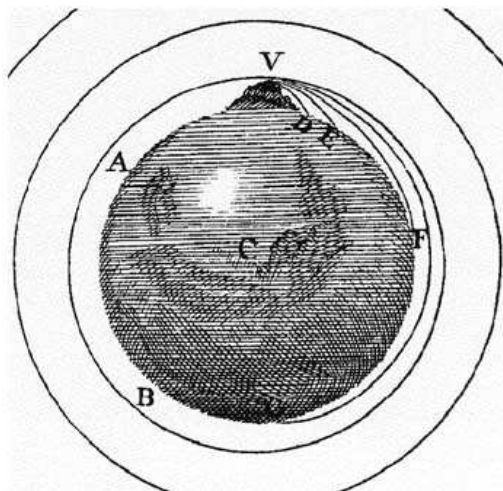


Name _____ Date _____ Partners _____

LAB 4: PROJECTILE MOTION



A famous illustration from Newton's *Principia* showing the relationship between projectile motion and orbital motion

OVERVIEW

We learned in our study of kinematics that two-dimensional motion is a straightforward extension of one-dimensional motion. Projectile motion under the influence of gravity is a subject with which we are all familiar. We learn to shoot basketballs in an arc to swish through the basket or to bounce off the backboard. We learn how to lob in volleyball and tennis. These are examples of projectile motion. The force of gravity acts in the vertical direction, and air resistance acts in both the horizontal and vertical directions, but we often neglect air resistance for small objects.

In this experiment we will explore how the motion depends upon the body's initial velocity and elevation angle. Consider a body with an initial speed v_0 at angle α with respect to the horizontal axis. We analyze the body's motion in two independent coordinates x (horizontal) and y (vertical). [As we are free to choose this origin of our coordinate system, we choose $x = 0$ and $y = 0$ when $t = 0$ so as to simplify our calculations.]

Figure 1 then shows that the components of the velocity vector along the x - and y -axes are respectively:

$$v_{0x} = v_0 \cos \alpha \quad \text{and} \quad v_{0y} = v_0 \sin \alpha \quad (1)$$

If we neglect air resistance, the only force affecting the motion of the object is gravity,

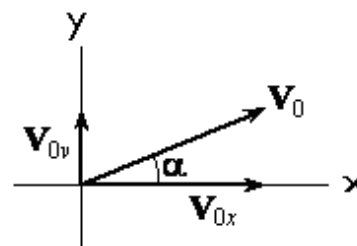


Figure 1. Velocity vector

which near the Earth's surface acts purely in the vertical direction ($\mathbf{F}_g = m\mathbf{g} = -mg\hat{\mathbf{y}}$).

There is no force at all in the horizontal direction.

Since there is no horizontally applied force, there is no acceleration in the horizontal direction; hence the x -component of the velocity will remain unchanged “forever”. The horizontal position of the body is then described by the expression for constant velocity:

$$x(t) = v_{0x}t = (v_0 \cos \alpha)t \quad (2)$$

The force in the y (vertical) direction is gravitation ($F_y = -mg$). Since $\mathbf{F} = m\mathbf{a}$, $a_y = -g$.

Integrating this with respect to time yields the vertical component of the velocity:

$$v_y(t) = v_{0y} - gt = v_0 \sin \alpha - gt. \quad (3)$$

The vertical component of the position can be obtained by integrating Eq. (3) with respect to time, t , yielding the result:

$$y(t) = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin \alpha)t - \frac{1}{2}gt^2. \quad (4)$$

We can use these equations to determine the actual trajectory of the body in terms of the x and y variables, with no explicit reference to the time t . The equation has a parabolic form

$$y = C_1x - C_2x^2 \quad (5)$$

where $C_1 = \tan(\alpha)$ and $C_2 = g/2v_0^2 \cos^2(\alpha)$. We are interested in the **height** h and the **range** R , the horizontal distance that the projectile travels on level ground. We can get the range from Eq. (5) by noting that y starts at zero when $x = 0$ and is once again zero when $x = R$. We obtain (see your textbook for details)

$$R = \frac{v_0^2 \sin(2\alpha)}{g} \quad (6)$$

Although we won't measure them in this experiment, we can similarly show that the time of flight (T) is given by

$$T = \frac{2v_0}{g} \sin(\alpha) \quad (7)$$

and the maximum height $h = y_{\max}$ is the value of y when the time is $T/2$. It also occurs when $v_y = 0$. The value of h is given by

$$h = \frac{v_0^2 \sin^2(\alpha)}{2g} \quad (8)$$

We show the trajectory in Figure 2 with the initial angle α , range R , and maximum height h indicated.

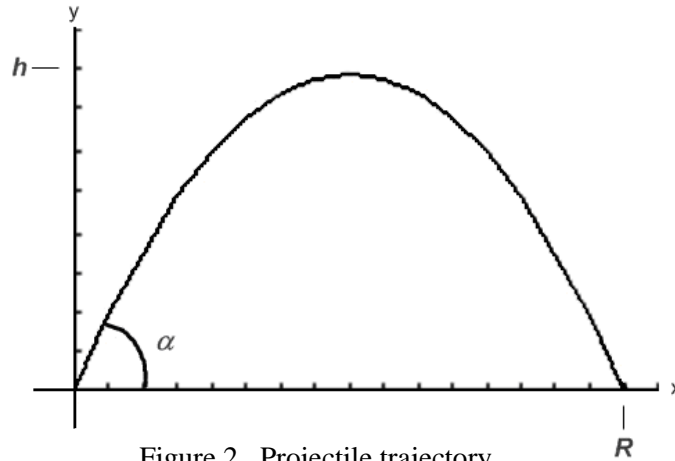


Figure 2. Projectile trajectory

We have prepared an Excel file to help you immensely in this experiment. Open Excel, find the TeachingLabs directory, open 635, and open the file **L04.Projectile Motion**. Note at the bottom there are three tabs (called *sheets*): Table 1-1, Table 1-2, and Table 1-3 for the three different measurements you will be making. You will be entering your data into the yellow background areas. Do nothing with the other cells, because Excel will do the calculations for you.

INVESTIGATION 1: MEASUREMENT OF RANGE

You will need the following materials:

- projectile motion apparatus
- optical bench
- meter stick
- level
- spark (pressure-sensitive) paper
- landing pad
- digital photogate timer
- masking tape
- 3 m tape measure

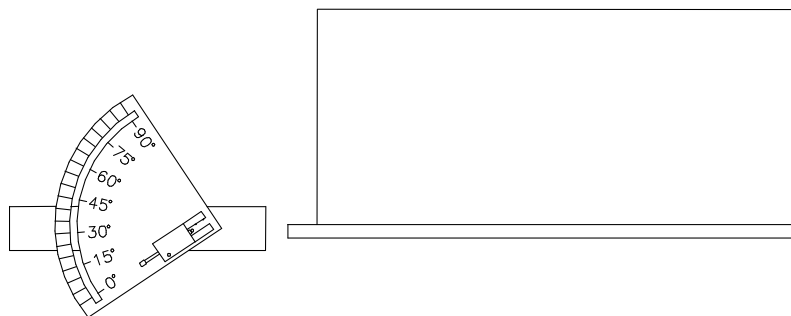


Figure 3. Projectile motion apparatus and landing pad.

1. Before doing anything, **listen while the Instructor goes through a description of the apparatus and how to level it.** It is crucial for the apparatus to be level or you will not obtain good results. The wooden table is heavy and can easily slip during the measurements.

2. After the brief Instructor discussion, familiarize yourself with the apparatus. The central feature of the setup is a “spring-loaded gun” mounted on a plate marked in degrees, as shown in Fig. 3. The plate can be rotated around a horizontal axis to set the initial direction of motion: a desired angle can be set by loosening the knurled screw attached to the plate, aligning the required angle mark with the horizontal mark on the stand and then tightening the screw.
3. The initial speed of the projectile (a steel ball) can be set to three different values by drawing back the spring-loaded firing pin until it is locked in one of the three preset positions. Notice the cable-release trigger and make sure that its plunger is fully withdrawn (this is achieved by pressing on the small ring near the end of the trigger cable), then try drawing back the firing pin and listen to the three distinct “clicks” corresponding to the three positions. Unfortunately, sometimes the cable-release becomes disconnected, something you want to avoid, because it is difficult to reconnect it! Press the trigger’s plunger and the pin will “fire”.
4. Now you can practice launching the projectile: try different angles and different velocities to get a feeling of which angular range will land the projectile onto the table. When doing your measurements, record the landing spot of the ball on the table by means of a pressure sensitive paper taped to the board. **Do not tape the paper in position until you are ready to start your measurements, because it is expensive, and we can’t afford to waste any.**
5. Now you will want to verify that the apparatus is level and that the landing table is at the same height as the pivot point (which is where we believe the ball also leaves the spring). First, place the level on the back of the gun apparatus and, if needed, use the two screws holding up the apparatus to level the gun apparatus. Use a meter stick and the level to make sure that the table top surface is at the same height as the pivot point of the angle mark (the timing apparatus can easily be pulled out to see where the pivot point actually is) **and** that the table is level. Ask your Instructor for assistance if this procedure is not clear. **IT IS CRUCIAL THAT YOU HAVE DONE THIS CORRECTLY!**

ACTIVITY 1-1: DETERMINING THE INITIAL SPEED

1. As the final preparation, we want to measure the initial speed of the projectile. As it leaves the gun, the projectile crosses the light beam of two “light bulbs” (actually light emitting diodes or LEDs), shining onto two photocells, which are connected to a timer unit. When in *pulse* mode, the timer will start counting when the first light beam is interrupted, and it will keep counting until the second beam is obscured. Make sure the timer is set to 0.1 ms resolution and set the launcher angle to 0° . This will give you the best possible time resolution.
2. Withdraw the firing pin to one of the three positions and place the ball into the gun. If the timer is running, press the RESET button. Launch the ball and use the Styrofoam cup to catch the ball. **The timer will record the time in seconds** it took the projectile to go from the first to the second light. If you know the distance d between the two (it

is marked on the apparatus and should be 20 mm), you can determine the average speed of the ball as it travels between the lights. Without more knowledge, we assume the uncertainty σ_d in d is one half of the least significant digit in d or 0.5 mm.

$$d = 20 \pm 0.5 \text{ mm}$$

Question 1-1: Can you think of a different way to measure the initial speed? Describe at least one other method. Discuss among your group why we use the procedure suggested (0° angle) to determine the initial speed. Write your conclusions here.

3. You should have the Excel file **L04.Projectile Motion** open (see paragraph above) and make sure you are looking at Table 1-1 (if not there, click on the proper sheet near the bottom of the screen). Note that even though the timer is set to 0.1 ms, this refers to its resolution. **The timer still displays in seconds. It is important to work together.** Measure the time (t) for at least four trials for each “click” setting. One student should enter the data into the Excel file while the other students take the data. Take turns during the lab doing each. Although your primary data record is in Excel, it is wise to also write your data into Table 1-1 shown here in the journal in case something happens to your computer file. **Do not calculate the quantities in Table 1-1 by hand – Excel does this for you.**
4. The Excel file will perform the calculations for you, but it is absolutely necessary that you understand what is being calculated. You can click on a given cell in Excel and look above the column headings to see the function being calculated. For example in cell F5, you will see the following: =AVERAGE(B5:E5)*1000 This means that we are finding the average time in seconds between cells B5 and E5 and then converting the time to milliseconds (ms) by multiplying by 1000. You should click on each cell in a row to understand what is being calculated.

For each setting the Excel file calculates the average time ($t_{\text{avg}} = \bar{t}$, Column F), the standard deviation σ_t (sigt, Column G), the “standard error in the mean” ($\sigma_{\bar{t}}$) (stan error mean, Column H), and one-half of the least significant digit in the timer (0.05 ms, Column I). Let your uncertainty in \bar{t} ($\sigma_{\bar{t}}$), be the **larger** of the “standard error in the mean” or one-half of the least significant digit (0.05 ms) and that is in the next Column J headed by “largest sigtbar”. In Column M we calculate the initial speed, $v_0 = d / \bar{t}$.

In order to calculate the uncertainty in the velocity we need to use the propagation of errors, because we have uncertainties in both distance d and time t . Therefore in

Columns K and L we find the fractional errors in the time and distance, respectively. These values are used to determine the uncertainty in velocity given in Column N. Finally in Column O we find the fractional uncertainty in the velocity, which we will use later to determine the uncertainty in a calculation of the range.

Table 1-1 Initial Speed

Position Click	t (s)				\bar{t} (ms)	σ_t (ms)	stand error mean $\sigma_{\bar{t}}$ (ms)	sig digit (ms)	largest sigtbar (ms)	frac error t	frac error d	vel v_0 (m/s)	uncer vel σ_{v_0} (m/s)	frac uncer vel
	Trial 1	Trial 2	Trial 3	Trial 4										
1														
2														
3														

5. Print out one copy of Table 1-1 from Excel for your group report.

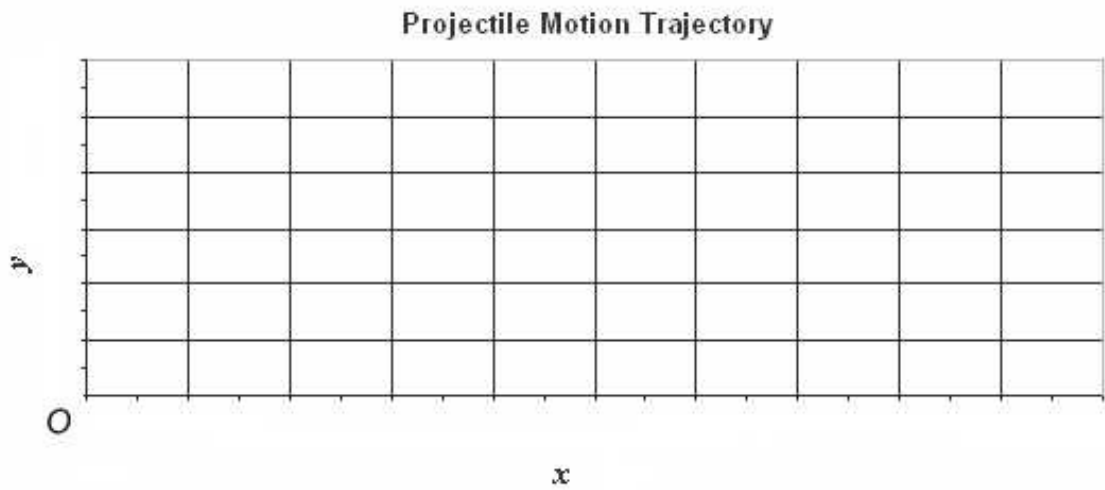
Question 1-2: Look at the function being calculated in cell N6 of the Excel file. What is being calculated? Explain the equation being used.

ACTIVITY 1-2: RANGE AS A FUNCTION OF INITIAL ANGLE

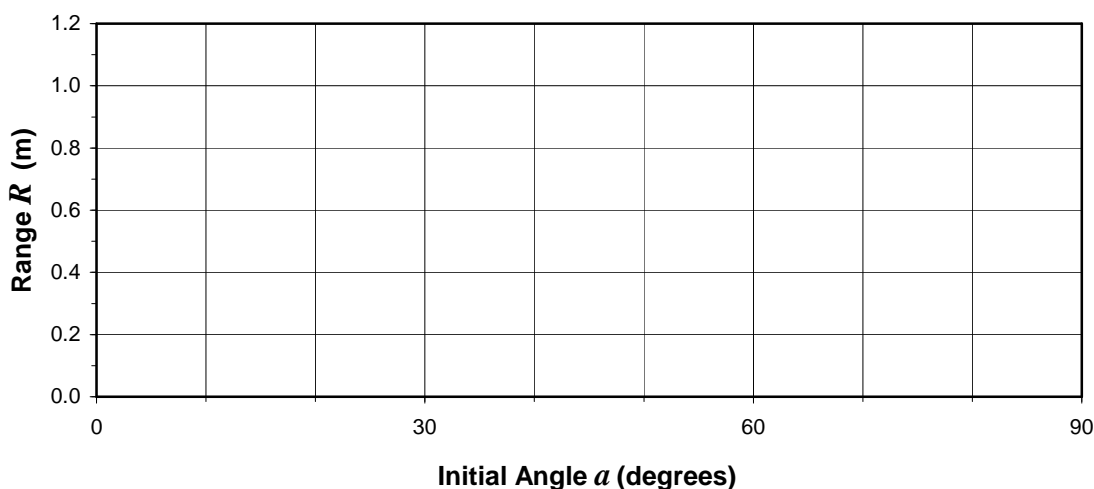
For this activity you will choose **one** of the three values of the launch speed and execute a set of launches for at least five different values of the launch angle: 15°, 30°, 45°, 60°, and 75°. We suggest that you take range data with the highest initial speed.

Question 1-3: Write down two reasons why it might be best to use the highest speed to take these data.

Prediction 1-1: Consider a coordinate system where the x and y coordinates are both zero when the ball passes through the pivot and lands on the top of the landing board at that same level ($y = 0$). The ball then starts at $y = 0$ and reaches $y = 0$ again when landing on the board. The range will simply be the x coordinate where it hits. Draw your **qualitative** predictions for the trajectory on the following graph for 15° , 30° , 45° , 60° , 75° . (These will be lines of motion.) **Do this before coming to lab.** [Qualitative means do not use actual numbers, but make the trajectories correct relative to each other: show which goes highest, which goes further, which have the same trajectory, etc.]



Prediction 1-2: Draw your theoretical predictions for the range R as a function of the initial angle α on the following graph. (These will be points.) You did a similar calculation in the prelab. You may include your data here from your prelab calculation. **Do this before coming to lab.**



1. Remove the photogate so that you can more easily measure the distances. Turn off the photogate's power.
2. For each launch, the range of the projectile will be recorded by a dot on the pressure-sensitive paper. You will notice that even if you keep the initial speed and inclination fixed, the landing positions will be spread over several millimeters. The actual range you will use for plotting your data will be the average of the measured points. To avoid confusions due to spurious marks on the paper caused by bounces of the projectile, it is advisable, first, to label each mark as the ball makes it and, second, to perform the measurements with the longest ranges first. You might, for example, **circle each mark to distinguish it from the newer ones. Do not take data until instructed to do so!** ***Ask your Instructor for permission to proceed***

Question 1-4: Can you determine from the formula given earlier (Eq. (6)) which angle corresponds to the longest range? Explain. That should be your first angle.

3. Record your initial speed: v_0 : _____. Enter this speed in Table 1-2 of your Excel file. You may have to click on the correct sheet in Excel at the bottom of the screen.
4. Now take your data for this same v_0 and insert your values into Table 1-2 of the Excel file. Your TA may have some suggestions as to how to most easily measure the individual ranges. It is easiest to measure in centimeters, and we will convert to meters in the Excel file. You may want to have one member of your group also include the range measurements in Table 1-2 of this journal for security reasons (what if something

happens to your Excel file?), but it is not required. Remember to work together. One student can input the data into Excel, while the other students perform the measurements. *Move quickly along to take these data.*

You do not need to do the calculations of \bar{R} , σ_R , and $\sigma_{\bar{R}}$ shown in Table 1-2. Excel will do this for you. We show you Table 1-2 here for purposes of knowing what is being done in your lab.

Table 1-2 Range for each projectile launch

Trial	Angle				
	15°	30°	45°	60°	75°
Range 1 (cm)					
Range 2 (cm)					
Range 3 (cm)					
Range 4 (cm)					
Range 5 (cm)					
Calculations	The calculations below are done for you in Excel				
\bar{R} (m)					
σ_R (m)					
Stan error mean $\sigma_{\bar{R}}$ (m)					
R_{theory} (m)					

Question 1-5: Think about your measurement of the range. How accurate do you think that measurement is? At the very most it can be no better than you can read the measurement device (meter stick, tape measure, whatever...) Write below what you think this limiting measurement value is and enter it into Table 1-2 and into row 14 of the Excel file.

Measurement limit: _____

Explain how you determined this value:

6. For each angle, Excel calculates the average range, \bar{R} , the standard deviation, σ_R , and the “standard error in the mean” $\sigma_{\bar{R}}$. Let your uncertainty in \bar{R} be the larger of the values of either the “standard error in the mean” or the value of your measurement limit from Question 1-5. The uncertainty in R is finally given in Row 15.

Question 1-6: Does it appear from your data that the ranges for any of the angles are about the same? If so, for which sets of data are this true? Discuss. Do you expect any angles to have the same range? Explain.

7. Look carefully at Table 1-2 in your Excel file to see what is being calculated. For example, the theoretical range is calculated in Column J depending on the angles shown in Column H and the velocity given in Cell I15. The average experimental range is determined in Column K. The graph shows the theoretical range R plotted versus $\sin(2\alpha)$ and is compared with your experimental measurements. The theory calculation depends on the velocity measurement you made in the previous activity.

Question 1-7: R_{theory} is being calculated in column J. Write down the equation used in terms of parameters (not cell numbers) and explain where the equation comes from.

Question 1-8: Explain what is being calculated in cell D13 of Table 1-2 in Excel. Write down the equation used in this cell below. What equation on page D-6 of Appendix D are we determining?

8. **Print** out Table 1-2 in Excel and include the graph with your report.

9. Try to add “error bars” by hand to the measured \bar{R} data on your graph. These should be short lines of your maximum uncertainty in \bar{R} that extend above and below the data point. Often these lines have “T” like ends. See the next question if you have a problem.

Question 1-9: Are your uncertainties in \bar{R} smaller than the size of your data points? If so, is that good? Explain.

Question 1-10: How well do your measurements agree with your predictions? Are the deviations consistent with the accuracy you expect of your measurements? Explain why or why not. Discuss possible sources of systematic errors that may be present.

ACTIVITY 1-3: RANGE AS A FUNCTION OF INITIAL SPEED

1. Now you want to choose one suitable angle and measure the range R for each of the three initial speeds that you determined earlier. Write down your chosen angle:

Launch angle: _____

Question 1-11: What criteria did you use for choosing your launch angle? Would it be better to use a shallow or steep angle? Does it matter for any reason?

2. Take five readings for each of the three initial speeds. Insert your results into Table 1-3 of the Excel file. Enter the data in centimeters, and we will convert it to meters in Excel. As a precaution against losing your data, perhaps one student should

write the data in Table 1-3 of this journal, but it is not required.

- The Excel file will calculate the average range data and uncertainties as before. This information is here so you will know what we are doing in the lab.
- Excel has plotted your range data versus v^2 . The plot also indicates the theoretical range calculated using the velocities you determined in Activity 1-1 and show in Table 1-1 of the Excel file. Because there are uncertainties in the velocity measurements (see Columns N and O of Table 1-1), there are also uncertainties in the calculation of our theoretical ranges. The error bars on the theory calculations in Table 1-3 are reflective of the velocity uncertainties. We do the range theoretical calculations in Row 13 and determine the uncertainty in Row 14.

Table 1-3
Range as a Function of Initial Speed

Trial	Initial Velocity		
	1:	2:	3:
Range 1 (cm)			
Range 2 (cm)			
Range 3 (cm)			
Range 4 (cm)			
Range 5 (cm)			
\bar{R} (m)			
σ_R (m)			
Stan error mean $\sigma_{\bar{R}}$ (m)			
Measurement limit (m)			
R_{theory} (m)			
Stan error in R_{theory} (m)			
v^2 (m/s) ²			

Question 1-12: Examine Cell D14 in Table 1-3 of the Excel file. Write down the equation in the cell below. What is being calculated? Explain each term in the equation.

5. **Print** out Table 1-3 from Excel and your graph. Add error bars to your range data (if larger than the data points), and attach the graph to your report.

Question 1-13: Do your experimental data agree reasonably well with your theoretical predictions? How many of your data points overlap your theoretical calculation? Is this what you expect? Do you believe your errors are statistical or systematic? Explain.

