Appendices and Spplemental Tables for Formal Home Health Care, Informal Care, and Family Decision Making

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Supplemental Table 1: Care Provision Tobit Coefficient Estimates						
Explanatory Variable	Estimate		Std. Err	Estimate		Std. Err
Constant	-1057.695	**	136.805	-1045.626	**	134.346
Parent Characteristics						
Age	8.436	**	1.857	8.339	**	1.821
Father	-88.501	**	26.226	-87.545	**	25.714
White	10.461		28.472	10.054		28.328
Married	-153.951	**	24.798	-159.370	**	24.620
Education	-12.478	**	3.003	-12.125	**	2.969
ADL Walk	119.355	**	23.727			
ADL Dress	76.543	**	27.456			
ADL Bathe	111.204	**	27.345			
ADL Eat	50.410	*	29.249			
ADL Bed	-23.699		28.871			
ADL Toilet	38.775		33.707			
Number of ADL Problems				65.503	**	5.562

Supplemental Table 1 (continued)						
Explanatory Variable	Estimate		Std. Err	Estimate		Std. Err
Child Characteristics						
Age	0.197		1.362	0.550		1.348
Male	4.731		18.857	7.456		18.661
Married	33.303		22.341	28.146		22.045
Education	-0.932		4.189	-1.093		4.160
Number of Children	-2.911		6.228	-2.884		6.166
Oldest Child	13.400		20.020	14.217		19.777
Wage	0.090		0.073	-0.095		0.072
Log Likelihood	-2221.6			-2231.0		
Number of Observations	7562			7562		

Notes:

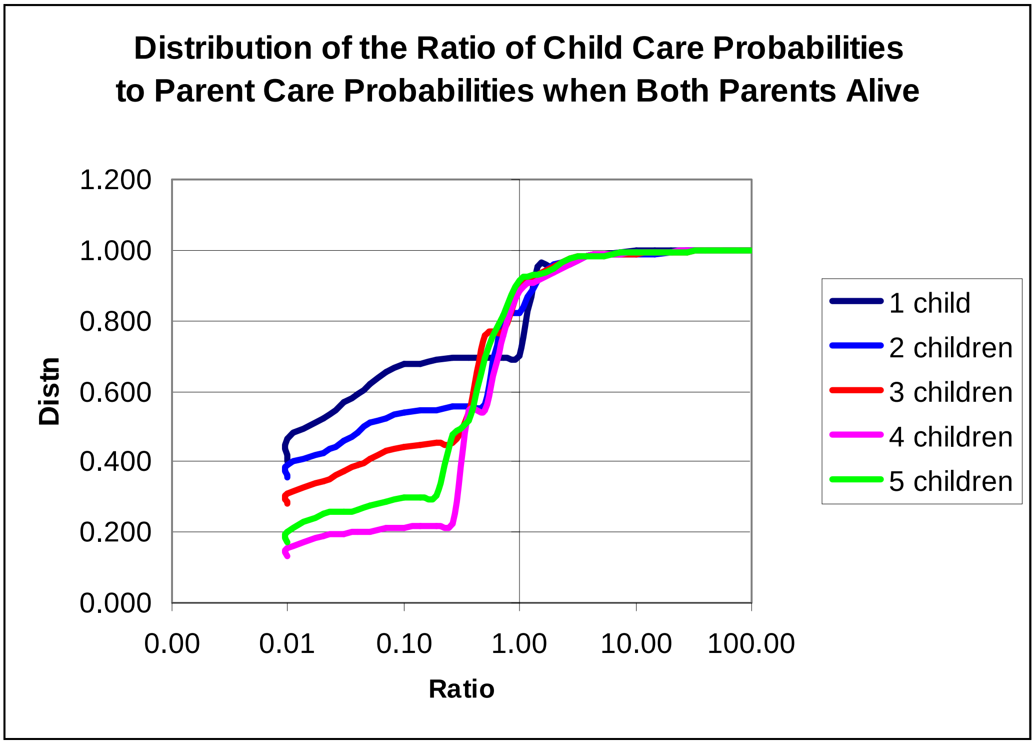
1. Dependent variable is the combined time the child and child's spouse (if applicable) spent helping the parent.

2. Single starred items are significant at the 10% level, and double starred items are significant at the 5% level.

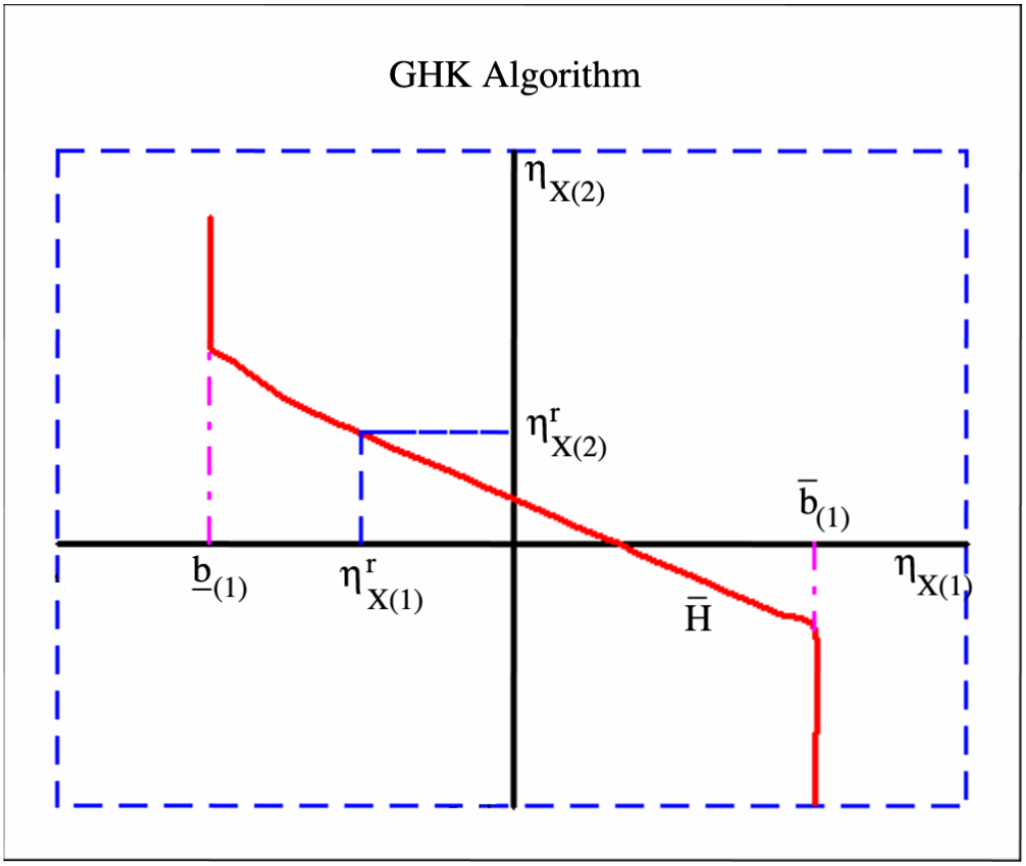
Supplemental Table 2: Happiness Coefficient Estimates			
Variable	Estimate		Std. Err
Constant	0.545	*	0.327
Father	0.109	**	0.052
Age	0.004		0.004
Education	0.019	**	0.007
White	0.144	**	0.072
Number of ADL Problems	-0.164	**	0.0187
Number of Children	0.014		0.017
Married	0.095	*	0.052
Log Likelihood = -1843.7			
Number of Observations =5242			

Notes:

1. Single starred items are significant at the 10% level, and double starred items are significant at the 5% level.



Supplemental Figure 1



Supplemental Figure 2

1 Appendix S1: Components of the Likelihood Function

Equation (15) in the main paper is a function of the terms $\Pr\left[u_0 \mid \widetilde{H_0}, t_0\right]$, $\Pr\left[t_{j0k}\right]$, $\Pr\left[t_i, L_i \mid \widetilde{H_i} = 0, \varepsilon_{Xi}\right]$, and $\Pr\left[t_i, L_i \mid \widetilde{H_i} = 1\right]$. These are defined below:

$$
\begin{aligned}
\Pr\left[\widetilde{H_0} = 0\right] &= \Phi\left[\frac{-\ln T_0^H}{\sigma_{\eta X}}\right], \qquad (1)\\
\Pr\left[u_0 \mid \widetilde{H_0}, t_0\right] &= \int \cdots \int \Pr\left[u_0 \mid \eta_{X0}, \eta_{t0}\right] f\left[\eta_{X0}, \eta_{t0} \mid \widetilde{H_0}, t_0\right] d\eta_{X0} d\eta_{t0},\\
\Pr\left[u_0 \mid \eta_{X0}, \eta_{t0}\right] &= \begin{cases} \Phi\left[\widetilde{U}_0\left(\varepsilon_{X0}, \varepsilon_{t0}\right)\right] & \text{if } u_0 = 1 \\ 1 - \Phi\left[\widetilde{U}_0\left(\varepsilon_{X0}, \varepsilon_{t0}\right)\right] & \text{if } u_0 = 0 \end{cases},\\
\widetilde{U}_0\left(\varepsilon_{X0}, \varepsilon_{t0}\right) &= \beta_0 + \beta_{10} \sum_{j \in m,p} \ln Q_j + \beta_{20} \varepsilon_{X0} \ln X_0\\
&\quad + \sum_{k \in m,p} \beta_{30k} \ln L_{0k} + \sum_{\substack{j,k \in m,p \\ j \neq k}} \left(\beta_{4j0k+} \varepsilon_{tj0k}\right) t_{j0k},
\end{aligned}
$$

$$
\Pr\left[t_{j0k}\right] = \begin{cases} \Phi\left[\frac{T_{0jk}^{t3}(0, \varepsilon_{L0k})}{\sigma_{\eta t}}\right] & \text{if } t_{j0k} = 0 \\ \frac{1}{\sigma_{\eta t}} \phi\left[\frac{T_{0jk}^{t3}\left(t_{j0k}, \varepsilon_{L0k}\right)}{\sigma_{\eta t}}\right] & \text{if } t_{j0k} > 0 \end{cases},
$$

and, for each child i, if $a_{ik} = 1$, $a_{il} = 0$ for $k, l = c, s; k \neq l$,

$$
\Pr\left[t_i, L_i \mid \cdot\right] = \begin{cases} \left|J_n\left(\eta_{Xi}\right)\right| \prod_{j=m,p} P_{ijc}^{t2}(\cdot)^{a_{0j}} \frac{1}{\sigma_{\eta L}} \phi\left(\frac{\ln T_{ic}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}}\right) & \\ & \text{if } W_{ic} = 1 \\ \int_{\ln T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty} \left|J_n\left(\eta_{Xi}, \eta_{Lc}\right)\right| \prod_{j=m,p} P_{ijc}^{t3}(\cdot)^{a_{0j}} \frac{1}{\sigma_{\eta L}} \phi\left(\frac{\eta_{Lc}}{\sigma_{\eta L}}\right) d\eta_{Lc} & \\ & \text{if } W_{ic} = 0 \end{cases}
$$

and, if $a_{ic} = a_{is} = 1$,

$$\Pr\left[t_i, L_i \mid \cdot\right] = \begin{cases} \left|J_n\left(\eta_{Xi}\right)\right| \prod_{j=m,p} R_{ij}^{22t}\left(\cdot\right)^{a_{0j}} B_{12}\left(\frac{\ln T_{ijc}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}}, \frac{\ln T_{ijs}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}}, \rho_L\right) \\ \qquad \text{if } H_i = 0,\, W_{ic} = W_{is} = 1 \\ \int_{\ln T_{ijs}^{L2}(\varepsilon_{Xi})}^{\infty} \left|J_n\left(\eta_{Xi}, \eta_{Ls}\right)\right| \prod_{j=m,p} R_{ij}^{23t}\left(\cdot\right)^{a_{0j}} \cdot \\ \qquad B_{12}\left(\frac{\ln T_{ijc}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}}, \frac{\eta_{Ls}}{\sigma_{\eta L}}, \rho_L\right) d\eta_{Ls} \\ \qquad \text{if } H_i = 0,\, W_{ic} = 1,\, W_{is} = 0 \\ \int_{\ln T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty} \left|J_n\left(\eta_{Xi}, \eta_{Lc}\right)\right| \prod_{j=m,p} R_{ij}^{32t}\left(\cdot\right)^{a_{0j}} \cdot \\ \qquad B_{12}\left(\frac{\eta_{Lc}}{\sigma_{\eta L}}, \frac{\ln T_{ijs}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}}, \rho_L\right) d\eta_{Lc} \\ \qquad \text{if } H_i = 0,\, W_{ic} = 0,\, W_{is} = 1 \\ \left|J_n\left(\eta_{Xi}\right)\right| \prod_{j=m,p} R_{ij}^{22t}\left(\cdot\right)^{a_{0j}} B_{12}\left(\frac{\ln T_{ijc}^{L1}}{\sigma_{\eta L}}, \frac{\ln T_{ijs}^{L1}}{\sigma_{\eta L}}, \rho_L\right) \\ \qquad \text{if } H_i = 1,\, W_{ic} = W_{is} = 1 \\ \int_{\ln T_{ijs}^{L2}(\varepsilon_{Xi})}^{\infty} \left|J_n\left(\eta_{Xi}, \eta_{Ls}\right)\right| \prod_{j=m,p} R_{ij}^{23t}\left(\cdot\right)^{a_{0j}} \cdot \\ \qquad B_{12}\left(\frac{\ln T_{ijc}^{L1}}{\sigma_{\eta L}}, \frac{\eta_{Ls}}{\sigma_{\eta L}}, \rho_L\right) d\eta_{Ls} \\ \qquad \text{if } H_i = 1,\, W_{ic} = 1,\, W_{is} = 0 \\ \int_{\ln T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty} \left|J_n\left(\eta_{Xi}, \eta_{Lc}\right)\right| \prod_{j=m,p} R_{ij}^{32t}\left(\cdot\right)^{a_{0j}} \cdot \\ \qquad B_{12}\left(\frac{\eta_{Lc}}{\sigma_{\eta L}}, \frac{\ln T_{ijs}^{L1}}{\sigma_{\eta L}}, \rho_L\right) d\eta_{Lc} \\ \qquad \text{if } H_i = 1,\, W_{ic} = 0,\, W_{is} = 1 \\ \int_{\log T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty} \int_{\log T_{ijs}^{L2}(\varepsilon_{Xi})}^{\infty} \left|J_n\left(\eta_{Xi}, \eta_{Lc}, \eta_{Ls}\right)\right| \prod_{j=m,p} R_{ij}^{33t}\left(\cdot\right)^{a_{0j}} \cdot \\ \qquad B_{12}\left(\frac{\eta_{Lc}}{\sigma_{\eta L}}, \frac{\eta_{Lc}}{\sigma_{\eta L}}, \rho_L\right) d\eta_{Ls} d\eta_{Lc} \\ \qquad \text{if } W_{ic} = W_{is} = 0 \end{cases} \tag{2}$$

$$P_{ijc}^{t2}(\cdot) = \begin{cases} \Phi\left(\frac{T_{ijc}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}=0,\ H_i=0 \\ \frac{1}{\sigma_{\eta t}}\phi\left(\frac{T_{ijc}^{t2}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}>0,\ H_i=0 \\ \Phi\left(\frac{T_{ijc}^{t1}(0)}{\sigma_{\eta t}}\right) & \text{if } t_{jic}=0,\ H_i=1 \\ \frac{1}{\sigma_{\eta t}}\phi\left(\frac{T_{ijc}^{t1}(t_{jic})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}>0,\ H_i=1 \end{cases}$$

$$P_{ijc}^{t3}(\cdot) = \begin{cases} \Phi\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}=0 \\ \frac{1}{\sigma_{\eta t}}\phi\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}>0 \end{cases}$$

where

$$R_{ij}^{22t}(\cdot) = \begin{cases} B\left(\frac{T_{ijc}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=t_{jis}=0,\ H_i=0 \\ B_1\left(\frac{T_{ijc}^{t2}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\ t_{jis}=0,\ H_i=0 \\ B_2\left(\frac{T_{ijc}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=0,\ t_{jis}>0,\ H_i=0 \\ B_{12}\left(\frac{T_{ijc}^{t2}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\ t_{jis}>0,\ H_i=0 \\ B\left(\frac{T_{ijc}^{t1}(0)}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=t_{jis}=0,\ H_i=1 \\ B_1\left(\frac{T_{ijc}^{t1}(t_{jic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\ t_{jis}=0,\ H_i=1 \\ B_2\left(\frac{T_{ijc}^{t1}(0)}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=0,\ t_{jis}>0,\ H_i=1 \\ B_{12}\left(\frac{T_{ijc}^{t1}(t_{jic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\ t_{jis}>0,\ H_i=1 \end{cases}$$

$$R_{ij}^{23t}(\cdot) = \begin{cases} B\left(\frac{T_{ijc}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0,\ H_i = 0 \\ B_1\left(\frac{T_{ijc}^{t2}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0,\ H_i = 0 \\ B_2\left(\frac{T_{ijc}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0,\ H_i = 0 \\ B_{12}\left(\frac{T_{ijc}^{t2}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0,\ H_i = 0 \\ B\left(\frac{T_{ijc}^{t1}(0)}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0,\ H_i = 1 \\ B_1\left(\frac{T_{ijc}^{t1}(t_{jic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0,\ H_i = 1 \\ B_2\left(\frac{T_{ijc}^{t1}(0)}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0,\ H_i = 1 \\ B_{12}\left(\frac{T_{ijc}^{t1}(t_{jic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0,\ H_i = 1 \end{cases}$$

$$R_{ij}^{32t}(\cdot) = \begin{cases} B\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0,\ H_i = 0 \\ B_1\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0,\ H_i = 0 \\ B_2\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0,\ H_i = 0 \\ B_{12}\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0,\ H_i = 0 \\ B\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0,\ H_i = 1 \\ B_1\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0,\ H_i = 1 \\ B_2\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0,\ H_i = 1 \\ B_{12}\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0,\ H_i = 1 \end{cases}$$

$$R_{ij}^{33t}(\cdot) = \begin{cases} B\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0 \\ B_1\left(\frac{T_{ijc}^{t3}(t_{mic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0 \\ B_2\left(\frac{T_{ijc}^{tc}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0 \\ B_{12}\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0 \end{cases} .$$

$B\left(\frac{x_1}{\sigma}, \frac{x}{\sigma}, \rho\right)$ is the standard bivariate normal distribution function with mean 0 and covariance matrix

$$\sigma^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

$$\begin{aligned} B_j\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right) &= \frac{\partial}{\partial x_j} B\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right) \\ B_{jk}\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right) &= \frac{\partial^2}{\partial x_j \partial x_k} B\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right), \end{aligned}$$

and J_n is the Jacobian corresponding to those errors with interior solutions.

2 Appendix S2: Simulation

In order to evaluate the likelihood contributions in equation (15) in the main paper, we must be able to simulate the last term,

$$\iiint\limits_{\substack{\eta_{Xi} \leq \ln T_i^H \\ i:\widetilde{H}_i=1}} 1\left(\sum_{i:\widetilde{H}_i=1} H_i\left(\eta_{Xi}\right) = \overline{H}\right) \prod_{i:\widetilde{H}_i=1} \Pr\left[t_i, L_i \mid \widetilde{H}_i = 1\right]^{1(i>0)} \bullet \frac{1}{\sigma_{\eta X}} \phi\left[\frac{\eta_{Xi}}{\sigma_{\eta X}}\right] d\eta_{Xi}. \quad (3)$$

Such a term is the probability of a vector of η_{Xi}'s (for those with $\widetilde{H}_i = 1$) conditional on each η_{Xi} being small enough to cause $\widetilde{H}_i = 1$ and also $\sum_{i:\widetilde{H}_i=1} H_i\left(\eta_{Xi}\right) = \overline{H}$. Consider the following GHK-type simulation algorithm:

1) Order $\left\{i : \widetilde{H}_i = 1\right\}$ according some criterion. Let (i) be the ith element of the ordered set. Let $I^* = \#\left\{i : \widetilde{H}_i = 1\right\}$.

2) Initialize $S_{(1)} = 0$ and $P^r = 1$.

3) For each $(i) < I^*$,

   a) Let

$$\overline{b}_{(i)} = H_{(i)}^{-1}\left(\max\left\{0, \Psi_{(i)}\right\}\right)$$

be an upper bound where $H_{(i)}^{-1}(\bullet)$ is the inverse of $H_{(i)}\left(\eta_{X(i)}\right)$ implied by equation (16):

$$H_{(i)}^{-1}(x) = \ln\left\{\frac{\beta_{1(i)}\mu}{\beta_{2(i)}q}\overline{Q}\left[\max\left(Y_{(i)}^{*}, Y_{(i)}^{**}\right) - qx\right]\right\}$$

where

$$\begin{aligned} Y_{(i)}^{*} &= \begin{cases} \sum_{k\in c,s} a_{(i)k}w_{(i)k}\left(1 - L_{(i)k} - \sum_{j\in m,p} t_{j(i)k}\right) & \text{if } i > 0 \\ Y_{(i)} & \text{if } i = 0 \end{cases}, \\ Y_{(i)}^{**} &= \begin{cases} \max\left(Y_{(i)}^{*}, Y_{(i)} + sY_{(i)}^{*}\right) & \text{if } i > 0 \\ Y_{(i)} & \text{if } i = 0 \end{cases}, \end{aligned}$$

and

$$\Psi_{(i)} = \overline{H} - S_{(i)} - \sum_{(l)>(i)} \max\left(Y_{(l)}^{*}, Y_{(l)}^{**}\right)$$

is the least player (i) can contribute; i.e., it is how much she would have to contribute if all remaining players used all resources for $\overline{H} - S_{(i)}$. Note that, if $0 \geq \Psi_{(i)}$,

$$\overline{b}_{(i)} = \ln\left(\frac{\beta_{1(i)}\mu p_{Xi}X_{(i)}\overline{Q}}{\beta_{2(i)}q}\right).$$

Also, let

$$\underline{b}_{(i)} = H_{(i)}^{-1}\left(\overline{H} - S_{(i)}\right)$$

be a lower bound.

b) Update

$$P^{r} = P^{r}\left[\Phi\left(\frac{\overline{b}_{(i)}}{\sigma_{\eta X}}\right) - \Phi\left(\frac{\underline{b}_{(i)}}{\sigma_{\eta X}}\right)\right]\cdot\prod_{i:\widetilde{H}_i=1}\Pr\left[t_i, L_i \mid \widetilde{H}_i = 1\right]^{1(i>0)}.$$

c) Simulate $\eta_{X(i)}$ conditional on $\underline{b}_{(i)} \leq \eta_{X(i)} \leq \overline{b}_{(i)}$ as

$$\eta_{X(i)}^{r} = \sigma_{\eta X}\Phi^{-1}\left\{\left[\Phi\left(\frac{\overline{b}_{(i)}}{\sigma_{\eta X}}\right) - \Phi\left(\frac{\underline{b}_{(i)}}{\sigma_{\eta X}}\right)\right]u^{r} + \Phi\left(\frac{\underline{b}_{(i)}}{\sigma_{\eta X}}\right)\right\}$$

where $u^r \sim U(0,1)$.

d) Compute $H_{(i)}\left(\eta^r_{X(i)}\right)$ using $\eta^r_{X(i)}$ and equation (16).

e) Compute $S_{(i+1)} = S_{(i)} + H_{(i)}\left(\eta^r_{X(i)}\right)$.

4) For $(i) = I^*$,

a) Let

$$\eta^r_{X(I^*)} = H^{-1}_{(I^*)}\left(\overline{H} - S_{(I^*)}\right).$$

b) Update

$$P^r = \frac{P^r}{\sigma_{\eta X}}\phi\left[\frac{\eta_{X(I^*)}}{\sigma_{\eta X}}\right] \cdot \prod_{i:\widetilde{H}_i=1} \Pr\left[t_i, L_i \mid \widetilde{H}_i = 1\right]^{1(i>0)}.$$

5) P^r is our simulator of equation (22). Note that, if there is only one player who satisfies the conditions in the integral in equation (22), then the equation becomes

$$\frac{1}{\sigma^2_{\eta X}}\phi\left[\frac{H^{-1}_{(i)}\left(\overline{H}\right)}{\sigma_{\eta X}}\right] \cdot \prod_{i:\widetilde{H}_i=1} \Pr\left[t_i, L_i \mid \widetilde{H}_i = 1\right]^{1(i>0)}$$

which can be evaluated analytically.

Consider how to interpret the GHK algorithm as an importance sampling simulator. Rewrite equation (22) as

$$\begin{aligned} & \iiint\limits_{\substack{\eta_{X(i)} \leq \ln T_i^H \\ \sum_{(i)=1}^{I^*} H_{(i)}\left(\eta_{X(i)}\right) = \overline{H}}} f(\eta_X)\, d\eta_X \\ = & \iiint\limits_{\substack{\eta_{X(i)} \leq \ln T_i^H \\ \sum_{(i)=1}^{I^*} H_{(i)}\left(\eta_{X(i)}\right) = \overline{H}}} \frac{f(\eta_X)}{g(\eta_X)} g(\eta_X)\, d\eta_X \end{aligned}$$

where $\eta_X = \left(\eta_{X(1)}, \eta_{X(2)}, .., \eta_{X(I)}\right)$,

$$
\begin{aligned}
f(\eta_X) &= \prod_{(i)=1}^{I^*} \frac{1}{\sigma^2_{\eta X}} \phi\left[\frac{\eta_{X(i)}}{\sigma_{\eta X}}\right] \prod_{(i)=1}^{I^*-1} 1\left(\underline{b}_{(i)} \leq \eta_{X(i)} \leq \bar{b}_{(i)}\right) \cdot \prod_{i:\widetilde{H}_i=1} \Pr\left[t_i, L_i \mid \widetilde{H}_i = 1\right]^{1(i>0)} \\
g(\eta_X) &= \prod_{(i)=1}^{I^*-1} \frac{\frac{1}{\sigma^2_{\eta X}} \phi\left[\frac{\eta_{X(i)}}{\sigma_{\eta X}}\right]}{\Phi\left(\frac{\bar{b}_{(i)}}{\sigma_{\eta X}}\right) - \Phi\left(\frac{\underline{b}_{(i)}}{\sigma_{\eta X}}\right)} 1\left(\underline{b}_{(i)} \leq \eta_{X(i)} \leq \bar{b}_{(i)}\right),
\end{aligned}
$$

which implies that

$$
\frac{f(\eta_X)}{g(\eta_X)} = \frac{1}{\sigma^2_{\eta X}} \phi\left[\frac{\eta_{X(I)}}{\sigma_{\eta X}}\right] \prod_{(i)=1}^{I^*-1} \left[\Phi\left(\frac{\bar{b}_{(i)}}{\sigma_{\eta X}}\right) - \Phi\left(\frac{\underline{b}_{(i)}}{\sigma_{\eta X}}\right)\right] \cdot \prod_{i:\widetilde{H}_i=1} \Pr\left[t_i, L_i \mid \widetilde{H}_i = 1\right]^{1(i>0)}.
$$

Note that we are simulating $E\frac{f(\eta_X)}{g(\eta_X)}$ with errors η_X simulated from density $g(\eta_X)$. The fact that we can write our simulator as an importance sampling simulator means that it is unbiased.

We want to minimize the variance of our simulator, especially because we are using maximum simulated likelihood estimation rather than the method of simulated moments. First, we can improve on the variance of our simulator by using antithetic acceleration. Second, we can use a criterion for ordering $\left\{i : \widetilde{H}_i = 1\right\}$ in Step 1 of the algorithm that reduces the variance. Consider a case with $I^* = 2$. In Figure 3, the $\overline{H}$ curve represents those values of $\left(\eta_{X(1)}, \eta_{X(2)}\right)$ that result in total formal care expenditures of $\overline{H}$. Note that the curve asymptotes at $\underline{b}_{(1)}$ and $\bar{b}_{(1)}$. At $\underline{b}_{(1)}$, as $\eta_{X(2)}$ increases, $H_{(2)}$ approaches 0 (and reaches 0); thus $\eta_{X(1)}$ must converge to that value such that child (1) will provide $\overline{H}$. On the other hand, as $\eta_{X(2)}$ decreases, $H_{(2)}$ converges to the income of child (2); thus $\eta_{X(1)}$ must converge to that value such that child (1) will provide $\overline{H}$ minus the income of child (2). Our GHK algorithm first computes the probability that $\underline{b}_{(1)} \leq \eta_{X(1)} \leq \bar{b}_{(1)}$. Next the GHK algorithm simulates a value of $\eta_{X(1)}$ conditional on $\underline{b}_{(1)} \leq \eta_{X(1)} \leq \bar{b}_{(1)}$. Then it computes the probability that $\eta_{X(2)}$ is such that the simulated values of $\eta_{X(1)}$ and $\eta_{X(2)}$ are on the $\overline{H}$

curve. The simulator is the product of the two probabilities. The variance of the simulator is proportional to the variance of the second probability as a function of the simulated value of $\eta_{X(1)}$. Thus, we should arrange $\left\{i : \widetilde{H}_i = 1\right\}$ in descending order of the variance of the contribution to the simulator with respect to the elements of η_X that precede it.

Such an ordering rule is too expensive to evaluate and may depend upon realizations of early elements of η_X. Instead, we want an alternative rule that approximates the rule described above but that is easy to employ and does not depend upon realizations of early elements of η_X. A simple example of such a rule is to order $\left\{i : \widetilde{H}_i = 1\right\}$ in ascending order with respect to

$$\left| \frac{\phi\left[\frac{\bar{b}_i}{\sigma_{\eta X}}\right] - \phi\left[\frac{b_i^*}{\sigma_{\eta X}}\right]}{\Phi\left(\frac{\bar{b}_i}{\sigma_{\eta X}}\right) - \Phi\left(\frac{b_i^*}{\sigma_{\eta X}}\right)} \right|$$

where

$$b_i^* = H_i^{-1}\left(\overline{H}\right).$$

We also need to simulate terms like the second term in equation (15):

$$\prod_{i:\widetilde{H}_i=0} \left\{ \int_{\eta_{Xi} \geq \ln T_i^H} \Pr\left[t_i, L_i \mid \widetilde{H}_i = 0, \varepsilon_{Xi}\right]^{1(i>0)} \frac{1}{\sigma_{\eta X}} \phi\left[\frac{\eta_{Xi}}{\sigma_{\eta X}}\right] d\eta_{Xi} \right\}$$

But this requires just drawing $\eta_{Xi} \mid \eta_{Xi} \geq \ln T_i^H$ and then evaluating the integrand conditional on the draw of η_{Xi}.

Finally, we need to be able to simulate

$$\Pr\left[u_0 \mid \widetilde{H}_0, t_0\right] = \int \cdots \int \Pr\left[u_0 \mid \eta_{X0}, \eta_{t0}\right] f\left[\eta_{X0}, \eta_{t0} \mid \widetilde{H}_0, t_0\right] d\eta_{X0} d\eta_{t0}.$$

This is a straightforward application of GHK.

3 Appendix S3: Correction for Measurement Error in Specification Tests

Let $y_i^* \sim iidF$, let

$$y_i = k1\left(c_k \leq y_i^* < c_{k+1}\right),$$

and let $g(\cdot,\cdot)$ be a continuous function of c_k and c_{k+1}, and let

$$\widehat{y_i^*} = \sum_k g\left(c_k, c_{k+1}\right) 1\left(y_i = k\right).$$

What are the moments of $z_i^* = \widehat{y_i^*} - y_i^*$?

$$\begin{aligned} Ez_i^* &= E\sum_k g\left(c_k, c_{k+1}\right) 1\left(y_i = k\right) - y_i^* \\ &= \sum_k g\left(c_k, c_{k+1}\right) \Pr\left(y_i = k\right) - Ey_i^* \\ &= \sum_k g\left(c_k, c_{k+1}\right) \left[F\left(c_{k+1}\right) - F\left(c_k\right)\right] - Ey_i^*, \end{aligned}$$

and

$$\begin{aligned} Var\left(z_i^*\right) &= Var\left[\sum_k g\left(c_k, c_{k+1}\right) 1\left(y_i = k\right) - y_i^*\right] \\ & \quad E\left[\sum_k g\left(c_k, c_{k+1}\right) 1\left(y_i = k\right) - y_i^* - Ez_i^*\right]^2 \\ &= \int \left[\sum_k g\left(c_k, c_{k+1}\right) 1\left(y_i = k\right) - y_i^* - Ez_i^*\right]^2 dF\left(y_i^*\right) \\ &= \sum_k \int_{c_k}^{c_{k+1}} \left[g\left(c_k, c_{k+1}\right) - y_i^* - Ez_i^*\right]^2 dF\left(y_i^*\right). \end{aligned}$$

If $y_i^* \sim iidU(0,1)$ and

$$g\left(c_k, c_{k+1}\right) = \frac{c_k + c_{k+1}}{2},$$

then

$$F\left(c_{k+1}\right)-F\left(c_k\right)=c_{k+1}-c_k,$$

$$\begin{aligned}
Ez_i^* &= \sum_k \frac{c_k+c_{k+1}}{2}\left(c_{k+1}-c_k\right)-\frac{1}{2} \\
&= \sum_k \frac{c_{k+1}^2-c_k^2}{2}-\frac{1}{2}=0,
\end{aligned}$$

and

$$\begin{aligned}
Var\left(z_i^*\right) &= \sum_k \int_{c_k}^{c_{k+1}} \left[\frac{c_k+c_{k+1}}{2}-y_i^*\right]^2 dy_i^* \\
&= \sum_k \int_{c_k}^{c_{k+1}} \left[\frac{c_k+c_{k+1}}{2}-y_i^*\right]^2 \frac{c_{k+1}-c_k}{c_{k+1}-c_k} dy_i^* \\
&= \sum_k \frac{\left(c_{k+1}-c_k\right)^3}{12}.
\end{aligned}$$

In the data, the values of c are $(0, 1/7, 1)$. Thus,

$$\begin{aligned}
Var\left(z_i^*\right) &= \sum_k \frac{\left(c_{k+1}-c_k\right)^3}{12} \\
&= \frac{\left(\frac{1}{7}\right)^3+\left(\frac{6}{7}\right)^3}{12} \\
&= .229^2.
\end{aligned}$$

We multiply $Var\left(z_i^*\right)$ by $(1.6/168)^2$ where 1.6 is the average value of π in equation (19) in the data. Thus, $\sigma_m = (1.6/168) \cdot .229 = .002$