Assume the value of an object j to individual i is $U_{1i}+U_{2j}$ where $f_1(u_1)$ is the density of U_1 and $f_2(u_2)$ is the density of U_2. We start off with a market with N individuals and M objects with $\mu = M/N$. Assume that each owner of an object knows his own value of U_1 and U_2, and potential buyers know only U_{1i} and the density $f_2(u_2)$. Let p be a proposed equilibrium price of the object. Then the number of sellers is

$$M \Pr\left[U_{1i}+U_{2j}-p<0\right], \tag{1}$$

and the number of buyers is

$$(N-M)\Pr\left[U_{1i}+U_{2j}-p>0 \mid U_{1k}+U_{2j}-p<0\right]. \tag{2}$$

Equation (1) is the number of owners times the probability of an owner being willing to sell at the equilibrium price. Equation (2) is the number of potential buyers times the probability that a buyer's U_{1i} is big enough so that the value of buying is positive even after taking into account the selection caused by voluntary selling. The probability in equation (1) can be written as

$$\begin{aligned}
& \int \Pr\left[U_{1i}<p-u_2\right] f_2(u_2)\,du_2 \\
= & \int F_1(p-u_2) f_2(u_2)\,du_2,
\end{aligned}$$

and the probability in equation (2) can be written as

$$\begin{aligned}
& \frac{\int \Pr\left[U_{1i}+u_2-p>0, U_{1k}+u_2-p<0 \mid u_2\right] f_2(u_2)\,du_2}{\int \Pr\left[U_{1k}+u_2-p<0 \mid u_2\right] f_2(u_2)\,du_2} \\
= & \frac{\int \Pr\left[U_{1i}>p-u_2\right]\Pr\left[U_{1k}<p-u_2\right] f_2(u_2)\,du_2}{\int \Pr\left[U_{1k}<p-u_2\right] f_2(u_2)\,du_2} \\
= & \frac{\int \left[1-F_1(p-u_2)\right] F_1(p-u_2) f_2(u_2)\,du_2}{\int F_1(p-u_2) f_2(u_2)\,du_2}.
\end{aligned}$$

Equilibrium requires that

$$M\int F_1(p-u_2) f_2(u_2)\,du_2 = (N-M)\frac{\int \left[1-F_1(p-u_2)\right] F_1(p-u_2) f_2(u_2)\,du_2}{\int F_1(p-u_2) f_2(u_2)\,du_2}. \tag{3}$$

Note that, as $p\to-\infty$, the left hand side of equation (3) goes to zero, and the right hand side has limit

$$\begin{aligned}
& \lim (N-M)\frac{\int \left[1-F_1(p-u_2)\right] F_1(p-u_2) f_2(u_2)\,du_2}{\int F_1(p-u_2) f_2(u_2)\,du_2} \\
= & (N-M)\lim \frac{\int \left[1-F_1(p-u_2)\right] F_1(p-u_2) f_2(u_2)\,du_2}{\int F_1(p-u_2) f_2(u_2)\,du_2} \\
= & (N-M)\lim \left[\frac{\int F_1(p-u_2) f_2(u_2)\,du_2}{\int F_1(p-u_2) f_2(u_2)\,du_2} - \frac{\int F_1^2(p-u_2) f_2(u_2)\,du_2}{\int F_1(p-u_2) f_2(u_2)\,du_2}\right] \\
= & (N-M)\lim \left[1-\frac{\int F_1^2(p-u_2) f_2(u_2)\,du_2}{\int F_1(p-u_2) f_2(u_2)\,du_2}\right] = 1.
\end{aligned}$$

Also note that, as $p \to \infty$, the left hand side of equation (3) goes to one, and the right hand side has limit

$$(N - M) \lim \left[1 - \frac{\int F_1^2 (p - u_2) f_2 (u_2) du_2}{\int F_1 (p - u_2) f_2 (u_2) du_2} \right] = 0.$$

Finally, note that

$$\frac{\partial}{\partial p} M \int F_1 (p - u_2) f_2 (u_2) du_2 = M \int f_1 (p - u_2) f_2 (u_2) du_2 > 0$$

and continuous, and

$$\frac{\partial}{\partial p} (N - M) \left[1 - \frac{\int F_1^2 (p - u_2) f_2 (u_2) du_2}{\int F_1 (p - u_2) f_2 (u_2) du_2} \right]$$

is continuous. Thus there is an interior price, p^* where equilibrium is satisfied.