Tax Burden Distribution

Tax incidence has been an active topic in economics for centuries. Its general equilibrium models were first developed in mid-20th century. Harberger (1962) examines a two-sector (both competitive) general equilibrium model, in which the two sectors employ capital and labor and each produces one final consumption good; Shoven (1976), extends Harberger’s work and computationally analyzes a twelve-sector model. In these models there are no input-output links among the sectors. Ballard, Fullerton, Shoven and Whalley (1985) writes a model with 19 producer goods and 15 consumer goods. Yet to my knowledge, no special attention has been paid to the role played by network structure. I thereby introduce the role of network links by comparing a parallel-structure economy and a chain-structure economy.¹

The burden of labor income tax on each sector is defined as the sector’s workers’ utility loss compared with no-tax utility level.² Figure 1 plots the utilities as functions of tax rate on one sector³, with the left graph for a two-parallel-sector economy and the right graph for a two-sector chain structured economy. Figure 1 shows that the two different structures produce two distinct patterns of tax burden (i.e. utility loss) distribution: under the parallel structure, the taxed sector has larger utility loss; under the chain structure, the two sectors have exactly the same utility loss. The intuition is obvious: under the parallel structure, Sector 2’s utility loss is due to Sector 1 producing less consumption good; under the chain structure, in equilibrium labor supplies in two sectors have to be proportional to each other (equal to each other in the Figure 1 Model) as a result of Sector 2’s

![Figure 1: Utility level as a function of tax rate on Sector 1.](image)

Left: Sector 1 and Sector 2 are parallel; neither provides input for the other.

Right: Sector 1 produces input for sector 2

¹ My modes are not fully comparable to Harberger’s and Shoven’s, though. The key difference is that in their models, labor is perfectly mobile. To explore properties of networks, I consider non-mobile labor models. Both assumptions are plausible; in the short run an economist can’t become a rocket scientist but in the long run different generations can have very different cross-sector population distributions.

² Firms are competitive in labor market and goods market, and always make zero profit. Therefore there’s no tax burden on firms.

³ Parameter values are set as \( f_i = g_i = 1 \), any applicable \( i \).
production technology. So sector 1’s labor supply decision directly affects that of Sector 2. As the worker in Sector 1, after taxation, reduces labor supply and produces less input for Sector 2, the Sector 2 worker has to work less, too.

Solving for the general $n$-sector model shows that, in the $n$-sector-chain economy, taxing different combinations of the sectors achieves the identical equilibrium result: labor supply in each sector, utility loss, consumption, tax revenue, GDP…. For example, in the 5-sector-chain model$^4$, a 5% tax on any sector alone is equivalent to a 5% tax on any other sector alone, which is equivalent to a 1.04% tax on every sector and equivalent to a 1% tax on Sector 2 and a 4.1% tax on Sector 3. All these tax schedules have the same result: every sector’s utility decreases by the same amount; tax revenue is 0.026 (normalizing the price of final consumption good, $p_y$, to 1); every sector’s labor supply decreases by the same amount under any of these tax schedules; the after tax wage declines equally for everyone under every schedule; in sum, these seemingly very different tax schedules produce exactly the result (except the “nominal” tax rate levels).

IV. Revenue maximizing tax rate

The idea and estimation of revenue maximizing tax rate, or the “Laffer Curve”, dated back to 1970s and is always in controversy. Existing literature estimates this rate by using the estimate of the elasticity of taxable income. I explain in a subsequent section that, the network structure view casts doubt on estimation of elasticities. Here I make the point that the inter-sector links matters to where the maximizing rate lies. Coming back to the comparison of the two simple two-sector models, one parallel and one chain-structured; suppose the government imposes a uniform labor income tax on everyone, these two simple models produce significantly different results, as shown in Figure 2.

![Figure 2: Tax revenue ($R$) as a function of the same labor income tax rate ($t$) on everyone](image)

Left: Sector 1 and Sector 2 are parallel; neither provides input for the other.
Right: Sector 1 produces input for sector 2

From Figure 2, it’s obvious that in the chain structured economy, tax revenue rises more quickly to its maximum.

$^4$ Parameter values are set as $f_i = g_i = 1$, any applicable $i$. 
V. Threats to the validity of the direct before-and-after estimation of elasticity

As is made clear in Section 2, different tax rate changes can yield the same change in labor supply. As a result, when one observes individuals’ tax rate changes and labor supply changes and divides percent change in labor supply over percent change in tax rate, one could get elasticity estimates that differ from each other (and it’s not even clear what “elasticity” one is estimating). In fact, the estimations in the literature range from near zero to above 1 (Fullerton 2008) – which should not be surprising under the network structure view. Take, for instance, the three different tax schedules raised as examples in Section 2, with the same underlying economic structure and workers’ preference, data from the three tax schedule changes (all affecting the economy in exactly the same way as I mentioned) would yield distinct labor supply elasticities (defined as percentage change in labor supply over percentage change in net-of-tax rate) estimates: (1) the 5% tax on one sector alone will yield an estimate of 0.32;\(^5\) (2) the 1.04% tax on all sectors will yield an estimate of 1.52;\(^6\) (3) the 1% tax on Sector 2 and the 4.1% tax on Sector 3 will yield an estimate of 0.98.\(^7\) Rigorously speaking, they are all estimating different things; using one term “labor supply elasticity” to describe all of them is misleading. The same problem should exist for elasticity of taxable income. Another immediate implication is the following: when elasticity is estimated by using only one group of people from the population, one should be very careful in applying the result to suggest policies that involve the whole population of the economy.

VI. Threats to the validity of difference-in-difference estimation of elasticities

The underlying assumption for using a difference-in-difference method is that what the control group has done is what the treated group would have done without the treatment. However, in a network economy, everyone is somehow related, which reduces the credibility this assumption. For example, in the 5-sector model in Section 2, no matter which particular sector is taxed, all sectors will respond exactly the same way. In this case, what the untaxed (the control group) have done is exactly NOT what the taxed (the treated group) would have done without the treatment – although the control group are not directly taxed, the effect of the tax on the treated group is that both group reduce labor supply by the same amount. If one applies difference-in-difference method to this kind of situation, one would mistakenly conclude that the tax has no effect on labor supply since the difference-in-difference between the non-taxed and the taxed is 0.

VII. Conclusion: On Two stylized Facts – Laffer Curve and Taxable Income Elasticity

Where does the revenue maximizing tax rate lie? People use taxable income elasticity – which seem a very stylized concept under the network structure view – to infer the answer. The conclusion of this paper is simple: under all the concerns discussed above, estimation result is sensitive to both the underlying economic structure and the design of tax schedule; the populace and legislators should be cautious when relying on these existing estimates in proposing tax rates.

References

\(^5\) calculated as \( \frac{(0.4921 - 0.5)/0.5}{-0.05/1} \), where 0.4921 is the after tax labor supply calculated from the model  
\(^6\) Calculated as \( \frac{(0.4921 - 0.5)/0.5}{-0.01/1} \)  
\(^7\) Calculated as the average of \( \frac{(0.4921 - 0.5)/0.5}{-0.01/1} \) and \( \frac{(0.4921 - 0.5)/0.5}{-0.041/1} \)
