An Equilibrium Model of Adverse Selection in the Used Car Market*

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1 Model

We follow Engers, Hartmann, and Stern (2004a) and add some minor modifications. A car is characterized by a brand index b, an age t, a privately observed condition (net of age) e_t, and an observable car-specific condition ξ_1.[1] The utility one receives from owning a particular car is

$$U(b,t,e_t \mid \theta) + \xi_1 + \varepsilon_{bt}$$

where ε_{bt} is an iid extreme value error specific to the brand and age and θ is a vector of utility parameters varying over the population of car owners. The owner has the ability to improve e_t to $e_{t+1} \geq e_t$ at a cost $C(e_t, e_{t+1})$. Also the owner can sell the car at a price

$$p(b,t+1) + \phi(\xi_1, b, t+1)$$

where $\phi(\xi_1, b, t+1)$ is the equilibrium capitalization of ξ_1 and pay a proportional transactions cost with "tax rate" τ. Note that the price does not depend on e_t because e_t is observed only by the owner. Upon selling a car, the owner can purchase a new car with brand j and age s at price $p(j,s) + \phi(\xi, j, s)$. Let $V(b,t,e_t,\xi_1 \mid \theta)$ be the value function for owning a car with characteristics

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[1]The point of adding ξ is to explain some cars being scrapped while others are sold at positive prices. See Engers, Hartmann and Stern (2004b).

(b, t, e_t, ξ_1) for an owner with utility parameters θ:

$$\begin{aligned} V(b,t,e_t,\xi_1 \mid \theta) &= \max_{e_{t+1}} \Big[U(b,t,e_t \mid \theta) + \xi_1 + \varepsilon_{bt} - C(e_t, e_{t+1}) \\ &\quad + \beta E_\varepsilon \max\left\{ V(b, t+1, e_{t+1}, \xi_1 \mid \theta), W(b,t,\xi_1) \right\} \Big] \end{aligned} \tag{1}$$

where

$$\begin{aligned} W(b,t,\xi_1) &= \max_{j,s} E_{e,\xi} \left[V(j,s,e,\xi \mid \theta) \right. \\ &\quad + \max\left[(p(b,t+1) + \phi(\xi_1, b, t+1))(1-\tau), 0 \right] \\ &\quad \left. - (p(j,s) + \phi(\xi, j, s)) + \varepsilon_{js} \right] \end{aligned}$$

is the value of the best choice the owner can make conditional on selling his car. Note that the owner has the option to "scrap" the car at price 0 if $(p(b,t+1) + \phi(\xi_1, b, t+1))(1-\tau) < 0$.

Assume $b \in \{1, 2, ..., B\}$, $t \in \{1, 2, ..., T\}$, and $e \in \{e_1, e_2, ..., e_E\}$. Assume $\theta \sim iidH$ in the population of car owners. Let $V(\theta)$ be a vector of $V(b,t,e_t \mid \theta)$ stacked over b, t, and e_t. Note that we ignore the effect of ξ on the value function because it affects utility linearly and (we assume) it is perfectly capitalized. Assume its size is BTE. Define $U(\theta)$, C, ε, and p appropriately. Then we can write equation (1) as

$$\begin{aligned} V(\theta) &= U(\theta) + \xi + \varepsilon + \\ &\quad D(\theta)\left[-C + \beta \Pi V(\theta) - \beta(p+\phi) - \beta\tau^*(p+\phi) \right] + \beta(p+\phi) \end{aligned} \tag{2}$$

where $D(\theta)$ is a matrix of size $BTE \times (E + BT)$ that maps elements of (b,t,e_t) into either (b,t,e_{t+1}) or (j,s) (assume the first E columns of $D(\theta)$ corresponds to keeping the same car) after integrating over the density of ε, Π is the matrix of probabilities of size $(1 + BT) \times BTE$ implicit in E_e, and τ^* is a vector with τ in every element except for the one corresponding to keeping the same car (where it is zero).

Define

$$\begin{aligned} V^*(j,s) &= E_{e,\xi} V(j,s,e,\xi \mid \theta) + \\ &\quad [p(b,t) + \phi(\xi_1, b, t+1)](1-\tau) - [p(j,s) + \phi(\xi, j, s)] - \varepsilon_{js} \end{aligned}$$

for all choices where $(j,s) \neq (b,t)$ and

$$\begin{aligned} V^{**}(b,t) &= \max \Big[E_{e,\xi} \left[V(b,t+1,e,\xi \mid \theta) - (p(b,t+1) + \phi(\xi, b, t+1)) \right] \\ &\quad + (1-\tau)(p(b,t+1) + \phi(\xi_1, b, t+1)) - \varepsilon_{bt}, \\ &\quad V(b, t+1, e_{t+1} \mid \theta) - \varepsilon_{bt} \Big]. \end{aligned}$$

For those rows of $D(\theta)$ corresponding to selling one's car and purchasing a vehicle of a different brand j and/or age s, each element is a multinomial logit term,

$$\frac{\exp\{V^*(j,s)\}}{\exp\{V^{**}(b,t)\}+\sum_{j',s'}\exp\{V^*(j',s')\}}, \tag{3}$$

for that row of $D(\theta)$ corresponding to selling one's car and purchasing a vehicle of the same brand b and age t, the element is a multinomial logit term,

$$\frac{\exp\{E_e V(b,t+1,e\mid\theta)-(1-\tau)\,p(b,t)-p(b,t+1)-\varepsilon_{bt}\}}{\exp\{V^{**}(b,t)\}+\sum_{j',s'}\exp\{V^*(j',s')\}}, \tag{4}$$

and for those rows of $D(\theta)$ corresponding to keeping one's car, each element is a multinomial logit term,

$$\frac{\exp\{V(b,t+1,e_{t+1}\mid\theta)-\varepsilon_{bt}\}}{\exp\{V^{**}(b,t)\}+\sum_{j',s'}\exp\{V^*(j',s')\}}. \tag{5}$$

The first E rows of Π (corresponding to keeping the same car) have a 1 in the element corresponding to the choice of e_{t+1} and 0 in each other element, and the other rows have probabilities depending on the distribution of e corresponding to a car purchased with brand j and age s. Equation (2) can be written as

$$V(\theta)=[I-\beta D(\theta)\,\Pi]^{-1}\,[U(\theta)+\varepsilon+\beta p-D(\theta)\{C+\beta p+\beta\tau^* p\}].$$

2 Equilibrium Behavior

[To be written]

3 Computation of Model Solution

The consumer's goal is to find the value of $D(\theta)$ that maximizes $V(\theta)$. We should be able to find an iterative solution using the following algorithm:

1. Set $V^1(\theta)=U(\theta)$ and set $k=1$.
2. For each element of $V^k(\theta)$, compute $D^k(\theta)$ using equations (3), (4), and (5).
3. Use $D^k(\theta)$ to update $V^{k+1}(\theta)$ using equation (2) for each element of $V^k(\theta)$, and increment k by 1.
4. Check for convergence. If not, return to step (2).

We also need to compute Π. Given $H(\theta)$ and a guess of the optimal $D(\theta)$, we can compute $q[b,t,e \mid \theta]$, the probability that a type-θ person owns a brand b car of age t and condition e. For each value of (b,t,e,θ), we can write

$$q[b,t,e \mid \theta] = \sum_{j,s,d} [D(\theta)\Pi]_{jsd,bte}\, q[j,s,c \mid \theta] \tag{6}$$

where $[D(\theta)\Pi]_{jsd,bte}$ is the jsd row and bte column of the product of $D(\theta)\Pi$. Solving equation (6) for each value of (b,t,e,θ) in terms of the other relevant values of (b,t,e,θ) provides the steady density of car holdings. Define $q(\theta)$ to be the vector of $q[b,t,e \mid \theta]$ stacked over values of (b,t,e) for each θ, and construct the appropriate matrix of the elements of $[D(\theta)\Pi]$. Equation (6) can be written in matrix form using these objects and solved by inverting a single matrix.

We now can compute the distribution of e conditional on being for sale with brand b and age t. This implies Π. In particular, the element of Π corresponding to purchasing a brand j car of age s and and having condition e is

$$\frac{\int 1[D_{jse}(\theta) > 1]\, q[j,s,e \mid \theta]\, dH(\theta)}{\int \sum_x 1[D_{jsx}(\theta) > 1]\, q[j,s,x \mid \theta]\, dH(\theta)}.$$

We can iterate between evaluating V, q, Π.

4 Estimation

Our goal is to estimate $\gamma = (H, C, \beta)$; i.e., find $\widehat{\gamma}$ that minimizes the sum of squared differences between supply and demand for each brand and age of car given equilibrium prices (from Kelley Blue Book) and fits the hazard model for time until sale.

4.1 Moments

We have hazard rate moments and price moments. While the hazard rate moments are pretty straightfoward to use, the price data moments have to be adjusted for endogenous scrapping. In particular, we need to compute

$$E[p(b,t) + \phi(\xi_1, b, t+1) \mid p(b,t) + \phi(\xi_1, b, t+1) \geq 0] \tag{7}$$

rather than without conditioning on $p(b,t) + \phi(\xi_1, b, t+1) \geq 0$. It is obvious that equation (7) is positive, and Engers, Hartmann, and Stern (2004b) show that conditioning on $p(b,t) + \phi(\xi_1, b, t+1) \geq 0$ with a price function linear in age explains used car prices very well.

4.2 Identification

Compare the number of parameters to the number of moments. Let

$$U(b,t,e_t \mid \theta) = \theta_b' \widetilde{b} + t\left(diag\widetilde{b}\right)\theta_{tb} + \theta_e e$$

where θ_b is a $B \times 1$ vector of brand dummies, $\widetilde{b}$ is a $B \times 1$ vector with $\underset{\sim}{1}$ in the bth position and zero elsewhere, $diag\widetilde{b}$ is a $B \times B$ diagonal matrix with $\widetilde{b}$ on the diagonal, θ_{tb} is a $B \times 1$ vector of brand-specific age effects, and θ_e is a scalar condition effect. Let there be n_θ different combinations of $\theta = (\theta_b, \theta_{tb}, \theta_e)$ in the population ordered θ^k, $k = 1, 2, .., n_\theta$, and let

$$\delta_{\theta k} = \Pr\left[\theta = \theta^k\right].$$

Let

$$C\left(e_t, e_{t+1}\right) = c_1 \Delta e_{t+1} + c_2 \left(\Delta e_{t+1}\right)^2$$

where $\Delta e_{t+1} = e_{t+1} - e_t$. Then there are $2Bn_\theta + n_\theta - 1 + 4$ parameters.[2] There are 26 brands. Table 1 shows how the number of parameters changes with n_θ.

Table 1
Parameters

n_θ	# Parameters
5	268
10	533
20	1063

Since the number of brands is 26 and the number of (interesting) ages is 10, there are 260 brand/age moments and another 260 hazard rate moments. Also there are two unobserved heterogeneity hazard rate parameters to match. Thus, there are 522 moments to match. This suggests either picking n_θ to be small or reparamerizing.

References

[1] Engers, Maxim, Monica Hartmann, and Steven Stern (2004a). "Are Lemons Really Hot Potatoes?" Unpublished manuscript.

[2] Engers, Maxim, Monica Hartmann, and Steven Stern (2004b). "Mileage and Used Car Prices." Unpublished manuscript.

[2] For each type k, there are $2B$ coefficients and a probability subject to the condition that the probabilities add up to one. Also, there are two c terms, a discount factor β, and a transactions cost τ.