

Name _____ Date _____ Partners _____

LAB 10:

HARMONIC MOTION AND THE PENDULUM



Galileo reportedly began his study of the pendulum in 1581 while watching this chandelier swing in Pisa, Italy

OVERVIEW

A body is said to be in a position of stable equilibrium if, after displacement in any direction, a force restores it to that position. If a body is displaced from a position of stable equilibrium and then released, it will oscillate about the equilibrium position. We shall show that a restoring force that is proportional to the displacement from the equilibrium position will lead to a sinusoidal oscillation; in that case the system is called a **simple harmonic oscillator** and its motion is called a **harmonic motion**. The simple harmonic oscillator is of great importance in physics because many more complicated systems can be treated to a good approximation as harmonic oscillators. Examples are the pendulum, a stretched string (indeed all musical instruments), the molecules in a solid, and the atoms in a molecule.

The example that you will be studying in this session is the pendulum: it has a position of stable equilibrium and undergoes a simple harmonic motion for small displacements from the equilibrium position. We will first analyze the motion theoretically before testing the theory experimentally.

THEORY

A simple pendulum consists of a mass m suspended from a fixed point by a string of length L . If the mass is pulled aside and released, it will move along the arc of a circle, as shown in Figure 1. Let s be the distance from the equilibrium position measured along that arc. While the force of gravity, mg , points downward, only its tangential component along the arc, $F_{\text{tan}} = -mg \sin \theta$, acts to accelerate the mass. The minus sign indicates that F_{tan} is a **restoring force**, i.e. one that points in a direction opposite to that of the displacement s . From Newton's second law, $F_{\text{tan}} = ma$, we then obtain:

$$-mg \sin \theta = ma \quad (1)$$

The speed of the mass bob is determined by $v = \Delta s / \Delta t$, and the acceleration by $a = \Delta v / \Delta t$. We can easily express s in terms of θ by noting that $s = L\theta$. So we have

$$s = L\theta \quad \text{and} \quad \theta = \frac{s}{L} \quad (2)$$

Equation (1) is normally studied for small angles, because the solution is particularly simple if we assume small arc length s , and therefore small angles $\theta \ll 1$ (in radians).

We expand $\sin \theta$ in terms of θ to obtain the series expansion $\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$,

(valid when θ is measured in *radians*). From this series expansion, we see that we can approximate $\sin \theta$ by the angle θ itself, as long as we keep θ so small that the higher order terms are much smaller than θ . In other words, when $\theta \ll 1$, $\sin \theta \cong \theta$. This is called the **small angle approximation**.

If we use the small angle approximation and Eq. (2), Eq. (1) becomes

$$F = -mg \sin \theta \approx -mg\theta = -\left(\frac{mg}{L}\right)s \quad (3)$$

This is the equation of motion of a simple harmonic oscillator, describing a simple harmonic motion. Note the similarity between this equation for the pendulum and what we found earlier for the mass and spring. In that case the solution for θ is solved for arbitrary values of the **amplitude**, θ_{max} , and the **phase**, δ , by:

$$\theta = \theta_{\text{max}} \cos(\omega t + \delta) \quad (4)$$

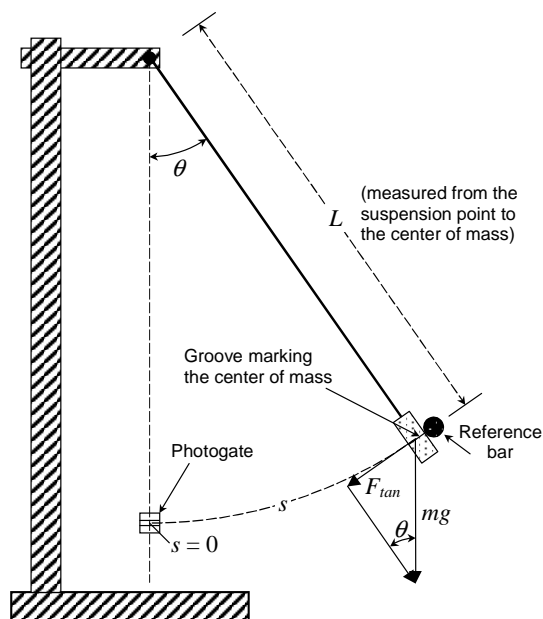


Figure 1

For the case of the mass and spring, $F = -kx$, and we found $\omega = \sqrt{k/m}$. In this case, we have $k = mg/L$ by comparison with Eq. (3), and our **angular frequency** ω becomes

$$\omega = \sqrt{\frac{g}{L}} \quad (5)$$

The cosine oscillates between -1 and 1; hence, according to Equation (4), the angle θ will oscillate over time between $-\theta_{\max}$ and θ_{\max} . The time to execute one complete cycle is called the **period** T and is related to ω by

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}} \quad (6)$$

NOTE: What *is not* in an equation is often as revealing as what *is* in it. Equation (6) does *not* contain the *mass* of the pendulum. *This means that the period is independent of the mass.* That can be true only if the mass on the left side of Equation (1) cancels the mass on the right side. You might think that this is obvious, but it is not. The mass on the left side is a measure of the resistance that the pendulum bob offers to acceleration, which we call the **inertial mass**. The mass on the right side is a measure of the gravitational force acting on the bob, and we call it the **gravitational mass**. While one would expect the period to remain unchanged if one replaces the bob with one of the same material but a different mass, it cannot be taken for granted that the ratio of inertial to gravitational mass is the same for all materials. Newton already worried about this point and did an experiment to uncover a possible difference, but found none. Modern experiments have established that changing the material of the bob changes the ratio of two kinds of mass by no more than 1 part in 10^{11} ! It was this observation that led to Einstein's *general theory of relativity*.

What is the significance of θ_{\max} and δ ? They are not determined by the previous equations but are fixed by the initial conditions. θ_{\max} is the maximum value of θ during the motion. It is determined by how far the pendulum was pulled back initially. The phase δ depends on when the clock was started that is used to measure t .

INVESTIGATION 1: MASS INDEPENDENCE OF PERIOD

In this activity you will measure the period of a simple pendulum.

The materials you will need are:

- photogate timer
- large and small support stands
- thread (string)
- pendulum string clamp
- brass and aluminum pendulum bobs
- meter stick
- step stool
- 2 right angle clamps
- long aluminum rod
- short aluminum rod
- digital mass scale
- protractor

ACTIVITY 1-1: PERIOD OF ALUMINUM BOB

We will measure the value of the oscillation period T that we found in Eq. (6) to be

$$T = 2\pi \sqrt{\frac{L}{g}} \quad (7)$$

Note that the mass of the bob does not appear in this expression. To see if this is true, we want to *measure* the periods of an aluminum and a brass bob having different masses, but having the same length.

NOTE: To make sure that you **start successive measurements with the same amplitude**, mount an aluminum rod (the reference bar) at an appropriate distance from the equilibrium position and start each measurement with the bob touching the rod.

Question 1-1: Discuss the conditions that need to be met to test Equation (7). Discuss, for example, the initial angle of displacement. What are its limitations? Why should the initial angle always be the same?

NOTE: To make the pendulums the same length suspend them side by side in the same clamp (*held* between the clamp and the bar) and adjust their lengths so that the center-of-mass grooves of the two bobs are at exactly the same level. **The lab instructor will show you how to do this. Have the instructor check your setup before proceeding!**

1. Set the pendulum lengths (measured from the pivot point to the line marking the bob's center of mass) to about 90 cm. There are several ways to measure the length. One way is to elevate the string clamp so you can place the meter stick between it and the table with the 0 of the meter stick flush with the pivot point. This also makes many of the length measurements at eye level rather than down near the table. The string clamp can then be moved down the support rod to where the bob is between the photogates. Record the measured length:

Pendulum length: _____

Prediction 1-1: What do you predict the period of the pendulum motion to be? Use the local value of the acceleration of gravity $g = 9.809 \text{ m/s}^2$. Show your calculation.

2. Move the brass bob out of the way by draping it over the stand, and measure the period of the aluminum bob. To make sure that you always start with the same amplitude, use a reference bar to set the initial angle (θ_{max}) to about 5° . The timing light is best placed at the equilibrium position $s = 0$. Align the photogate such that the center of the bob breaks the “beam”. The timer should be in the **pendulum mode** at 0.1 ms resolution. In pendulum mode, three triggers are necessary. The first trigger starts the timer. [In this case, the bob passing through the photogate the first time causes this first trigger.] The second trigger is ignored by the timer. [The bob swinging back through the photogate causes the second trigger.] The third trigger stops the timer (which corresponds to the bob passing through the photogate a third time – after one complete period). If the memory switch is on, additional triggers will not change the displayed time unless you push the memory switch to READ. Practice releasing the bob a few times.
3. Pull back the bob and release it. Wobbling will result in a poor measurement. Try to minimize the wobbling.
4. Measure the period (three times) for the aluminum bob and put your results in Table 1-1.

Table 1-1. Period of Pendulum

Trial	Aluminum Time (s)	Brass Time (s)
# 1		
# 2		
# 3		

ACTIVITY 1-2: PERIOD OF BRASS BOB

1. Hang the bobs side by side once again and see if they are still the same length. If the length has changed, you may need to measure the length again and repeat your measurements. Move the aluminum bob aside so you can measure the period of the brass bob.
2. Measure the period of the brass pendulum (again, three times) and enter your data into Table 1-1.

Question 1-2: What are your experimental errors in the pendulum measurement? Discuss them and make estimates here:

3. Average your measurements of the period for each mass and find the **standard error**. Excel and most standard deviation calculations return the **sample standard deviation**. The *sample standard deviation* represents the statistical error of each given measurement. The *standard error of the mean* is the uncertainty in the **mean** of the measurement values. See Appendix D for more information, especially the last page, which is a summary of the appropriate equations.

Aluminum pendulum period (mean): _____ Standard error _____

Brass pendulum period (mean): _____ Standard error _____

Question 1-3: Is there a significant difference between the two periods? How do you know?

Question 1-4: How well do your measurements for the period agree with Prediction 1-1? Explain any differences.

INVESTIGATION 2: DEPENDENCE ON ANGLE AND LENGTH

ACTIVITY 2-1: DEPENDENCE ON ANGLE

The simple theory says the period does not depend on the amplitude θ_{\max} , but that statement is based on the assumption that θ is small. In this experiment we want to investigate the dependence of the period on the initial angle θ . We want to see if there is

a dependence on the magnitude of the initial angle and to see the effect of assuming small angles in this measurement.

Do not allow the bobs to strike the photogate, ESPECIALLY DURING THE LARGER OSCILLATIONS!!

1. You want to use the brass bob and measure the period T for several initial angles θ as shown in Figure 1. Make sure that you have aligned the protractor properly with the bob in its equilibrium position.
2. Use $\theta = 5^\circ, 10^\circ, 20^\circ, 30^\circ,$ and 40° . Set the length L to *about* 60 cm, measure, and write it down below. For each angle, measure T three times and make sure the results are consistent. If not, remeasure until they are and average the results. Put your results into Table 2-1 and find the average and the standard deviation.

Length of Pendulum: _____

Table 2-1. Period Measurements as a Function of Angle

Angle	Period				Standard Error $\sigma_{\bar{T}}$
	T_1	T_2	T_3	Average \bar{T}	
5°					
10°					
20°					
30°					
40°					

Question 2-1: Describe what you observe in the raw data. Is the period constant as the angle increases?

3. Use Excel to plot your average period, \bar{T} , versus θ_{\max} . Your plot should show one half of a shallow parabola opening upward.
4. **Print** one copy of your graph.

Question 2-2: Note that in this experiment we have had you repeat measurements. Why do you think this is so? In general how well have your measurements agreed?

ACTIVITY 2-2: DEPENDENCE ON PENDULUM LENGTH

1. You will now measure T for several lengths, say 30, 60, and 90 cm for one angle. We suggest $\theta_{\max} = 5^\circ$.

Question 2-3: What criteria should be used when determining what initial angle to use for this experiment? Discuss these criteria.

2. Take three trials at each length and enter your data into the following table. Average your results. It is not necessary for you to have the lengths set right at 30, 60, and 90 cm. Set it close to these lengths and measure the actual length.
3. Put all your data and the calculations into Excel as shown in Table 2-2.
4. Plot the *square* of the period, \bar{T}^2 , vs. the pendulum length, L , using Excel. Fit a straight line to the data and display your fit on the graph.
5. **Print** a copy for your report. Include data and graph.

Table 2-2. Period Measurements as a Function of Length

Trial	Period T (s) for each length L (cm)		
	30 cm	60 cm	90 cm
Nominal length:			
Actual length:			
Time 1			
Time 2			
Time 3			
Mean \bar{T}			
Standard deviation σ_T			
Standard Error $\sigma_{\bar{T}}$ (Std. Error in Mean)			
\bar{T}^2			

Question 2-4: From your fit to \bar{T}^2 vs. L , calculate g . Show your work.

INVESTIGATION 3: CONSERVATION OF ENERGY

The conservation of energy is one of the most fundamental laws of physics. We test it more than once in this course. The total mechanical energy, i.e. the sum of potential and kinetic energy, should be a *constant*. On the other hand, both kinetic and potential energies change with time. Conservation of energy ($|\Delta KE| = |\Delta PE|$) requires that:

$$\frac{1}{2}mv^2 = mg\Delta h = mg(h_2 - h_1) \quad (8)$$

We will use the motion of the pendulum to check Equation (8). The change in kinetic energy must be equal to the change in potential energy along the entire path of motion of the pendulum. We measure the change in potential energy by simply measuring the change in height of the pendulum bob. To check the conversion of potential energy into kinetic energy, we need to measure the maximum speed v_{\max} of the bob as it passes through the equilibrium position.

It will be even easier if we just measure the ratio of the change of the two energies, which must be equal to one:

$$1 = \frac{\Delta KE}{\Delta PE} = \frac{0.5 mv^2}{mg\Delta h} = \frac{v^2}{2g(h_2 - h_1)} \quad (9)$$

The initial speed is zero. The mass has canceled out, and we only need to measure the bob's initial height h_2 , the bob's lowest height h_1 , and the maximum speed v_{\max} of the bob.

ACTIVITY 3-1: MEASUREMENT OF SPEED

To measure the maximum speed of the bob, we will use the motion detector to measure the maximum speed of the bob at the bottom. We could check Equations (8) and (9) anywhere along the path, but it is easiest to measure the bob's speed at the bottom of the swing, where the speed is a maximum.

The motion detector does not easily detect the bob's swing, because it does not move directly towards or away from the detector. We have found the bobs we have been using up to now in this lab to be too small, so we are going to change our experiment somewhat to use a larger bob. You will need the following equipment for this measurement:

- motion detector
 - large pendulum bob and string
 - long and short metal rods and clamps
 - hook clamp to suspend pendulum string
 - 2-m stick
1. Place the motion detector on the table in front of the computer as shown in Fig. 2. Put the detector on broad beam.
 2. Clamp the long metal rod vertically near the end of the table. Place the shorter metal rod horizontally and slide on the hook clamp. See Fig. 2.
 3. Hook the pendulum string on the hook clamp. The length L of the pendulum should be about 100 cm.



Figure 2. Pendulum setup for conservation of energy

Question 3-1: Explain why you do not need to know precisely the mass of the bob and the pendulum length for this measurement.

4. Adjust the short rod on the long rod so the center of the pendulum bob is *about* 1.0 m above the floor. The simplest way to do this is for one student to move the short rod on the long vertical rod while another student places the 2-m stick vertically next to the pendulum bob. Because the ball is spherical, you can easily tell where the outside of the sphere touches the 2-m stick. Adjust the height of the short rod. The height of the center of the ball above the floor will be the height h_1 , the lowest height of the pendulum bob. Write down the value of h_1 :

Height h_1 of pendulum bob: _____

5. The pendulum bob will be pulled back away from the computer and motion detector and let go. Adjust the geometry so the bob easily clears the motion detector. It should not come within 20 cm of the motion detector. Practice pulling back the bob a couple of times and letting it go. Use an initial angle of about 45° , but the value of the angle is not important.

Question 3-2: Explain why you do not need to know the precise value of the angle in this measurement.

6. Open the experiment file **L10.A3-1 Conservation of Energy** in the computer. The position and velocity of the bob will be visible.
7. Pull back the bob to an angle greater than 45° so that the bob will move directly towards the motion detector. Use the technique discussed in step 4 to measure the initial height of the bob h_2 . You might even hold the 2-m stick in position until you let go of the bob. Write down this initial height in Table 3-1 for trial 1.
8. **Start** taking data and let the pendulum swing through a couple of periods before stopping the computer.
9. You probably will notice the display indicates that the motion detector has several blips where the bob is not seen. This is normal because the motion detector is looking straight ahead, yet the bob swings in an arc. Near large angles the bob can

easily go out of view of the detector. However, we are only interested in measuring the bob's velocity at the bottom of its swing where the velocity is a maximum. You want to try to use the initial arc when the bob is swinging towards the motion detector.

Question 3-3: Why do we want to use the initial arc of the bob's swing? In this case is the velocity positive or negative? Explain.

10. Find the maximum speed of the bob during this first arc and write the speed v in Table 3-1. Complete Table 3-1 for trial 1.
11. Repeat your measurements and calculations for an initial angle less than 45° . This will be trial 2. Fill out Table 3-1 for this trial. Do not do trials 3 and 4 until you read Question 3-4.

Table 3-1. Conservation of Energy Measurement

Parameter	Trial			
	1	2	3	4
Height h_2				
Height h_1				
Height $\Delta h = h_2 - h_1$				
Speed v				
$\frac{\Delta KE}{\Delta PE}$				

Question 3-4: How well do your measurements agree with what you expect? If your results disagree with your expected answer by more than 5%, you should repeat them and be more careful. You do not need to do trials 3 and 4 if your measurements are accurate to better than 5%.

**PLEASE CLEAN UP YOUR LAB AREA!
TURN OFF THE PHOTOGATE TIMER.**