

## APPENDIX B

### GRAPHICAL ANALYSIS

#### INTRODUCTION

There are two basic ways to present data: the data table and the graph. In this appendix, we will try to acquaint you with some of the points to be considered in preparing a proper graph.

#### SELECTING THE GRAPH PAPER

For a first graph, made while the data are being taken, you might find it convenient to just use the square-ruled paper of your lab notebook. If you are like most people, you will be able to divide a small length into five equal parts by eye with sufficient accuracy. A typical notebook has squares  $1/4 \times 1/4$  inch. Allowing some margins for labeling that leaves you an area of about  $30 \times 40$  squares. Counting on your ability to interpolate to about  $1/5$  of the width of an individual square this will permit you to plot a graph containing  $150 \times 200$  units. Choose your scale so that the graph fills the page as much as possible without, however, going to strange units. Thus, if you want to plot 100 seconds along the 30 squares of the x-axis, you will be better off if you use units of 5 sec per division instead of 3.333 sec/div, even though the latter choice would have filled the available space. Do not use 8 div/inch because that forces you to use fractions instead of the more convenient decimal scale.

For a formal lab report as well as for a better looking journal, you might want to use regular *graph paper* ruled in either millimeters or  $1/10$  of an inch. Again, avoid the kind of graph paper that is ruled in  $1/2$ ,  $1/4$ , and  $1/8$  inches.

#### LOGARITHMIC PAPER

Sometimes it becomes *necessary* or *desirable* to plot the logarithm of a quantity instead of the quantity itself:

*Necessary:* In the case that a parameter varies over several orders of magnitude, the drastic compression of the scale by the logarithm ( $\log 10 = 1$ ,  $\log 100 = 2$ ,  $\log 1000 = 3$ , etc.) makes it possible to plot the data over a very wide range of the argument and/or the function.

*Desirable:* A logarithmic plot brings out the functional relation between the two variables plotted along the x and the y-axis. Consider, for example, the function

$$y = Cx^\alpha \quad (1)$$

For  $\alpha = 1$  it is represented by a straight line, for any other value of  $\alpha$  by some curve or other. If we take the logarithm of both sides of this equation we get

$$\log y = \log C + \alpha \log x . \quad (2)$$

This equation is represented by a straight line if we plot  $\log x$  along the **abscissa** (the  $x$ -axis) and  $\log y$  along the **ordinate** (the  $y$ -axis) of linear graph paper. Doubly logarithmic graph paper is ruled so that we do not need to take the logarithm, we simply plot  $x$  along the (logarithmically ruled)  $x$ -axis and  $y$  along the (logarithmically ruled)  $y$ -axis and get a straight line in the resulting **log-log plot**. The slope of that straight line is given by the exponent  $\alpha$ . Note that this is true only for a simple power law of the form given by Eq. 1. Even an additive constant will make the log-log plot non-linear.

If one wishes to plot an exponential function of the form

$$y = Ce^{\alpha x} , \quad (3)$$

it is expedient to use **semi-log paper**, i.e. graph paper on which the  $x$ -axis is ruled linearly and the  $y$ -axis logarithmically. If one plots Eq. 3 on such paper, a straight line with the slope  $\alpha$  will result. Again any additive term, even a simple constant will make the resulting plot non-linear.

To find the slope from a logarithmic graph, read off two points from your straight line and solve the relevant equation, solving for the slope. For example, working from Eq. 2,

$$\begin{aligned} \log y_1 &= \log C + \alpha \log x_1 \\ \log y_2 &= \log C + \alpha \log x_2 , \end{aligned} \quad (4)$$

so 
$$\log y_2 - \log y_1 = \alpha(\log x_2 - \log x_1) , \quad (5)$$

or 
$$\alpha = \frac{\log\left(\frac{y_2}{y_1}\right)}{\log\left(\frac{x_2}{x_1}\right)} , \quad (6)$$

where  $x_1, x_2, y_1,$  and  $y_2$  are read directly off the graph.

Just in case you do not remember what logarithms are all about, we list some useful formulas and definitions:

$$\ln x \equiv \text{natural log } x = \ln_e x$$

$$\ln(e^x) = x$$

$$\ln x = 2.3026 \log_{10} x$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$\log x \equiv \text{common log } x = \log_{10} x$$

$$\log(10^x) = x$$

$$\ln(ab) = \ln a + \ln b$$

$$\ln(a^x) = x \ln a$$

## PLOTTING THE GRAPH

Select a scale for the x and the y-axis so that the graph fills the available area as much as is practical. In a linear plot, you might consider to suppress zero. For example, if y varies only from 100 to 125 you might want to start the y-axis at  $y = 100$  and spread the interval from 100 to 125 over the entire length of the axis. Plot each individual point first with a pencil and, after a final check of all points, mark them permanently. A dot with a concentric circle around it will both mark a point precisely and draw attention to it. If you plot more than one curve on a graph, select different symbols for the data points of each curve. Indicate the size of the probable error by error bars; if appropriate show error bars in both the x and the y-direction.

One usually plots not only the measured data points on a graph but connects them also with some kind of curve. This curve can either be a graphic representation of a theory that (one hopes) describes the experiment or it can simply be a smooth line drawn by eye with a French curve that more or less follows the data points. Whatever it is should be clearly stated in the caption underneath the figure, e.g. "The curve shows the calculated values according to ... [the formula]," or "The curve was drawn to guide the eye."

Often the slope of a curve, i.e. the tangent of the angle with the x-axis, conveys useful information. This does not mean that you can learn anything by taking a protractor to your curve, measuring its angle with the x-axis and then taking the tangent of that angle. Let us assume that you have plotted a distance  $x$  (measured in meters) against the time  $t$  (measured in seconds) it takes to travel that distance. In that case, the slope  $\Delta x / \Delta t$  of the resulting curve will be a measure of the velocity. This slope, however, will depend on the scales that you have used along the two axes. You will get the correct value of the velocity in m/s only if you divide the value of  $\Delta x$  (in meters) that you have read off the y-scale by the value of  $\Delta t$  (in seconds) read off the x-scale.

## LABELING THE GRAPH

Every graph should have a title that tells what is shown. You should also label both axes and give the units used. The customary way is to give the name of the variable followed by the dimension in square brackets, e.g.: time [sec]. A graph without proper labeling of the axes gives no useful information.

## AN EXAMPLE

Figures 1 and 2 (following page) show an example of the same set of data plotted on a linear and a log-log plot. The time it took a free falling steel sphere of 1 cm diameter to fall a distance  $S$  was measured at 20 cm intervals. The experimental errors were smaller than the size of the data points. The curve represents the equation  $t = \sqrt{2S/g}$  for  $g = 9.8 \text{ m/s}^2$ .

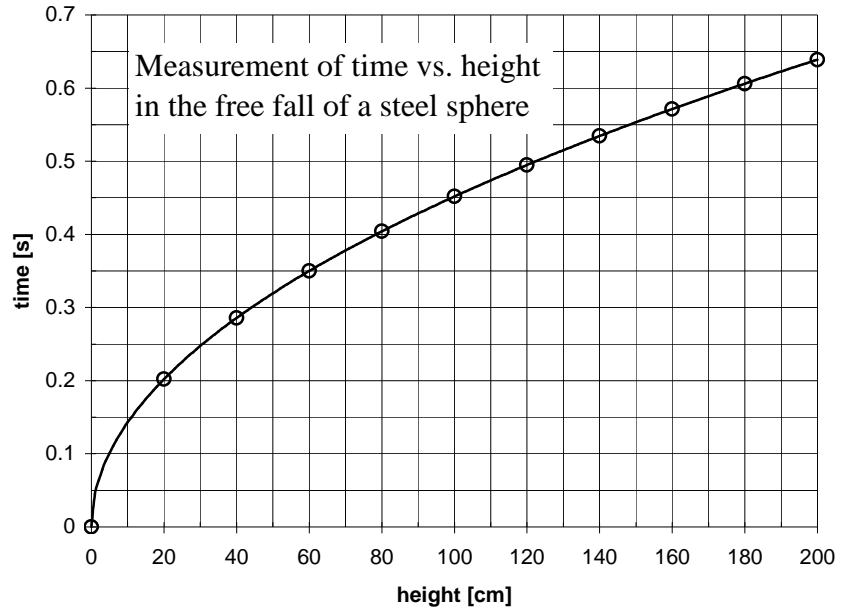


Figure 1. Time as a function of distance in free fall.

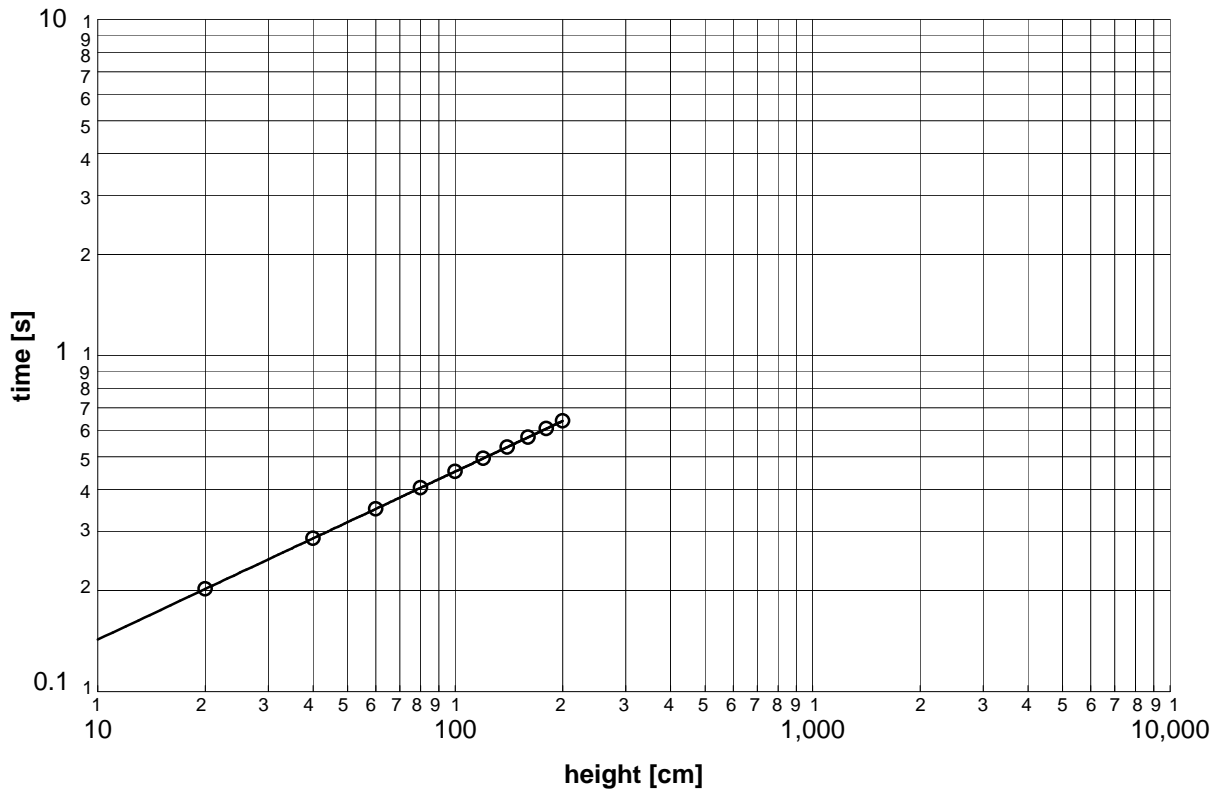


Figure 2. Same as Figure 1, but plotted on log-log plot.