Simulation of Policy Effects When Corner Solutions are Prevelant

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March 28, 2002

1 General Methodology

Consider the general problem where we have a set of agents indexed by i, each with a utility function

$$U_i(v_i, P)$$

with choice variables v_i and some (government) policy variable or environmental characteristic P. In general, we can solve for the derivatives of choice variables with respect to policy in the following way: Let v be the $m-$vector of choice variables, let ε be the $m-$vector of errors in the model, and let the set of first order conditions be written as

$$\underset{m_r \times m}{D_n'(\varepsilon)} \left[\underset{m\times 1}{\varepsilon} - \underset{m\times 1}{\psi} \left(\underset{m\times 1}{v}, P \right) \right] = 0 \tag{1}$$

where $D_n(\varepsilon)$ is a matrix that pulls the m_r rows of $\varepsilon - \psi(v, P)$ corresponding to interior solutions of first order conditions (conditional on ε) for family n. Conditional on ε, we can differentiate equation (1) to get

$$\begin{aligned} 0 &= \underset{m_r\times m}{D_n'(\varepsilon, P)} \left[\underset{m\times m}{\psi_v} \underset{m\times m_r}{D_n(\varepsilon, P)} \underset{m_r\times m}{D_n'(\varepsilon, P)} \underset{m\times 1}{dv} + \underset{m\times 1}{\psi_P} dP \right] \\ \Rightarrow D_n'(\varepsilon, P)\frac{dv}{dP} &= -\left[\underset{m_r\times m}{D_n'(\varepsilon, P)} \underset{m\times m}{\psi_v} \underset{m\times m_r}{D_n(\varepsilon, P)} \right]^{-1} \left[\underset{m_r\times m}{D_n'(\varepsilon, P)} \underset{m\times 1}{\psi_P} \right]. \end{aligned}$$

Now we are interested in

$$E\frac{dv}{dP} = \int \cdots \int D_n'(\varepsilon, P) \frac{dv}{dP} f(\varepsilon)\, d\varepsilon \tag{2}$$

where $f(\varepsilon)$ is the joint density of ε. Note that we do not have to worry about the term associated with $\partial D_n(\varepsilon, P)/\partial P$ because the relevant term is

$$\frac{\partial D_n(\varepsilon, P)}{\partial P} v(\varepsilon, P) f(\varepsilon)$$

which is zero because $v(\varepsilon, P) = 0$ at values of (ε, P) where $D_n(\varepsilon, P)$ changes.

We need to simulate equation (2) and in such a way that we "oversample" from that part of the support of ε where $\frac{dv}{dP} \neq 0$. Write equation (2) as

$$E\frac{dv}{dP} = E_k\frac{dv}{dP} = \underset{\widetilde{\varepsilon}_k}{\int \cdots \int} \int_{\varepsilon_k} D_n'(\varepsilon, P) \frac{dv}{dP} f(\varepsilon) \, d\varepsilon_k d\widetilde{\varepsilon}_k$$

where $E_k \frac{dv}{dP}$ reorders the order of integration in equation (2) so that the innermost integral is over the kth element of ε and $\widetilde{\varepsilon}_k = (\varepsilon_1, .., \varepsilon_{k-1}, \varepsilon_{k+1}, .., \varepsilon_m)'$ is the $(m-1)$ −vector of ε excluding ε_k. Note that

$$E\frac{dv}{dP} = E_k\frac{dv}{dP} \ \forall k.$$

Then we can write equation (2) as

$$\begin{aligned} E\frac{dv}{dP} &= \frac{1}{m}\sum_{k=1}^{m} E_k\frac{dv}{dP} \qquad (3) \\ &= \frac{1}{m}\sum_{k=1}^{m} \underset{\widetilde{\varepsilon}_k}{\int \cdots \int} \int_{\varepsilon_k} D_n'(\varepsilon, P) \frac{dv}{dP} f(\varepsilon) \, d\varepsilon_k d\widetilde{\varepsilon}_k \\ &= \frac{1}{m}\sum_{k=1}^{m} \underset{\widetilde{\varepsilon}_k}{\int \cdots \int} \int_{\varepsilon_k} D_n'(\varepsilon, P) \frac{dv}{dP} f(\varepsilon) \, d\varepsilon_k d\widetilde{\varepsilon}_k. \end{aligned}$$

Let

$$A_k(\widetilde{\varepsilon}_k) = \{\varepsilon_k : v(\varepsilon, P) > 0 \mid \widetilde{\varepsilon}_k\}$$

and

$$B_k(\widetilde{\varepsilon}_k) = \{\varepsilon_k : v(\varepsilon, P) = 0 \mid \widetilde{\varepsilon}_k\}.$$

Then equation (3) can be written as

$$\begin{aligned} = \ & \frac{1}{m}\sum_{k=1}^{m} \underset{\widetilde{\varepsilon}_k}{\int \cdots \int} \left[\int_{A_k(\widetilde{\varepsilon}_k)} D_n'(\varepsilon, P) \frac{dv}{dP} f(\varepsilon) \, d\varepsilon_k d\widetilde{\varepsilon}_k \right. \\ & \left. + \int_{B_k(\widetilde{\varepsilon}_k)} D_n'(\varepsilon, P) \frac{dv}{dP} f(\varepsilon) \, d\varepsilon_k d\widetilde{\varepsilon}_k \right]. \end{aligned}$$

It can be simulated as

$$\begin{aligned} & \frac{1}{mR}\sum_{r=1}^{R}\sum_{k=1}^{m} \left[D_n'\left(\varepsilon_{A_k}^r, P\right) \frac{dv\left(\varepsilon_{A_k}^r, P\right)}{dP} \Pr\left(\varepsilon_k \in A_k\left(\widetilde{\varepsilon}_k^r\right) \mid \widetilde{\varepsilon}_k^r\right) \right. \qquad (4) \\ & \left. + D_n'\left(\varepsilon_{B_k}^r, P\right) \frac{dv\left(\varepsilon_{B_k}^r, P\right)}{dP} \Pr\left(\varepsilon_k \in B_k\left(\widetilde{\varepsilon}_k^r\right) \mid \widetilde{\varepsilon}_k^r\right) \right] \end{aligned}$$

where $\varepsilon^r_{A_k}$ is a draw from $f(\cdot)$ conditional on the kth element $\varepsilon^r_{A_k k} \in A_k(\widetilde{\varepsilon}^r_k)$ and $\varepsilon^r_{B_k}$ is a draw from $f(\cdot)$ conditional on the kth element $\varepsilon^r_{B_k k} \in B_k(\widetilde{\varepsilon}^r_k)$. In practice, one uses the following algorithm to simulate equation (4):

For each draw $r = 1, 2, .., R$:

1) Draws ε^r from $f(\cdot)$.

2) For each $k = 1, 2, .., m$:

a) Pull out $\widetilde{\varepsilon}^r_k$ from ε^r;

b) Find the boundary along ε_k between $A_k(\widetilde{\varepsilon}^r_k)$ and $B_k(\widetilde{\varepsilon}^r_k)$ and call it ε^*_k;

c) Compute $F_k(\varepsilon_k \mid \widetilde{\varepsilon}^r_k)$ analytically and assign appropriate probabilities to $\Pr(\varepsilon_k \in A_k(\widetilde{\varepsilon}^r_k) \mid \widetilde{\varepsilon}^r_k)$ and $\Pr(\varepsilon_k \in B_k(\widetilde{\varepsilon}^r_k) \mid \widetilde{\varepsilon}^r_k)$;

d) Simulate $\varepsilon^r_{A_k}$ and evaluate $dv\left(\varepsilon^r_{A_k}, P\right)/dP$;

e) Simulate $\varepsilon^r_{B_k}$ and evaluate $dv\left(\varepsilon^r_{B_k}, P\right)/dP$;

f) Plug the simulated values of $dv\left(\varepsilon^r_{A_k}, P\right)/dP$ and $dv\left(\varepsilon^r_{B_k}, P\right)/dP$ into equation (4) and add;

3) Divide by mR.

2 Application

In our application, the utility function is

$$U_i(Q, X_i, L_i, t_i)$$

where $Q = (Q_m, Q_f)$, $L_i = (L_{im}, L_{if})$ if $i = 0$ and $L_i = (L_{ic}, L_{is})$ if $i > 0$, and $t_i = (t_{imf}, t_{ifm})$ if $i = 0$ and $t_i = (t_{imc}, t_{ims}, t_{ifc}, t_{ifs})$ if $i > 0$. We are interested in the effects of some policy P on all of the choice variables, quality of life, on utility, on probability of offering informal care, and probability of offering financial help for formal care. We can think of them as

$$\frac{dU_i}{dP} = \begin{cases} \frac{\partial U_i}{\partial X_i}\frac{dX_i}{dP} + \sum_{j=m}^{f}\left[\frac{\partial U_i}{\partial Q_j}\frac{dQ_j}{dP} + \left(\frac{\partial U_i}{\partial t_{ijj'}} - \frac{\partial U_i}{\partial L_{ij}}\right)\frac{dt_{ijj'}}{dP}\right] & \text{if } i = 0 \\ \sum_{j=m}^{f}\frac{\partial U_i}{\partial Q_j}\frac{dQ_j}{dP} + \sum_{j=c}^{s}\left[\frac{\partial U_i}{\partial X_{ij}}\frac{dX_j}{dP} + \frac{\partial U_i}{\partial L_{ij}}\frac{dL_{ij}}{dP} + \sum_{k=m}^{f}\frac{\partial U_i}{\partial t_{ijk}}\frac{dt_{ijk}}{dP}\right] & \text{if } i > 0 \end{cases};$$

$$\frac{dQ_j}{dP} = \frac{\partial Q_j}{\partial t_{0jj'}}\frac{dt_{0jj'}}{dP} + \frac{\partial Q_j}{\partial X_0}\frac{dX_0}{dP} + \sum_{i>0}\left[\left(\sum_{k=c}^{s}\frac{\partial Q_j}{\partial t_{ijk}}\frac{dt_{ijk}}{dP} + \frac{\partial Q_j}{\partial L_{ik}}\frac{dL_{ik}}{dP}\right) + \frac{\partial Q_j}{\partial X_i}\frac{dX_i}{dP}\right];$$

$$\begin{aligned}\frac{d\Pr\left[\bigcup_{j,k} t_{ijk} > 0\right]}{dP} &= -\frac{d\Pr\left[\bigcap_{j,k} t_{ijk} = 0\right]}{dP} \\ &= -\frac{d\prod_j \Pr\left[\bigcap_k t_{ijk} = 0\right]}{dP} \\ &= -\prod_j \Pr\left[\bigcap_k t_{ij'k} = 0\right] \frac{d\Pr\left[\bigcap_k t_{ijk} = 0\right]}{dP}\end{aligned}$$

with

$$\frac{d\Pr\left[\bigcap_k t_{ijk} = 0\right]}{dP} = \sum_{i'} \left[\frac{\partial \Pr\left[\bigcap_k t_{ijk} = 0\right]}{\partial X_{i'}} \frac{dX_{i'}}{dP} \right];$$

$$\frac{d\Pr\left[H_i > 0\right]}{dP} = \frac{\partial \Pr\left[H_i > 0\right]}{\partial t_{0jj'}} \frac{dt_{0jj'}}{dP} + \sum_{i'>0} \left[\sum_{k=c}^{s} \frac{\partial \Pr\left[H_i > 0\right]}{\partial t_{i'jk}} \frac{dt_{i'jk}}{dP} + \frac{\partial \Pr\left[H_i > 0\right]}{\partial L_{i'k}} \frac{dL_{i'k}}{dP} \right] + \overline{Q}$$

where $j' = m$ if $j = f$ and $j' = f$ if $j = m$. Using equation (1) from the other paper, we can substitute in

$$\begin{aligned}\frac{\partial Q_j}{\partial t_{0jj'}} &= \alpha_{0jj'} \left(1 + 2\gamma t_{0jj'}\right); \\ \frac{\partial Q_j}{\partial X_0} &= -\frac{p_{X_0}\mu}{q}; \\ \frac{\partial Q_j}{\partial t_{ijk}} &= \alpha_{ijk} \left(1 + 2\gamma t_{ijk}\right) - \frac{w_{ik}\mu}{q} \widetilde{H}_i; \\ \frac{\partial Q_j}{\partial L_{ik}} &= -\frac{w_{ik}\mu}{q} \widetilde{H}_i; \\ \frac{\partial Q_j}{\partial X_i} &= -\frac{p_{X_i}\mu}{q}\end{aligned}$$

to get

$$\begin{aligned}\frac{dQ_j}{dP} = {} & \frac{\partial Q_j}{\partial P} + \alpha_{0jj'} \left(1 + \gamma t_{0jj'}\right) \frac{dt_{0jj'}}{dP} - \frac{p_{X_0}\mu}{q} \frac{dX_0}{dP} + \\ & \sum_{i>0} \left[\left(\sum_{k=c}^{s} \left(\alpha_{ijk} \left(1 + \gamma t_{0jj'}\right) - \frac{w_{ik}\mu}{q} \widetilde{H}_i \right) \frac{dt_{ijk}}{dP} - \frac{w_{ik}\mu}{q} \widetilde{H}_i \frac{dL_{ik}}{dP} \right) \right. \\ & \left. - \frac{p_{X_i}\mu}{q} \frac{dX_i}{dP} \right].\end{aligned} \tag{5}$$

Using the corner conditions for formal and informal help, we can substitute in

$$
\begin{aligned}
\frac{\partial \Pr\left[\bigcap_k t_{ijk} = 0\right]}{\partial X_{i'}} &= \left\{ \begin{array}{ll} \sum_k B_k\left(\tau_1, \tau_2\right) \frac{\tau_k}{Q_j} \frac{\partial Q_j}{\partial X_{i'}} - \sum_k B_k\left(\tau_1, \tau_2\right) \frac{\tau_k}{X_i} 1\left(i = i'\right) & \text{if } i > 0 \\ \sum_k B_k\left(\tau_1, \tau_2\right) \frac{\tau_k}{Q_j} \frac{\partial Q_j}{\partial X_{i'}} & \text{if } i = 0 \end{array} \right. ; \\
\frac{\partial \Pr\left[H_i > 0\right]}{\partial t_{i'jk}} &= \phi\left(\frac{\log \frac{\beta_{1i}\mu p_{Xi} X_i \overline{Q}}{\beta_{2i} q}}{\sigma_{\eta X}} \right) \frac{1}{\overline{Q}} \frac{\partial \overline{Q}}{\partial t_{i'jk}}; \\
\frac{\partial \Pr\left[H_i > 0\right]}{\partial L_{i'k}} &= \phi\left(\frac{\log \frac{\beta_{1i}\mu p_{Xi} X_i \overline{Q}}{\beta_{2i} q}}{\sigma_{\eta X}} \right) \frac{1}{\overline{Q}} \frac{\partial \overline{Q}}{\partial L_{i'k}}
\end{aligned}
$$

where

$$
\tau_k = \left\{ \begin{array}{ll} \frac{\beta_{2i}\varepsilon_{xi} s_i^* w_{ik}}{p_{Xi} X_i} - \frac{\beta_{1i}}{Q_j} \alpha_{ijk} + \beta_{4ijk} & \text{if } i > 0 \\ -\beta_{1i} \frac{\alpha_{ijk}}{Q_j} + \beta_{4ijk} & \text{if } i = 0 \end{array} \right.
$$

and $B_k\left(\tau_1, \tau_2\right)$ is the derivative of the bivariate normal distribution with respect to the kth argument.

In order to compute dv/dP, we need all of the derivatives of each of the first order conditions with respect to each of the choice variables and the relevant policy variables. The first order conditions and derivatives are listed below:

Condition for ε_{xi}:

$$
\varepsilon_{xi} - \frac{\beta_{1i}\mu p_{Xi} X_i \overline{Q}}{\beta_{2i} q} = 0.
$$

Derivatives:

$$
\begin{aligned}
X_{i'} &: \quad -\frac{\beta_{1i}\mu p_{Xi} \overline{Q}}{\beta_{2i} q} 1\left(i = i'\right) + \frac{\beta_{1i}\mu p_{Xi} X_i}{\beta_{2i} q} \sum_{j=m}^{f} \frac{\delta_j}{Q_j^2} \frac{\partial Q_j}{\partial X_{i'}}; \\
L_{i'k'} &: \quad -\frac{\beta_{1i}\mu p_{Xi} X_i}{\beta_{2i} q} \sum_{j=m}^{f} \frac{\delta_j}{Q_j^2} \frac{\partial Q_j}{\partial L_{i'k'}}; \\
t_{i'j'k'} &: \quad \frac{\beta_{1i}\mu p_{Xi} X_i}{\beta_{2i} q} \frac{\delta_j}{Q_{j'}^2} \frac{\partial Q_{j'}}{\partial t_{i'j'k'}}.
\end{aligned}
$$

Condition for ε_{Lik} if $H_i > 0$:

$$
\varepsilon_{Lik} - \frac{\beta_{1i} s_i^* w_{ik} L_{ik} \mu \overline{Q}}{\beta_{3ik} q} = 0.
$$

Derivatives:

$$
\begin{aligned}
X_{i'} &: \quad \frac{\beta_{1i}s_i^* w_{ik}L_{ik}\mu}{\beta_{3ik}q}\sum_{j=m}^{f}\frac{\delta_j}{Q_j^2}\frac{\partial Q_j}{\partial X_{i'}}; \\
L_{i'k'} &: \quad -\frac{\beta_{1i}s_i^* w_{ik}\mu\overline{Q}}{\beta_{3ik}q}1\left(ik=i'k'\right)+\frac{\beta_{1i}s_i^* w_{ik}L_{ik}\mu}{\beta_{3ik}q}\sum_{j=m}^{f}\frac{\delta_j}{Q_j^2}\frac{\partial Q_j}{\partial L_{i'k'}}; \\
t_{i'j'k'} &: \quad \frac{\beta_{1i}s_i^* w_{ik}L_{ik}\mu}{\beta_{3ik}q}\frac{\delta_{j'}}{Q_{j'}^2}\frac{\partial Q_{j'}}{\partial t_{i'j'k'}}.
\end{aligned}
$$

Condition for ε_{Lik} if $H_i = 0$:

$$
\varepsilon_{Lik}-\frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik}L_{ik}}{\beta_{3ik}p_{Xi}X_i}=0.
$$

Derivatives:

$$
\begin{aligned}
X_{i'} &: \quad \frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik}L_{ik}}{\beta_{3ik}p_{Xi}X_i^2}1\left(i=i'\right); \\
L_{i'k'} &: \quad -\frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik}}{\beta_{3ik}p_{Xi}X_i}1\left(ik=i'k'\right); \\
t_{i'j'k'} &: \quad 0.
\end{aligned}
$$

Condition for ε_{tijk} if $H_i > 0$ or $i = 0$:

$$
\varepsilon_{tijk}-\beta_{1i}\left[\frac{\mu s_i^* w_{ik}\overline{Q}}{q}1\left(i>0\right)-\frac{\alpha_{ijk}}{Q_j}\left(1+2\gamma t_{ijk}\right)\right]-\beta_{4ijk}=0.
$$

Derivatives:

$$
\begin{aligned}
X_{i'} &: \quad \beta_{1i}\left[\frac{\mu s_i^* w_{ik}}{q}1\left(i>0\right)\sum_{j=m}^{f}\frac{\delta_j}{Q_j^2}\frac{\partial Q_j}{\partial X_{i'}}-\frac{\alpha_{ijk}}{Q_j^2}\left(1+2\gamma t_{ijk}\right)\frac{\partial Q_j}{\partial X_{i'}}\right]; \\
L_{i'k'} &: \quad \beta_{1i}\left[\frac{\mu s_i^* w_{ik}}{q}1\left(i>0\right)\sum_{j=m}^{f}\frac{\delta_j}{Q_j^2}\frac{\partial Q_j}{\partial L_{i'k'}}-\frac{\alpha_{ijk}}{Q_j^2}\left(1+2\gamma t_{ijk}\right)\frac{\partial Q_j}{\partial L_{i'k'}}\right]; \\
t_{i'j'k'} &: \quad \beta_{1i}\left[\frac{\mu s_i^* w_{ik}}{q}1\left(i>0\right)\frac{\delta_{j'}}{Q_{j'}^2}\frac{\partial Q_{j'}}{\partial t_{i'j'k'}}-\frac{\alpha_{ijk}}{Q_j^2}\left(1+2\gamma t_{ijk}\right)\frac{\partial Q_j}{\partial t_{i'jk'}}1\left(j=j'\right)\right. \\
&\qquad \left.+2\gamma\frac{\alpha_{ijk}}{Q_j}1\left(ijk=i'j'k'\right)\right].
\end{aligned}
$$

Condition for ε_{tijk} if $H_i = 0$ and $i > 0$:

$$
\varepsilon_{tijk}-\frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik}}{p_{Xi}X_i}+\frac{\beta_{1i}}{Q_j}\alpha_{ijk}\left(1+2\gamma t_{ijk}\right)-\beta_{4ijk}=0.
$$

Derivatives:

$$\begin{aligned}
X_{i'} &: \quad \frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik}}{p_{Xi}X_i^2}1\left(i=i'\right)-\frac{\beta_{1i}}{Q_j^2}\alpha_{ijk}\left(1+2\gamma t_{ijk}\right)\frac{\partial Q_j}{\partial X_{i'}}; \\
L_{i'k'} &: \quad -\frac{\beta_{1i}}{Q_j^2}\alpha_{ijk}\left(1+2\gamma t_{ijk}\right)\frac{\partial Q_j}{\partial L_{i'k'}}; \\
t_{i'j'k'} &: \quad -\frac{\beta_{1i}}{Q_j^2}\alpha_{ijk}\left(1+2\gamma t_{ijk}\right)\frac{\partial Q_j}{\partial t_{i'jk'}}1\left(j=j'\right)+2\frac{\beta_{1i}}{Q_j}\alpha_{ijk}\gamma 1\left(ijk=i'j'k'\right).
\end{aligned}$$

In order to evaluate partial derivatives with respect to policy variables, we must first decide what the policy variables are. Consider the following six policies:

1. Provide a subsidy of qF to each parent that must be used for formal care (formal care stamps). This changes This changes the Q function to

$$\begin{aligned}
Q_m &= \alpha_{0mf}\left(t_{0mf}+\gamma t_{0mf}^2\right)+\sum_{i=1}^{M}\sum_{k=c}^{s}\alpha_{imk}\left(t_{imk}+\gamma t_{imk}^2\right) \\
&\quad +\mu\left[F+\sum_{i=0}^{M}H_i\right]+Z_m; \\
Q_f &= \alpha_{0fm}\left(t_{0fm}+\gamma t_{0fm}^2\right)+\sum_{i=1}^{M}\sum_{k=c}^{s}\alpha_{ifk}\left(t_{ifk}+\gamma t_{ifk}^2\right) \\
&\quad +\mu\left[F+\sum_{i=0}^{M}H_i\right]+Z_f.
\end{aligned}$$

It changes first order conditions and therefore derivatives in the following way: The condition for ε_{xi} is

$$\varepsilon_{xi}-\frac{\beta_{1i}\mu p_{Xi}X_i\overline{Q}}{\beta_{2i}q}=0$$

with derivative

$$\frac{\beta_{1i}p_{Xi}X_i}{\beta_{2i}}\left(\frac{\mu}{q}\right)^2\sum_{j=m}^{f}\frac{\delta_j}{Q_j^2}$$

(because only $\overline{Q}$ changes). The condition for ε_{Lik} if $H_i>0$ is

$$\varepsilon_{Lik}-\frac{\beta_{1i}s_i^* w_{ik}L_{ik}\mu\overline{Q}}{\beta_{3ik}q}=0$$

with derivative

$$\frac{\beta_{1i}s_i^* w_{ik}L_{ik}\mu}{\beta_{3ik}q}\sum_{j=m}^{f}\frac{\delta_j}{Q_j^2}\frac{\mu}{q}.$$

The condition for ε_{Lik} if $H_i = 0$:

$$\varepsilon_{Lik} - \frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik} L_{ik}}{\beta_{3ik} p_{Xi} X_i} = 0$$

with derivative zero. The condition for ε_{tijk} if $H_i > 0$ or $i = 0$ is

$$\varepsilon_{tijk} - \beta_{1i}\left[\frac{\mu s_i^* w_{ik}\overline{Q}}{q} 1(i > 0) - \frac{\alpha_{ijk}}{Q_j}(1 + 2\gamma t_{ijk})\right] - \beta_{4ijk} = 0$$

with derivative

$$\frac{\beta_{1i}\mu}{q}\left[\frac{\mu s_i^* w_{ik}}{q} 1(i > 0) \sum_{j=m}^{f} \frac{\delta_j}{Q_j^2} - \frac{\alpha_{ijk}}{Q_j^2}(1 + 2\gamma t_{ijk})\right].$$

The condition for ε_{tijk} if $H_i = 0$ and $i > 0$:

$$\varepsilon_{tijk} - \frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik}}{p_{Xi} X_i} + \frac{\beta_{1i}}{Q_j}\alpha_{ijk}(1 + 2\gamma t_{ijk}) + \beta_{4ijk} = 0$$

with derivative

$$-\frac{\beta_{1i}}{Q_j^2}\alpha_{ijk}(1 + 2\gamma t_{ijk})\frac{\mu}{q}$$

(because Q_j increases by μ/q). Note that, in equation (5), $\partial Q_j/\partial P = \mu$.

2. Provide a subsidy of F to each child for each unit of time they provide informal care. This changes the child's budget constraint to

$$\max[Y_i^*, Y_i^{**}] \geq p_{Xi}X_i + qH_i - F\sum_{j=m,f}\sum_{k=c}^{s} t_{ijk}.$$

It changes the first order conditions and therefore derivatives in the following way: The condition for ε_{xi} if $t_{ijk} > 0$ and $i > 0$ is

$$\varepsilon_{xi} - \frac{\beta_{1i}\mu p_{Xi} X_i \overline{Q}}{\beta_{2i} q} = 0$$

with derivative zero. The condition for ε_{xi} if $t_{ijk} = 0$ or $i = 0$ is

$$\varepsilon_{xi} - \frac{\beta_{1i}\mu p_{Xi} X_i \overline{Q}}{\beta_{2i} q} = 0$$

with derivative zero. The condition for ε_{Lik} if $H_i > 0$ is

$$\varepsilon_{Lik} - \frac{\beta_{1i}s_i^* w_{ik} L_{ik}\mu\overline{Q}}{\beta_{3ik} q} = 0$$

with derivative zero. The condition for ε_{Lik} if $H_i = 0$ is

$$\varepsilon_{Lik} - \frac{\beta_{2i}\varepsilon_{xi}s_i^* w_{ik} L_{ik}}{\beta_{3ik} p_{Xi} X_i} = 0$$

with derivative zero. The condition for ε_{tijk} if $H_i > 0$ or $i = 0$ is

$$\varepsilon_{tijk} - \frac{\beta_{1i}}{Q_j}\left[\alpha_{ijk}\left(1 + 2\gamma t_{ijk}\right) - \frac{\mu s_i^* \left(w_{ik} - F\right)\overline{Q}}{q} 1\left(i > 0\right)\right] - \beta_{4ijk} = 0$$

with derivative

$$-\frac{\mu}{q}\frac{\beta_{1i}}{Q_j}s_i^*\overline{Q}1\left(i > 0\right).$$

The condition for ε_{tijk} if $H_i = 0$ and $i > 0$ is

$$\varepsilon_{tijk} - \frac{\beta_{1i}}{Q_j}\left[\alpha_{ijk}\left(1 + 2\gamma t_{ijk}\right)\right] - \frac{\beta_{2i}\varepsilon_{xi}}{X_i}\frac{\left(w_{ik} - F\right)}{p_{Xi}} - \beta_{4ijk} = 0$$

with derivative

$$\frac{\beta_{2i}\varepsilon_{xi}}{p_{Xi}X_i}.$$

3. Provide a subsidy of F for each dollar spent on formal care. This changes the child's budget constraint to

$$\max\left[Y_i^*, Y_i^{**}\right] \geq p_{Xi}X_i + \left(q - F\right)H_i.$$

It changes the first order conditions and therefore derivatives in the following way: The condition for ε_{xi} if $t_{ijk} > 0$ and $i > 0$ is

$$\varepsilon_{xi} - \frac{\beta_{1i}\mu p_{Xi}X_i\overline{Q}}{\beta_{2i}\left(q - F\right)} = 0$$

with derivative

$$-\frac{\beta_{1i}\mu p_{Xi}X_i\overline{Q}}{\beta_{2i}\left(q - F\right)^2}.$$

The condition for ε_{xi} if $t_{ijk} = 0$ or $i = 0$ is

$$\varepsilon_{xi} - \frac{\beta_{1i}\mu p_{Xi}X_i\overline{Q}}{\beta_{2i}\left(q - F\right)} = 0$$

with derivative

$$-\frac{\beta_{1i}\mu p_{Xi}X_i\overline{Q}}{\beta_{2i}\left(q - F\right)^2}.$$

The condition for ε_{Lik} if $H_i > 0$ is

$$\varepsilon_{Lik} - \frac{\beta_{1i} s_i^* w_{ik} L_{ik} \mu \overline{Q}}{\beta_{3ik} (q - F)} = 0$$

with derivative

$$-\frac{\beta_{1i} s_i^* w_{ik} L_{ik} \mu \overline{Q}}{\beta_{3ik} (q - F)^2}.$$

The condition for ε_{Lik} if $H_i = 0$ is

$$\varepsilon_{Lik} - \frac{\beta_{2i} \varepsilon_{xi} s_i^* w_{ik} L_{ik}}{\beta_{3ik} p_{Xi} X_i} = 0$$

with derivative zero. The condition for ε_{tijk} if $H_i > 0$ or $i = 0$ is

$$\varepsilon_{tijk} - \frac{\beta_{1i}}{Q_j} \left[\alpha_{ijk} (1 + 2\gamma t_{ijk}) - \frac{\mu s_i^* w_{ik} \overline{Q}}{(q - F)} 1 (i > 0) \right] - \beta_{4ijk} = 0$$

with derivative

$$-\frac{\beta_{1i}}{Q_j} \frac{\mu s_i^* w_{ik} \overline{Q}}{(q - F)^2} 1 (i > 0).$$

The condition for ε_{tijk} if $H_i = 0$ and $i > 0$ is

$$\varepsilon_{tijk} - \frac{\beta_{1i}}{Q_j} [\alpha_{ijk} (1 + 2\gamma t_{ijk})] - \frac{\beta_{2i} \varepsilon_{xi}}{X_i} \frac{w_{ik}}{p_{Xi}} - \beta_{4ijk} = 0$$

with derivative zero.

4. Provide a lump sum of F to the parent. This changes the first order conditions and therefore derivatives in the following way: The condition for ε_{xi} if $t_{ijk} > 0$ and $i > 0$ is

$$\varepsilon_{xi} - \frac{\beta_{1i} \mu p_{Xi} X_i \overline{Q}}{\beta_{2i} q} = 0$$

with derivative zero. The condition for ε_{xi} if $t_{ijk} = 0$ or $i = 0$ is

$$\varepsilon_{xi} - \frac{\beta_{1i} \mu [p_{Xi} X_i + F 1 (i = 0)] \overline{Q}}{\beta_{2i} q} = 0$$

with derivative

$$-\frac{\beta_{1i} \mu \overline{Q}}{\beta_{2i} q} 1 (i = 0).$$

The condition for ε_{Lik} if $H_i > 0$ is

$$\varepsilon_{Lik} - \frac{\beta_{1i} s_i^* w_{ik} L_{ik} \mu \overline{Q}}{\beta_{3ik} q} = 0$$

with derivative zero. The condition for ε_{Lik} if $H_i = 0$ is

$$\varepsilon_{Lik} - \frac{\beta_{2i} \varepsilon_{xi} s_i^* w_{ik} L_{ik}}{\beta_{3ik} p_{Xi} X_i} = 0$$

with derivative zero. The condition for ε_{tijk} if $H_i > 0$ or $i = 0$ is

$$\varepsilon_{tijk} - \frac{\beta_{1i}}{Q_j} \left[\alpha_{ijk} \left(1 + 2\gamma t_{ijk}\right) - \frac{\mu s_i^* w_{ik} \overline{Q}}{(q - F)} 1\left(i > 0\right) \right] - \beta_{4ijk} = 0$$

with derivative zero. The condition for ε_{tijk} if $H_i = 0$ and $i > 0$ is

$$\varepsilon_{tijk} - \frac{\beta_{1i}}{Q_j} \left[\alpha_{ijk} \left(1 + 2\gamma t_{ijk}\right) \right] - \frac{\beta_{2i} \varepsilon_{xi}}{X_i} \frac{w_{ik}}{p_{Xi}} - \beta_{4ijk} = 0$$

with derivative zero.

5. Increase Ψ, the income limit for Medicaid. This is the same as experiment 1 times the density of income at Ψ.

6. Provide a subsidy of F to each parent for each ADL the parent has that must be used for formal care (formal care stamps). This has the same effect as experiment 1 multiplied by the number of ADLs.