Name ______________________________ Date _________ Partners ____________________________

LAB 11:

FREE, DAMPED, AND FORCED OSCILLATIONS

OBJECTIVES

- To understand the free oscillations of a mass and spring.
- To understand how energy is shared between potential and kinetic energy.
- To understand the effects of damping on oscillatory motion.
- To understand how driving forces dominate oscillatory motion.
- To understand the effects of resonance in oscillatory motion.

OVERVIEW

You have already studied the motion of a mass moving on the end of a spring. We understand that the concept of mechanical energy applies and the energy is shared back and forth between the potential and kinetic energy. We know how to find the angular frequency of the mass motion if we know the spring constant. We will examine in this lab the mass-spring system again, but this time we will have two springs — each having one end fixed on either side of the mass. We will let the mass slide on an air track that has very little friction. We first will study the free oscillation of this system. Then we will use magnets to add some damping and study the motion as a function of the damping coefficient. Finally, we will hook up a motor that will oscillate the system at practically any frequency we choose. We will find that this motion leads to several interesting results including wild oscillations.

Harmonic motions are ubiquitous in physics and engineering - we often observe them in mechanical and electrical systems. The same general principles apply for atomic, molecular, and other oscillators, so once you understand harmonic motion in one guise you have the basis for understanding an immense range of phenomena.

INVESTIGATION 1: FREE OSCILLATIONS

An example of a simple harmonic oscillator is a mass m which moves on the x-axis and is attached to a spring with its equilibrium position at $x = 0$ (by definition). When the mass is moved from its equilibrium position, the restoring force of the spring tends to bring it back to the equilibrium position. The spring force is given by

$$F_{spring} = -kx \tag{1}$$

where k is the spring constant. The equation of motion for m becomes

$$m\frac{d^2x}{dt^2} = -kx \tag{2}$$

This is the equation for *simple harmonic motion*. Its solution, as one can easily verify, is given by:

$$x = A_F \sin\left(\omega_F t + \delta_F\right) \tag{3}$$

where

$$\omega_F = \sqrt{k/m} \tag{4}$$

Note: The subscript "*F*" on ω_F, etc. refers to the *natural* or *free* oscillation.

A_F and δ_F are constants of integration and are determined by the initial conditions. [For example, if the spring is maximally extended at $t = 0$, we find that A_F is the displacement from equilibrium and $\delta = \pi/2$.]

We can calculate the velocity by differentiating with respect to time:

$$v = \frac{dx}{dt} = \omega_F A_F \cos\left(\omega_F t + \delta_F\right) \tag{5}$$

The kinetic energy is then:

$$KE = \tfrac{1}{2} mv^2 = \tfrac{1}{2} kA_F^2 \cos^2\left(\omega_F t + \delta_F\right) \tag{6}$$

The potential energy is given by integrating the force *on the spring* times the displacement:

$$PE \;=\; \int_0^x kx\,dx = \tfrac{1}{2} kx^2 = \tfrac{1}{2} kA_F^2 \sin^2\left(\omega_F t + \delta_F\right) \tag{7}$$

We see that the sum of the two energies is constant:

$$KE \;+\; PE = \tfrac{1}{2} kA_F^2 \tag{8}$$

ACTIVITY 1-1: MEASURING THE SPRING CONSTANT

We have already studied the free oscillations of a spring in a previous lab, but let's quickly determine the spring constants of the two springs that we have. To determine the spring constants, we shall use the method that we used in Lab 8. We can use the force probe to measure the force on the spring and the motion detector to measure the corresponding spring stretch.

To perform this laboratory you will need the following equipment:

- force probe
- motion detector
- mechanical vibrator
- air track and cart glider
- two springs with approximately equal spring constants
- electronic balance
- four ceramic magnets
- masking tape
- string with loops at each end, ~30 cm long

1. Turn on the air supply for the air track. Make sure the air track is level. Check it by placing the glider on the track and see if it is motionless. Some adjustments may be

necessary on the feet, but be careful, because it may be impossible to keep the track level over its entire length.

2. Tape four ceramic magnets to the top of the glider cart and measure the mass of the glider cart on the electronic balance.

glider cart mass ________________ kg(with four magnets)

Never move items on the air track unless the air is flowing! You might scratch the surfaces and create considerable friction.

3. Set up the force probe, glider, spring, motion detector, and mechanical vibrator as shown below on the air track. If not already done, tie a loop at each end of a string, so that it ends up about 30 cm long. Loop one end around the force probe hook and the other end around the metal flag on the glider cart. Note the spring you put on the apparatus as *Spring 1*. Make sure the mechanical vibrator-oscillator driver is in the locked position.

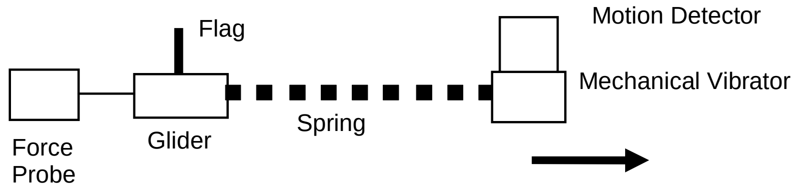

Figure 1

Open the experiment file called **L11.A1-1 Spring Constant**.

4. **Zero** the force probe with no force on it.

5. Pull the mechanical vibrator back slowly until the spring is barely extended.

6. **Start the computer and begin graphing**. Use your hand to slowly pull the mechanical vibrator so that the spring is extended about 30 cm. Hold the vibrator still and **stop the computer**.

7. The data will appear a little jagged, because your hand cannot pull back smoothly, but overall you should see a straight line. Use the mouse to highlight the region of good data. Then use the **fit routine** in the software to find the line that fits your data, and determine the spring constant from the fit equation (the slope). Include your best estimate of the uncertainty (the fit routine reports this).

k_1 ________________ N/m

8. **Print** out one set of graphs for your group that includes the fit and include it in your group report.

Question 1-1: Was the force exerted by the spring proportional to the displacement of the spring? Explain.

Question 1-2: What kind of a spring would have a large spring constant (large value of k)? A small spring constant?

9. Repeat the same measurement for the other spring and write down its spring constant (including uncertainty):

k_2: ________________ N/m

Question 1-3: How well do the two spring constants agree? Is this a reasonable difference? Discuss.

ACTIVITY 1-2: FINDING THE EFFECTIVE SPRING CONSTANT FOR THE MOTION OF THE TWO SPRING SYSTEM

For the rest of the experiment, we will be using the two-spring system with the springs connected on either side of the glider cart. You do not need any new equipment.

It is straightforward to see that the *effective* spring constant of the multi-spring system is simply the sum of the individual spring constants:

$$F_{eff} = \sum F_i = \sum -k_i(x - x_i) = -\left[\left(\sum k_i\right)x - \sum k_i x_i\right] = -k_{eff}\left(x - x_{eff}\right) \tag{9}$$

where the effective spring constant is:

$$k_{eff} = \sum k_i \tag{10}$$

and new equilibrium position is:

$$x_{eff} = \sum x_i k_i / k_{eff} \tag{11}$$

1. Set up the system on the air track as the diagram (Figure 2) below indicates. Ask your TA if you have any questions. **Never move items on the air track unless the air is flowing! You might scratch the surfaces and create considerable friction.** Both springs should be slightly stretched in equilibrium. Move the mechanical vibrator to the right so that the distance from end-to-end of the springs is about 76 cm. Note that the flag is mounted on top of the cart in a position that the motion detector will see it. One spring is connected to a fixture on the left and the other spring will be connected to the mechanical vibrator. The mechanical vibrator is sitting on top of a small glider cart and should no longer be moved on the air track.

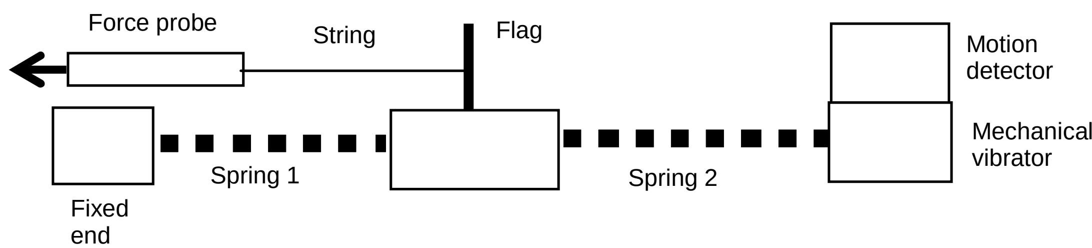

Figure 2

2. You will continue to use the same experimental file used previously in **Activity 1-1**, **L11.A1-1 Spring Constant**. Delete any data showing.
3. Make sure the air is on for the air track. A string is connected near the bottom of the flag on the glider cart to the force probe. In this experiment you will be holding the force probe in your hand and pulling the force probe to the left.
4. Zero the force probe and hold on to the force probe.
5. Let a colleague start the computer taking data. When you hear the motion detector clicking (or see the green light), start pulling the force probe slowly to the left about 10 cm or so. Spring 1 should still be extended. Stop the computer.
6. Do the same analysis that you did in **Activity 1-1** to determine the spring constant of the combined two-spring system.

   k : ________________ N/m

7. **Print** out one graph with the linear fit showing and include with your group report.

Question 1-4: How well does this value of the spring constant agree with the individual ones you found previously? Explain any differences you found.

Question 1-5: What relationship exists between the effective spring constant for the two spring system and the individual spring constants?

ACTIVITY 1-3: FREE MOTION OF THE TWO SPRING SYSTEM

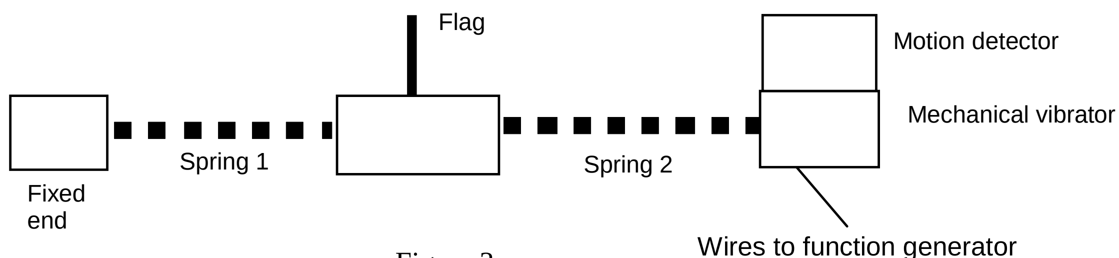

Figure 3

1. Remove the force probe and string connected to the flag from the previous experiment. The setup is like that shown in Figure 3. Now we want to examine the free oscillations of this system.
2. Open the experimental file **L11.A1-3 Two Spring System**.
3. Make sure the air is on in the air track. Verify the function generator is off. Let the glider remain at rest.
4. **Start the computer** and take data for 2-3 seconds with the glider at rest so you can obtain the equilibrium position. Then pull back the glider about 10-15 cm and let the glider oscillate until you have at least ten complete cycles. Then **stop the computer**.
5. **Print** out this graph and include it with your report.
6. We can find the angular frequency by measuring the period T_F. Use the **Smart Tool** to find the time for the left most complete peak (write it down), count over several more peaks (hopefully at least 10) to the right and find the time for another peak

(write it down). Subtract the times for the two peaks and divide by the number of *complete cycles* to find the period. Then determine f_F and ω_F .

First peak ___________ s Last peak ___________ s # cycles ___________

Period T_F ___________ s Frequency f_F ____________ Hz

Angular frequency ω_F ____________ rad/s

7. Now determine the spring constant from Equation (4).

k : ________________ N/m

Question 1-6: Compare the values of this spring constant with the ones you found in **Activity 1-2**. Are they similar? Do you expect them to be? Explain.

Question 1-7: Describe the motion that you observed in this activity. Does it look like it will continue for a long time? Did you observe significant damping?

INVESTIGATION 2: DAMPED OSCILLATORY MOTION

Equation (2) in the previous section describes a periodic motion that will last forever. The only force acting on the mass is the restoring force, F_{spring} . Most motions in nature do not have such simple "free" oscillations. It is more likely there will be some kind of friction or resistance to damp out the free motion. In this investigation we will start the free oscillations like we did in the previous experiment, but we will add some damping. Air resistance is a good example of damping in nature. Automobile springs, for example, are damped (by shock absorbers) to reduce oscillations caused by rough road surfaces.

In general, the friction which occurs between oiled surfaces (or in liquids and gases) is not constant but depends on the velocity. The simplest form for this velocity dependent friction is:

$$F_v = -bv \tag{12}$$

Dry friction $(F_d = \mu N)$ is also found in mechanical systems; but in electrical oscillations the damping term is almost always of the form of Equation (12).

The new equation of motion becomes:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{13}$$

A simple sinusoid will not satisfy this equation. In fact, the form of the solution is strongly dependent upon the value of b.

If $b > 2\sqrt{mk} = 2m\omega_F$, the system is "over damped" and the mass will not oscillate. The solution will be a sum of two decaying exponentials.

If $b = 2m\omega_F$, the system is "critically damped" and, again, no oscillations occur. The solution is a simple decaying exponential. Shock absorbers in cars are so constructed that the damping is nearly critical. One does not increase the damping beyond critical because the ride would feel too hard.

For sufficiently small damping ($b < 2m\omega_F$), the solution is given by:

$$x = A_D e^{-t/\tau} \sin\left(\omega_D t + \delta_D\right) \tag{14}$$

where the time for the amplitude to drop to $1/e$ of its initial value is given by:

$$\tau = \frac{2m}{b} \tag{15}$$

and the angular frequency is given by:

$$\omega_D = \sqrt{\omega_F^2 - 1/\tau^2} \tag{16}$$

The subscript ("D", for "damping") helps to distinguish this angular frequency from the natural or free angular frequency, ω_F. Note that frequency of the damped oscillator, ω_D, is shifted slightly from the natural frequency. For small damping one may neglect the shift.

A_D and δ_D are constants of integration and are again determined by the initial conditions.

An example of Equation (14), with $\delta_D = \pi/2$, is shown in Figure 4:

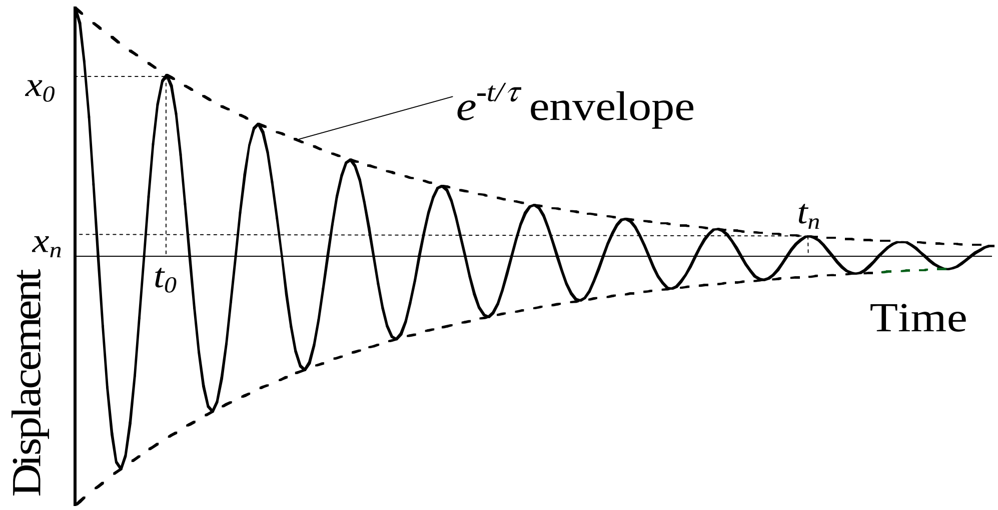

Figure 4.

ACTIVITY 2-1: DESCRIPTION OF DAMPED HARMONIC MOTION

We can add damping by attaching one or more strong ceramic magnets to the side of the moving cart. In your study of electromagnetism, you will learn that a moving magnetic field sets up a second magnetic field to oppose the effects of the original magnetic field. This is explained by Lenz's Law. The magnets we attach to the moving cart will cause magnetic fields to be created in the aluminum air track that will oppose the motion of the cart. The effect will be one of damping. The opposing magnetic fields are caused by induced currents, called *eddy currents,* and they will eventually dissipate in the aluminum track due to resistive losses. The motion carts themselves are made of non-magnetic material that allows the magnetic field to pass through (iron carts would not work). We can change the magnitude of this damping force by the number of magnets we attach.

NOTE: Do not place the magnets near the bottom of the cart. The magnets' interaction with steel support rods would perturb the motion.

1. Place a ceramic magnet symmetrically on each side of the glider cart, in the middle. Leave the other magnets taped to the top of the cart so they will not damp out the motion. Use a small piece of tape to keep the magnets in place.
2. Open the experiment file called **L11.A2-1 Spring Oscillations.**
3. Make sure the air is on for the air track, the glider is at equilibrium and not moving on the air track.
4. **Start the computer**. Let the glider be at rest for a couple of seconds, so you can obtain the equilibrium position. Then pull back the glider about 10 – 15 cm and release it.
5. Let the glider oscillate through at least ten cycles before stopping the computer.

Question 2-1: Does the motion seem to agree with Equation (14)? Explain.

6. **Print** out the graphs, *but do not erase the data* (keep this as **Run 1**).
7. Determine the angular frequency for damping ω_D by measuring the time for ten cycles.

   Time for 10 cycles: ____________ s, Period T_D ___________ s;

   Frequency f_D ____________ Hz., ω_D ________________ rad/s (2 magnets)

Question 2-2: How well does this value of the angular frequency agree with ω_F of step 5 of the previous activity? Do you expect it to agree or not? Explain.

8. Repeat steps **3 - 6** for 4 magnets (two placed symmetrically on each side) taped to the side of the cart. Make sure they are placed symmetrically on the cart. Find the frequency and angular frequency:

   Time for 10 cycles: ____________ s; Period T_D ____________ s

   Frequency f_D ____________ Hz., ω_D ________________ rad/s (4 magnets)

9. **Print** out the graph, but keep the data.

Question 2-3: What is the biggest difference between the observed motion of the 2 and 4 magnets. Can you observe the damping? Describe it.

ACTIVITY 2-2: DETERMINATION OF DAMPING COEFFICIENTS

We can use Equation (14) and our data to determine the damping time τ and then the damping coefficient b from Equation (15). We measure the amplitude x_0 at some time t_0 corresponding to a peak. We then measure the amplitude x_N at t_N (the peak N periods later). [Remember that the period is given by $T = 1/f = 2\pi/\omega$.] From Equation (14) we can see that the ratio of the amplitudes will be given by:

$$x_0/x_N = e^{NT/\tau} = e^{(t_N - t_0)/\tau} \quad (17)$$

From this we can get the decay time:

$$\tau = (t_N - t_0)/\ln(x_0/x_N) \quad (18)$$

1. Look at the data you took in the previous activity that has two magnets on each side of the cart. Use the **Smart Tool** and find the equilibrium position of the cart. Then place the cursor on top of the first complete peak. Note both the peak height and time, and subtract the equilibrium position from the peak height to find the amplitude. Click on another peak that is about a factor of two smaller than the first peak. Determine again the amplitude and time.

   Equilibrium position ________________ m

	Peak 0	Peak *N*
Peak Height	________________ m	________________ m
Amplitude	________________ m	________________ m
Time	________________ s	________________ s

2. Use Equation (18) to find the decay time.

   τ : ________________ s

3. Use this value for τ and Equation (15) to find the damping coefficient b.

   b : ________________ kg/s

4. Now use Equation (16) to determine the theoretical value of the angular frequency.

ω_D : ________________ rad/s

Question 2-4: How well does this value of the angular frequency agree with the experimental value you determined in the previous activity? Consider Equation (16) and discuss.

INVESTIGATION 3: FORCED OSCILLATORY MOTION

In addition to the restoring and damping forces, one may have a force, which keeps the oscillation going. This is called a driving force. In many cases, especially in the interesting cases, this force will be sinusoidal in time.

$$F_{driving} = F_0 \sin \omega t \tag{19}$$

The equation of motion becomes

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin \omega t \tag{20}$$

This equation differs from Equation (13) by the term on the right, which makes it *inhomogeneous*. The theory of linear differential equations tells us that *any* solution of the inhomogeneous equation added to any solution of the homogeneous equation will be the general solution. The driving term forces the general solution to be oscillatory. In addition, there will be a phase difference between the driving term and the response $x(t)$. Consequently, a solution with sine or cosine alone will not do.

Without proof, we will state that a solution to the inhomogeneous equation can be written as:

$$x_{ss} = A(\omega)\sin(\omega t + \delta) \tag{21}$$

Adding this to the previously found solution to the homogeneous equation, we obtain the general solution:

$$x = A_D e^{-t/\tau} \sin(\omega_D t + \delta_D) + A(\omega)\sin(\omega t + \delta) \tag{22}$$

We recognize the first term as the damped oscillation which, after a time, goes to zero. This is called the *transient* solution.

The second term is the *steady state* solution and describes the motion after the transient part has faded away. It is this motion that we will now consider.

The amplitude of the steady state motion is given by:

$$A(\omega) = \frac{F_0/m}{\sqrt{\left(\omega_F^2 - \omega^2\right)^2 + \left(\frac{b\omega}{m}\right)^2}} \tag{23}$$

and the phase shift is given by

$$\delta = -\tan^{-1}\left(\frac{b\omega/m}{\omega_F^2 - \omega^2}\right) \tag{24}$$

To discuss Equation (21), we look at its variation with the driving frequency ω. First consider very low driving frequencies. The phase shift goes to zero (the displacement is in phase with the driving force) and the amplitude is essentially constant:

$$A(\omega \ll \omega_F) \approx A(0) = F_0/m\omega_F^2 = F_0/k \tag{25}$$

This is just what you should expect if you exert a force F_0 on a spring with spring constant k.

For very high driving frequencies, the phase shift goes to 180°. The displacement will again vary as $\sin\omega t$ but it will be 180° out of phase with the driving force. The amplitude drops off rapidly with increasing frequency:

$$A(\omega \gg \omega_F) \approx F_0/m\omega^2 = \left(\frac{\omega_F}{\omega}\right)^2 A(0) \tag{26}$$

At intermediate frequencies, the amplitude reaches a maximum at

$$\omega_R = \sqrt{\omega_F^2 - \frac{1}{2}\left(\frac{b}{m}\right)^2} = \sqrt{\omega_F^2 - \frac{2}{\tau^2}} \tag{27}$$

[where the denominator of Equation (23) reaches a minimum]. The system is said to be at *resonance* and we denote this condition with the subscript "*R*". Note that ω_R is slightly lower than ω_F and slightly larger than ω_D.

Question 3-1: We claim that for our purposes, we can take $\omega_R \approx \omega_D \approx \omega_F$. Use your measured values of ω_F and τ and discuss the validity of this claim.

At resonance (actually at ω_F), the displacement is 90° out of phase with the driving force. The amplitude at resonance is given by:

$$A_R = A(\omega_R) = \frac{F_0}{b\omega_F} = \frac{F_0}{b}\sqrt{\frac{m}{k}}\,. \tag{28}$$

From Equation (28) we see that the amplitude would go to infinity if b, the damping coefficient, were zero. In many mechanical systems one must build in damping, otherwise resonance could destroy the system. [You may have experienced this effect if you've ever ridden in a car with bad shock absorbers!]

The *resonant amplification* (also known as "Quality Factor") is defined to be the ratio of the amplitude at resonance to the amplitude in the limit of zero frequency:

$$Q \equiv \frac{A_R}{A(0)} = \frac{\sqrt{mk}}{b} = \frac{m}{b}\omega_F \; . \tag{29}$$

$A(\omega)$ is shown for various values of Q in Figure 5 below.

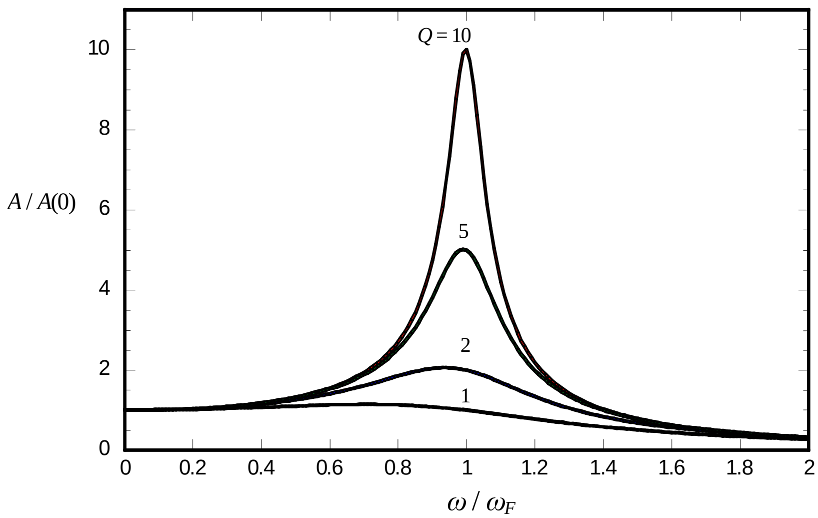

Figure 5. Amplitude as a function of frequency for various values of Q

We can find another useful interpretation of Q by observing the motion of this damped oscillator with no driving force. Instead imagine simply pulling the mass away from its equilibrium position and releasing it from rest. As we have seen, the mass will oscillate with continuously decreasing amplitude. Recall that in a time $\tau = 2m/b$ the amplitude will decay to $1/e$ of its original value. The period of oscillation is given by $T_D = 2\pi/\omega_D$, hence:

$$Q = \pi \frac{\tau}{T_D} \tag{30}$$

This says that Q is just π times the number of cycles of the oscillation required for the amplitude to decay to $1/e$ of its original value.

We finally are ready to study what happens to simple harmonic motion when we apply a force to sustain the oscillation .

ACTIVITY 3-1: OBSERVING DRIVEN OSCILLATIONS

The only extra equipment you will need is the

- Digital Function Generator – Amplifier

You will be able to continue using the same experimental file that you have been using. The equipment is the same, but you will need to utilize the mechanical vibrator to drive the spring. Keep the four magnets taped to the side of the cart or the resonant motion will be too large. We need to have the damping.

1. Unlock the oscillation driver. Turn on the power for the Function Generator driver (switch on back). Adjust the amplitude knob to about halfway. **Ask your TA for help if you need it.** The variable speed motor oscillates one of the springs at some specific frequency. Note that the generator displays the frequency f, **not** the angular frequency ω.
2. Set the motor frequency to ~0.01 Hz. This frequency is low enough that we can use it to measure A_0, the amplitude for zero frequency.
3. Make sure the air is on the air track. The vibrator should be moving left and right very slowly.

Question 3-2: Considering your measured values of ω_F and τ, about how long do you need to wait for the transient part of the motion to die out?

4. Open the file **L11.3-1 DrivenOscillations.** Make sure that the generator is connected to the driver and that the voltage probe is connected to the generator. [The red leads go to the red jacks and the black to black.]
5. **Start the computer** and take data for at least three cycles of the motion. Note visually the phase relation between the driver and the motion of the cart. Ask your Instructor for help if you have problems.
6. Once you have a good set of data, **stop the computer** and save these data.

Question 3-3: Describe the motion of the cart, especially the phase relation between the driver and the motion of the cart.

7. Find the peak-to-peak values of the displacement for the oscillatory motion. Note that this is twice the amplitude! This measurement can be done easily using the Smart Tool technique of measuring between two points. Ask your TA if you are not familiar with this technique.

$2A_0$ ________________ [A_0 is your "zero frequency" amplitude.]

Question 3-4: We claim that for our purposes, we can take $\omega_R \approx \omega_F$. Use your measured values of ω_F and τ and discuss the validity of this claim.

Question 3-5: For what driving frequency do you expect to obtain maximum amplitude and what peak-to-peak amplitude distance do you expect? Show your work.

$f_{R,\,predicted}$: ________________ Hz $2A_{R,\,predicted}$: ________________ m

8. Set the driving frequency to your predicted resonant frequency. Wait until the transients die out and then **start the computer.** Slowly vary the frequency around $f_{R,\,predicted}$ until you find the maximum amplitude. What resonant frequency and amplitude did you find?

$f_{R,experimental}$: ________________ Hz $2A_{R,experimental}$: ________________ m

Question 3-6: How well do your two resonant frequencies agree? Would you expect them to be the same? Explain.

9. Set the driver frequency to twice the resonant frequency. Wait for the transients to decay and then **start the computer** and observe the motion. Take data for at least three cycles and then **stop the computer.**

Question 3-7: Describe the motion of the glider, especially the relative phase between the driver and the glider.

10. Find the peak-to-peak values of the amplitude for this motion. Include the actual frequency that you used.

$2A$: ________________ at ________________ Hz

ACTIVITY 3-2: EXAMINATION OF THE QUALITY FACTOR

Prediction 3-1: Use your experimental values of τ and T_D to predict the quality factor, Q. Show your work.

$Q_{predicted}$: ________________

Now we want to be able to produce a graph like that in Figure 5. You will want to find the amplitude for some frequencies around the resonant frequency.

1. Fill in Table 3-1 below. [Note: You may already have some of these data]. One group member should continue with the next step while taking these data.

Table 3-1

Frequency f	Peak-to-peak amplitude, $2A$
0.01Hz	
$0.80\ f_R\ =$	
$0.90\ f_R\ =$	
$0.95\ f_R\ =$	
$f_R =$	
$1.05\ f_R\ =$	
$1.10\ f_R\ =$	
$1.20\ f_R\ =$	
$2\ f_R\ =$	

2. Enter your data into Excel and **produce a graph** like that shown in Figure 5 for your data. Use $A(0) \approx A(0.01\text{ Hz})$ and your experimentally determined value of f_R.

3. **Print** one copy of your table and your graph in your report.

Question 3-8: Does your graph have the shape you expect? Discuss your graph.

Question 3-9: Calculate Q from your amplitude vs. frequency data. Discuss agreement with your prediction.