FOC and Likelihood Contribution Presentation

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1 First Order Conditions

FOCs for Children						
Cases				FOCs		
L_{ik}	t_{jik}	Work	H_i	H_i	t_{jik}	L_{ik}
I	I	I	I	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t1}(t_{jik})$	$\varepsilon_{Lik} = T_{ik}^{L1}$
I	I	I	C	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t2}(t_{jik}, \varepsilon_{Xi})$	$\varepsilon_{Lik} = T_{ik}^{L2}(\varepsilon_{Xi})$
I	I	C	I	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t3}(t_{jik}, \varepsilon_{Lik})$	$\varepsilon_{Lik} \geq T_{ik}^{L1}$
I	I	C	C	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} = T_{ijk}^{t3}(t_{jik}, \varepsilon_{Lik})$	$\varepsilon_{Lik} \geq T_{ik}^{L2}(\varepsilon_{Xi})$
I	C	I	I	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t1}(0)$	$\varepsilon_{Lik} = T_{ik}^{L1}$
I	C	I	C	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t2}(0, \varepsilon_{Xi})$	$\varepsilon_{Lik} = T_{ik}^{L2}(\varepsilon_{Xi})$
C	C	C	I	$\varepsilon_{Xi} = T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t3}(0, \varepsilon_{Lik})$	$\varepsilon_{Lik} \geq T_{ik}^{L1}$
C	C	C	C	$\varepsilon_{Xi} \geq T_i^H$	$\varepsilon_{tjik} \leq T_{ijk}^{t3}(0, \varepsilon_{Lik})$	$\varepsilon_{Lik} \geq T_{ik}^{L2}(\varepsilon_{Xi})$

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\begin{aligned}
T_i^H &= \frac{\beta_{1i}\mu p_{Xi} X_i}{q\beta_{2i}} \overline{Q} \\
T_{ijk}^{t1}(t_{jik}) &= \beta_{1i}\left[\frac{s_i^* w_{ik}\mu}{q}\overline{Q} - \frac{\alpha_{jik}(1+2\gamma t_{jik})}{Q_j}\right] - \beta_{4jik} \\
T_{ijk}^{t2}(t_{jik}, \varepsilon_{Xi}) &= \beta_{2i}\varepsilon_{Xi}\frac{s_i^* w_{ik}}{p_{Xi}X_i} - \beta_{1i}\frac{\alpha_{jik}(1+2\gamma t_{jik})}{Q_j} - \beta_{4jik} \\
T_{ijk}^{t3}(t_{jik}, \varepsilon_{Lik}) &= \frac{\beta_{3ik}\varepsilon_{Lik}}{L_{ik}} - \frac{\beta_{1i}\alpha_{jik}(1+2\gamma t_{jik})}{Q_j} - \beta_{4jik} \\
T_{ik}^{L1} &= \frac{\beta_{1i}s_i^* w_{ik} L_{ik}\mu}{\beta_{3ik} q}\overline{Q} \\
T_{ik}^{L2}(\varepsilon_{Xi}) &= \frac{\beta_{2i}\varepsilon_{Xi}s_i^* w_{ik} L_{ik}}{\beta_{3ik} p_{Xi} X_i}
\end{aligned}
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FOCs for Parents			
Cases		FOCs	
t_{jik}	H_i	H_i	t_{jik}
I	I	$\varepsilon_{X0} = T_0^H$	$\varepsilon_{tj0k} = T_{0jk}^{t3}(t_{j0k}, \varepsilon_{L0k})$
I	C	$\varepsilon_{X0} \geq T_0^H$	$\varepsilon_{tj0k} = T_{0jk}^{t3}(t_{j0k}, \varepsilon_{L0k})$
C	I	$\varepsilon_{X0} = T_0^H$	$\varepsilon_{tj0k} \leq T_{0jk}^{t3}(0, \varepsilon_{L0k})$
C	C	$\varepsilon_{X0} \geq T_0^H$	$\varepsilon_{tj0k} \leq T_{0jk}^{t3}(0, \varepsilon_{L0k})$

$$\begin{aligned} T_0^H &= \frac{\beta_{10}\mu p_{X0} X_0 \overline{Q}}{\beta_{20} q} \\ T_{0jk}^{t3}(t_{j0k}, \varepsilon_{L0k}) &= -\frac{\beta_{10}}{Q_j}\alpha_{j0k}(1 + 2\gamma t_{j0k}) + \frac{\beta_{30k}\varepsilon_{L0k}}{L_{0k}} - \beta_{4j0k} \end{aligned}$$

2 Likelihood Contribution

$$\begin{aligned} \pounds_n = & \left\{ \Pr\left[u_0 \mid \widetilde{H_0}, t_0\right] \prod_{\substack{j \in m,p \\ k \neq j}} \Pr\left[t_{j0k}\right]^{a_{0k} a_{0j}} \right\} \bullet \\ & \prod_{i:\widetilde{H}_i = 0} \left\{ \int_{\eta_{Xi} \geq \zeta_i} \Pr\left[t_i, L_i \mid \widetilde{H_i} = 0, \varepsilon_{Xi}\right]^{1(i>0)} \frac{1}{\sigma_{\eta X}} \phi\left[\frac{\eta_{Xi}}{\sigma_{\eta X}}\right] d\eta_{Xi} \right\} \bullet \\ & \iiint_{\substack{\eta_{Xi} \leq \zeta_i \\ i:\widetilde{H}_i = 1}} 1\left(\sum_{i:\widetilde{H}_i = 1} H_i(\eta_{Xi}) = \overline{H} \right) \prod_{i:\widetilde{H}_i = 1} \Pr\left[t_i, L_i \mid \widetilde{H_i} = 1\right]^{1(i>0)} \bullet \\ & \frac{1}{\sigma_{\eta X}} \phi\left[\frac{\eta_{Xi}}{\sigma_{\eta X}}\right] d\eta_{Xi} \end{aligned}$$

$$\begin{aligned} \Pr\left[\widetilde{H_0} = 0\right] &= \Phi\left[\frac{-\log\left(T_0^H\right)}{\sigma_{\eta X}}\right], \qquad (1) \\ \Pr\left[u_0 \mid \widetilde{H_0}, t_0\right] &= \int \cdots \int \Pr\left[u_0 \mid \eta_{X0}, \eta_{t0}\right] f\left[\eta_{X0}, \eta_{t0} \mid \widetilde{H_0}, t_0\right] d\eta_{X0} d\eta_{t0}, \\ \Pr\left[u_0 \mid \eta_{X0}, \eta_{t0}\right] &= \begin{cases} \Phi\left[\widetilde{U}_0(\varepsilon_{X0}, \varepsilon_{t0})\right] & \text{if } u_0 = 1 \\ 1 - \Phi\left[\widetilde{U}_0(\varepsilon_{X0}, \varepsilon_{t0})\right] & \text{if } u_0 = 0 \end{cases}, \\ \widetilde{U}_0(\varepsilon_{X0}, \varepsilon_{t0}) &= \beta_0 + \beta_{10} \sum_{j \in m,p} \ln Q_j + \beta_{20} \varepsilon_{X0} \ln X_0 \\ & + \sum_{k \in m,p} \beta_{30k} \ln L_{0k} + \sum_{\substack{j,k \in m,p \\ j \neq k}} \left(\beta_{4j0k+}\varepsilon_{tj0k}\right) t_{j0k}, \end{aligned}$$

$$\Pr\left[t_{j0k}\right]=\begin{cases}\Phi\left[\frac{T_{0jk}^{t3}(0,\varepsilon_{L0k})}{\sigma_{\eta t}}\right] & \text{if } t_{j0k}=0\\ \frac{1}{\sigma_{\eta t}}\phi\left[\frac{T_{0jk}^{t3}(t_{j0k},\varepsilon_{L0k})}{\sigma_{\eta t}}\right] & \text{if } t_{j0k}>0\end{cases}$$

If $a_{is}=0$

$$\Pr\left[t_i,L_i\mid\cdot\right]=\begin{cases}\left|J_n\left(\eta_{Xi}\right)\right|\prod_{j=m,p}P_{ijc}^{t2}\left(\cdot\right)^{a_{0j}}\frac{1}{\sigma_{\eta L}}\phi\left(\frac{\log T_{ic}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}}\right)\\ \qquad\qquad\text{if } W_{ic}=1\\ \int_{\log T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty}\left|J_n\left(\eta_{Xi},\eta_{Lc}\right)\right|\prod_{j=m,p}P_{ijc}^{t3}\left(\cdot\right)^{a_{0j}}\frac{1}{\sigma_{\eta L}}\phi\left(\frac{\eta_{Lc}}{\sigma_{\eta L}}\right)d\eta_{Lc}\\ \qquad\qquad\text{if } W_{ic}=0\end{cases}$$

If $a_{is}=1$,

$$\Pr\left[t_i,L_i\mid\cdot\right]=\begin{cases}\left|J_n\left(\eta_{Xi}\right)\right|\prod_{j=m,p}R_{ij}^{22t}\left(\cdot\right)^{a_{0j}}B_{12}\left(\frac{\log T_{ijc}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}},\frac{\log T_{ijs}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}},\rho_L\right)\\ \qquad\qquad\text{if } H_i=0,\ W_{ic}=W_{is}=1\\ \int_{\log T_{ijs}^{L2}(\varepsilon_{Xi})}^{\infty}\left|J_n\left(\eta_{Xi},\eta_{Ls}\right)\right|\prod_{j=m,p}R_{ij}^{23t}\left(\cdot\right)^{a_{0j}}\cdot\\ \qquad B_{12}\left(\frac{\log T_{ijc}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}},\frac{\eta_{Ls}}{\sigma_{\eta L}},\rho_L\right)d\eta_{Ls}\\ \qquad\qquad\text{if } H_i=0,\ W_{ic}=1,\ W_{is}=0\\ \int_{\log T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty}\left|J_n\left(\eta_{Xi},\eta_{Lc}\right)\right|\prod_{j=m,p}R_{ij}^{32t}\left(\cdot\right)^{a_{0j}}\cdot\\ \qquad B_{12}\left(\frac{\eta_{Lc}}{\sigma_{\eta L}},\frac{\log T_{ijs}^{L2}(\varepsilon_{Xi})}{\sigma_{\eta L}},\rho_L\right)d\eta_{Lc}\\ \qquad\qquad\text{if } H_i=0,\ W_{ic}=0,\ W_{is}=1\\ \left|J_n\left(\eta_{Xi}\right)\right|\prod_{j=m,p}R_{ij}^{22t}\left(\cdot\right)^{a_{0j}}B_{12}\left(\frac{\log T_{ijc}^{L1}}{\sigma_{\eta L}},\frac{\log T_{ijs}^{L1}}{\sigma_{\eta L}},\rho_L\right)\\ \qquad\qquad\text{if } H_i=1,\ W_{ic}=W_{is}=1\\ \int_{\log T_{ijs}^{L2}(\varepsilon_{Xi})}^{\infty}\left|J_n\left(\eta_{Xi},\eta_{Ls}\right)\right|\prod_{j=m,p}R_{ij}^{23t}\left(\cdot\right)^{a_{0j}}\cdot\\ \qquad B_{12}\left(\frac{\log T_{ijc}^{L1}}{\sigma_{\eta L}},\frac{\eta_{Ls}}{\sigma_{\eta L}},\rho_L\right)d\eta_{Ls}\\ \qquad\qquad\text{if } H_i=1,\ W_{ic}=1,\ W_{is}=0\\ \int_{\log T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty}\left|J_n\left(\eta_{Xi},\eta_{Lc}\right)\right|\prod_{j=m,p}R_{ij}^{32t}\left(\cdot\right)^{a_{0j}}\cdot\\ \qquad B_{12}\left(\frac{\eta_{Lc}}{\sigma_{\eta L}},\frac{\log T_{ijs}^{L1}}{\sigma_{\eta L}},\rho_L\right)d\eta_{Lc}\\ \qquad\qquad\text{if } H_i=1,\ W_{ic}=0,\ W_{is}=1\\ \int_{\log T_{ijc}^{L2}(\varepsilon_{Xi})}^{\infty}\int_{\log T_{ijs}^{L2}(\varepsilon_{Xi})}^{\infty}\left|J_n\left(\eta_{Xi},\eta_{Lc},\eta_{Ls}\right)\right|\prod_{j=m,p}R_{ij}^{33t}\left(\cdot\right)^{a_{0j}}\cdot\\ \qquad B_{12}\left(\frac{\eta_{Lc}}{\sigma_{\eta L}},\frac{\eta_{Lc}}{\sigma_{\eta L}},\rho_L\right)d\eta_{Ls}d\eta_{Lc}\\ \qquad\qquad\text{if } W_{ic}=W_{is}=0\end{cases}$$

$$P^{t2}_{ijc}(\cdot) = \begin{cases} \Phi\left(\frac{T^{t2}_{ijc}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}=0,\, H_i=0 \\ \frac{1}{\sigma_{\eta t}}\phi\left(\frac{T^{t2}_{ijc}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}>0,\, H_i=0 \\ \Phi\left(\frac{T^{t1}_{ijc}(0)}{\sigma_{\eta t}}\right) & \text{if } t_{jic}=0,\, H_i=1 \\ \frac{1}{\sigma_{\eta t}}\phi\left(\frac{T^{t1}_{ijc}(t_{jic})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}>0,\, H_i=1 \end{cases}$$

$$P^{t3}_{ijc}(\cdot) = \begin{cases} \Phi\left(\frac{T^{t3}_{ijc}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}=0 \\ \frac{1}{\sigma_{\eta t}}\phi\left(\frac{T^{t3}_{ijc}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}\right) & \text{if } t_{jic}>0 \end{cases}$$

$$R^{22t}_{ij}(\cdot) = \begin{cases} B\left(\frac{T^{t2}_{ijc}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t2}_{ijs}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=t_{jis}=0,\, H_i=0 \\ B_1\left(\frac{T^{t2}_{ijc}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t2}_{ijs}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}=0,\, H_i=0 \\ B_2\left(\frac{T^{t2}_{ijc}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t2}_{ijs}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=0,\, t_{jis}>0,\, H_i=0 \\ B_{12}\left(\frac{T^{t2}_{ijc}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t2}_{ijs}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}>0,\, H_i=0 \\ B\left(\frac{T^{t1}_{ijc}(0)}{\sigma_{\eta t}}, \frac{T^{t1}_{ijs}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=t_{jis}=0,\, H_i=1 \\ B_1\left(\frac{T^{t1}_{ijc}(t_{jic})}{\sigma_{\eta t}}, \frac{T^{t1}_{ijs}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}=0,\, H_i=1 \\ B_2\left(\frac{T^{t1}_{ijc}(0)}{\sigma_{\eta t}}, \frac{T^{t1}_{ijs}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=0,\, t_{jis}>0,\, H_i=1 \\ B_{12}\left(\frac{T^{t1}_{ijc}(t_{jic})}{\sigma_{\eta t}}, \frac{T^{t1}_{ijs}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}>0,\, H_i=1 \end{cases}$$

$$R^{23t}_{ij}(\cdot) = \begin{cases} B\left(\frac{T^{t2}_{ijc}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=t_{jis}=0,\, H_i=0 \\ B_1\left(\frac{T^{t2}_{ijc}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}=0,\, H_i=0 \\ B_2\left(\frac{T^{t2}_{ijc}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=0,\, t_{jis}>0,\, H_i=0 \\ B_{12}\left(\frac{T^{t2}_{ijc}(t_{jic},\varepsilon_{Xi})}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}>0,\, H_i=0 \\ B\left(\frac{T^{t1}_{ijc}(0)}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=t_{jis}=0,\, H_i=1 \\ B_1\left(\frac{T^{t1}_{ijc}(t_{jic})}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}=0,\, H_i=1 \\ B_2\left(\frac{T^{t1}_{ijc}(0)}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}=0,\, t_{jis}>0,\, H_i=1 \\ B_{12}\left(\frac{T^{t1}_{ijc}(t_{jic})}{\sigma_{\eta t}}, \frac{T^{t3}_{ijs}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic}>0,\, t_{jis}>0,\, H_i=1 \end{cases}$$

$$R_{ij}^{32t}(\cdot) = \begin{cases} B\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0,\ H_i = 0 \\ B_1\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(0,\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0,\ H_i = 0 \\ B_2\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0,\ H_i = 0 \\ B_{12}\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t2}(t_{jis},\varepsilon_{Xi})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0,\ H_i = 0 \\ B\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0,\ H_i = 1 \\ B_1\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(0)}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0,\ H_i = 1 \\ B_2\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0,\ H_i = 1 \\ B_{12}\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t1}(t_{jis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0,\ H_i = 1 \end{cases}$$

$$R_{ij}^{33t}(\cdot) = \begin{cases} B\left(\frac{T_{ijc}^{t3}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = t_{jis} = 0 \\ B_1\left(\frac{T_{ijc}^{t3}(t_{mic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(0,\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} = 0 \\ B_2\left(\frac{T_{ijc}^{tc}(0,\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} = 0,\ t_{jis} > 0 \\ B_{12}\left(\frac{T_{ijc}^{t3}(t_{jic},\varepsilon_{Lic})}{\sigma_{\eta t}}, \frac{T_{ijs}^{t3}(t_{jis},\varepsilon_{Lis})}{\sigma_{\eta t}}, \rho_t\right) & \text{if } t_{jic} > 0,\ t_{jis} > 0 \end{cases},$$

$B\left(\frac{x_1}{\sigma}, \frac{x}{\sigma}, \rho\right)$ is the standard bivariate normal distribution function,

$$\begin{aligned} B_j\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right) &= \frac{\partial}{\partial x_j} B\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right) \\ B_{jk}\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right) &= \frac{\partial^2}{\partial x_j \partial x_k} B\left(\frac{x_1}{\sigma}, \frac{x_2}{\sigma}, \rho\right). \end{aligned}$$