The Production Function for Surgeries with Special Emphasis on Anesthesia Provision*

Lorraine Jordan
AANA

Elizabeth Merwin
University of Virginia

Steven Stern
University of Virginia

June 4, 2004

1 Introduction

In 2000, there were approximately 25 million surgeries performed in the United States. A significant majority of these required anesthesia, and in the majority of those cases, anesthesia was administered by an anesthesiologist (MD), a certified registered nurse anesthetist (CRNA), or some combination of the two. In a US sample of 1100 hospitals in 2002, 21% used only CRNAs, 21% used only MDs, 21% used some of each but a majority of MDs, and 37% used some of each but a majority of CRNAs (Merwin, Stern, and Jordan 2004). Yet, there is little discussion in the literature about the relative productivity of MDs and CRNAs or how they interact in the production of surgeries.

This work estimates a production function for surgeries focusing on the provision of anesthesia. We use a functional form for the production function that places no restrictions (other than continuity and differentiability) on how MDs and CRNAs add to output. Despite the lack of restrictions on functional form, the estimates of the production function are extremely well behaved and credible. We find that MDs and CRNAs are approximately equally productive in the production of surgeries, and both exhibit decreasing returns. The first result has strong implications for the way that MDs and CRNAs should be compensated. The second has implications for optimal mixes of MDs and CRNAs in hospital settings.

---

*The investigators would like to thank the Internal and External Advisory Panels for their many insights and useful contributions to this study. The members of these panels provided invaluable assistance in designing and implementing a study relevant to workforce issues faced in different areas of the country, in different settings, and within different practice arrangements. The investigators appreciate the responsive support of many AANA staff members who provided information for the study. Luis A. Rivera, Director of Information Systems was particularly helpful. The investigators would like to thank Jeffery Beutler, Executive Director for his support of the study.

Most of the literature on estimating production functions or cost functions for hospitals has estimated cost functions with multiple outputs and multiple input prices.[1] The concern in this paper is with anesthesia provision which, for the most part, is limited to surgeries.[2] Thus, we focus on the single output: surgeries. RNs are the only input we control for that have other major responsibilities besides surgery. Since RNs are not the focus of the work, we ignore biases caused by the multiple roles of RNs. We deviate from most of the literature as well in not using input prices because a) salary information at the hospital level (or county level) is not available for some of the inputs and b) salary is probably not the appropriate measure of cost for the inputs given complicated Medicare and private insurance financing rules. Thus, we estimate the production function directly. The reason most researchers avoid this is because of potential endogeneity of the inputs. Standard theory of the firm implies that the inputs will be correlated with the error in the production function if the error represents hospital characteristics observed by the hospital decision maker and unobserved by the econometrician. However, we show that, for a large, widely used[3] class of production functions, while the estimated marginal products are biased upwards, the ratio of estimated marginal products is consistent. Thus, we feel comfortable comparing marginal products of different anesthesia providers.

2 Financing Rules for Anesthesia Provision

Since 1986, under the Omnibus Budget Reconciliation Act, CRNAs received direct reimbursement for anesthesia and related services under Medicare (now Center of Medicare and Medicaid Services, CMS). Today CRNAs are required to bill under Medicare Part B which provides reimbursement for all physician and specific allowable non-physician services. Therefore, Medicare services provided by CRNAs must be billed under Medicare Part B, and the CRNAs must accept the assignment of benefits.

Also CRNA services may be covered under Medicare Part A. Under Medicare Part A, hospitals, facilities, and ambulatory surgery costs incurred by Medicare beneficiaries are covered. The only exception under Part A is rural hospital cost pass-through funding for CRNA services.

Under Medicare billing, there are two payment schedules: (1) CRNAs medically directed and medically supervised by anesthesiologists and (2) CRNAs non-medically directed, working with physicians other than anesthesiologists. Payment for CRNA services is fixed at 50% of the allowable anesthesia fee schedule [42 U.S.C. $13851 (1) Sec.§ 1833] when working with anesthesiologists.

Medical direction occurs when an anesthesiologist is involved in the case. In billing medical direction, an anesthesiologist is involved in one through four

---

[1] See NHS (1996) or Smet (2002) for surveys of hospital cost studies.

[2] However, anesthesia providers are becoming more involved in other areas of medicine such as pain control and radiology.

[3] See Barnett and Binner (2004), Chapter 13.

concurrent cases and must meet the seven conditions of participation set forth under 1982 Tax Equity and Fiscal Responsibility Act (TEFRA). Medical supervision occurs when an anesthesiologist is involved in more than four concurrent cases and two of the seven TEFRA conditions do not apply. Services rendered by CRNAs who are medically directed are reimbursed at 50% of the rate for their anesthesia services. The anesthesiologist is reimbursed at a lesser fee.

Reimbursement for non-medically directed cases indicate anesthesiologists are not involved in the case and the CRNA receives 100% of the payment for the anesthesia services. The seven conditions of participation required by TEFRA do not apply.

Non-Medicare insurance payers' payment systems differ greatly for CRNA services. Each insurance carrier varies greatly in regards to the payment schedule and system. However, some insurance carriers adopted the provisions for payment using Medicare's definitions of medical direction and medical supervision by anesthesiologists. In any case, a necessary condition for (societal) cost minimization is that reimbursement rates be proportional to marginal products of surgical inputs. Below we discuss how to estimate relative marginal products and provide estimates of relative marginal products.

3 Production Function

The basic idea in the theory of firm behavior is that a firm purchases, hires, or rents a number of inputs to produce an output. The firm chooses inputs (implying output) that maximize profit and, as a side effect, minimize the cost of production. The basic theory is very powerful in predicting firm behavior in many dimensions. In the production of surgeries, some of the basic assumptions underlying the theory of the firm are not true. In particular, it is probably inappropriate to assume that hospitals and other providers of surgical services attempt to only maximize profit. However, even under different, more reasonable assumptions about the objectives of hospitals, much of the basic theory still holds. In particular, once a hospital chooses a quantity and quality of surgical services to provide, it still should attempt to minimize the cost of providing that combination of quantity and quality. For example, if its objective were to maximize the number of surgeries, it would do so by performing surgeries with the smallest number of staff that could still achieve acceptable levels of quality. Thus, for most of the analysis, it is assumed that hospitals try to minimize the cost of providing the quantity and quality of surgeries they have chosen to supply.

Hospitals use a set of inputs including surgeons, RNs, MDs, and CRNAs to produce surgeries. It is assumed that the addition of each extra unit of each input increases the number of surgeries that can be performed. One might assume alternatively that the production of a surgery requires a fixed combination of a set of inputs and that increases in one input without proportionate increases in the other inputs would have no effect on the supply of surgeries. However, hospitals can use various inputs more intensely than others and thus change the proportion used to produce surgeries. Consider two examples: a) a hospital with a shortage of operating rooms can make sure that all of the operating rooms are used as often as possible even if it means scheduling surgeries at inconvenient times or using temporary employees (e.g., locum tenums); b) a hospital may substitute between MDs and CRNAs.

3.1 Cost Minimization

Let

$$y_i = \theta_i g\left(X_i\right) \tag{1}$$

be a production function for hospital i given a $J-$element vector of inputs X_i and a scalar random productivity parameter θ_i.[4] Without loss of generality, we can assume that $E\theta_i = 0$. Let

$$C\left(y_i\right) = w_i' X_i$$

be a cost function. The goal of the hospital decision maker is to minimize cost subject to a quantity demand:

$$\begin{aligned} & \min_{X_i} C\left(y_i\right) \\ \text{st} \quad y_i &= \theta_i g\left(X_i\right). \end{aligned}$$

We can write the firm's optimization problem as a Lagrangean equation as

$$L = w_i' X_i + \lambda\left[y_i - \theta_i g\left(X_i\right)\right]$$

with derivates,

$$\frac{\partial L}{\partial X_i} = w_i - \lambda\left(y_i\right)\theta_i g_X\left(X_i\right) = 0 \tag{2}$$

(where $g_X\left(X_i\right)$ is the vector of J "marginal products" not including the effect of θ_i)[5] and

$$\frac{\partial L}{\partial \lambda} = y_i - \theta_i g\left(X_i\right).$$

Equation (2) implies that, if θ_i is known at the time of the input decision, then

$$X_i = g_X^{-1}\left(\frac{w_i}{\lambda\left(y_i\right)\theta_i}\right) \tag{3}$$

---

[4] CRNAs and MDs perform some tasks not associated with surgery (e.g., pain management and radiology). This causes an estimation problem only to the degree that the frequency of such activities varies across observations and the variation is correlated with the explanatory variables already included in the analysis.

[5] In most of the analysis, we ignore the case of corner solutions even though they occur quite frequently in the data. In fact, the analysis is quite similar: equation (2) is replaced by a similar term truncated from below at zero.

and, if θ_i is not known at the time of the input decision, then

$$X_i = g_X^{-1}\left(\frac{w_i}{\lambda(y_i)\,E\theta_i}\right) \tag{4}$$

(where $g_X^{-1}(z)$ is the solution to the set of equations in equation (2) for

$$z_j = \frac{w_j}{\lambda(y_i)\,\theta_i} \text{ or } \frac{w_j}{\lambda(y_i)\,E\theta_i},$$

depending on whether θ_i is known).[6]

3.2 The Hospital's Optimization Problem and $\partial X_i/\partial\theta_i$

In the next section, asymptotic bias results will depend upon the sign of $\partial X_i/\partial\theta_i$. We consider a number of different hospital optimization problems to determine the sensitivity of the sign of $\partial X_i/\partial\theta_i$ to assumptions about the objective of the hospital decision maker. Throughout the section, we consider only the case where the decision maker observes θ_i prior to making an input decision.

First, consider the case where the hospital decision maker maximizes profits,

$$\pi_i = p\theta_i g(X_i) - wX_i.$$

Then,

$$\frac{dX_i}{d\theta_i} = -\frac{g_X(X_i)}{\theta_i g_{XX}(X_i)} > 0 \tag{5}$$

(assuming $g_{XX}(X_i) < 0$ which is necessary for the second order condition). On the other hand, if the decision maker has a fixed, exogenous output to produce and only minimizes cost, then $\partial X_i/\partial\theta_i < 0$ because, as θ_i increases, the same input bundle results in more output.

Consider a more interesting case where the hospital decision maker has a utility function $U(\pi_i, py_i)$ that depends upon both profits π_i and revenue py_i.[7] Then,

$$\frac{dX_i}{d\theta_i} = -\frac{p\left(1+\frac{U_2}{U_1}\right)g_X(X_i) + p\theta_i g_X(X_i)\,g(X_i)\,U^*}{p\left(1+\frac{U_2}{U_1}\right)\theta_i g_{XX}(X_i) + p\left[\theta_i g_X(X_i)\right]^2 U^*} \tag{6}$$

where

$$U^* = \frac{\partial}{\partial\pi_i}\left(\frac{U_2}{U_1}\right) + \frac{\partial}{\partial py_i}\left(\frac{U_2}{U_1}\right) = \frac{U_1U_{12} - U_2U_{11}}{U_1^2} + \frac{U_1U_{22} - U_2U_{12}}{U_1^2}. \tag{7}$$

---

[6] One might worry that it is important to allow for unobserved heterogeneity across hospitals in more than just a multiplicative term. While this is a valid concern, it is not one that has been considered much by the literature.

[7] Note that it would not make sense for the decision maker to have a utility function depending on y_i instead of py_i. If it depended on y_i, then an irrelevant change in units would have large effects on the behavior of the decision maker.

Note that, if $U_2 = 0$, then equation (6) reduces to equation (5). Further note that, in general, U^* can not be signed.[8] In the special case where

$$U\left(\pi_i, py_i\right) = \log \pi_i + \alpha \log py_i,$$

equation (7) becomes

$$U^* = \frac{\alpha \pi^2 w X}{\left(py\right)^2} > 0$$

which implies that $dX_i/d\theta_i > 0$. However, in general, one can not sign $dX_i/d\theta_i$.

For the remainder of the analysis, we *assume* that $dX_i/d\theta_i > 0$. The results for the other case are obvious.

3.3 Nonparametric Consistency

Let $\widehat{g}\left(X\right)$ be a nonparametric kernel estimator of $g\left(\cdot\right)$. For example, we might set

$$\widehat{g}\left(X\right) = \frac{\sum y_i K\left(X_i - X\right)}{\sum K\left(X_i - X\right)}. \tag{8}$$

Define

$$g_{Xj}\left(X_i\right) = \frac{\partial g\left(X_i\right)}{\partial X_{ij}}.$$

Going back to Mundlak (1978), there is concern that an estimate of $g_{Xj}\left(X_i\right)$ will be biased if θ_i is correlated with X_i. Theorem 1 makes the same point in a nonparametric setting.

Theorem 1

$$plim \widehat{g}_{Xj}\left(X\right) > g_{Xj}\left(X\right) \quad \forall j$$

if equation (3) is the appropriate input decision, and

$$plim \widehat{g}_{Xj}\left(X\right) = g_{Xj}\left(X\right) \quad \forall j$$

if equation (4) is the appropriate input decision.

Proof. Think of equation (8) as

$$\widehat{g}\left(X\right) = \sum y_i K^*\left(X_i - X\right) \tag{9}$$

---

[8] However, we can assume the denominator is negative because it is the second order condition.

where

$$K^*\left(X_i - X\right) = \frac{K\left(X_i - X\right)}{\sum K\left(X_i - X\right)}.$$

Substituting equation (1) into equation (9) leads to

$$\widehat{g}\left(X\right) = \sum \theta_i g\left(X_i\right) K^*\left(X_i - X\right).$$

A first order Taylor series approximation implies

$$\begin{aligned} \widehat{g}\left(X\right) &= \sum \theta_i \left[g\left(X_i\left(E\theta_i\right)\right) + g_X^{'}\left(X_i\left(E\theta_i\right)\right) X_{i\theta}\left(E\theta_i\right)\left(\theta_i - E\theta_i\right)\right] \cdot \\ &\quad K^*\left(X_i - X\right) \\ &= \sum \theta_i g\left(X_i\left(E\theta_i\right)\right) K^*\left(X_i - X\right) \\ &\quad + \sum \theta_i g_X^{'}\left(X_i\left(E\theta_i\right)\right) X_{i\theta}\left(E\theta_i\right)\left(\theta_i - E\theta_i\right) K^*\left(X_i - X\right) \end{aligned}$$

where

$$\begin{aligned} g_X^{'}\left(X_i\left(E\theta_i\right)\right) &= \frac{\partial}{\partial X'} g\left(X_i\left(E\theta_i\right)\right), \\ X_{i\theta} &= \frac{\partial X_i}{\partial \theta}. \end{aligned}$$

Taking *plim*s leads to

$$\begin{aligned} plim\widehat{g}\left(X\right) &= plim \sum \theta_i g\left(X_i\left(E\theta_i\right)\right) K^*\left(X_i - X\right) \qquad (10) \\ &\quad + plim \sum \theta_i g_X^{'}\left(X_i\left(E\theta_i\right)\right) X_{i\theta}\left(E\theta_i\right)\left(\theta_i - E\theta_i\right) K^*\left(X_i - X\right) \\ &= E\theta g\left(X\left(E\theta\right)\right) + g_X^{'}\left(X\left(E\theta\right)\right) E X_\theta \theta\left(\theta - E\theta\right). \end{aligned}$$

If X is exogenous, then $X_\theta = 0$, and

$$plim\widehat{g}_X\left(X\right) = E\theta g_X\left(X\left(E\theta\right)\right) = g_X\left(X\right).$$

If X is endogenous, then differentiation of equation (2) with respect to θ implies that

$$X_\theta = -\left[\theta g_{XX}\left(X\right)\right]^{-1} g_X\left(X\right) > 0.$$

Plugging X_θ into equation (10) leads to

$$\begin{aligned} plim\widehat{g}_X\left(X\right) &= E\theta g_X\left(X\left(E\theta\right)\right) + g_{XX}^{'}\left(X\left(E\theta\right)\right) E X_\theta \theta\left(\theta - E\theta\right) \\ &= E\theta g_X\left(X\left(E\theta\right)\right) - g_{XX}^{'}\left(X\left(E\theta\right)\right) E\left[g_{XX}\left(X\right)\right]^{-1} g_X\left(X\right)\left(\theta - E\theta\right) \\ &= E\theta g_X\left(X\left(E\theta\right)\right) - E\left[g_X\left(X\right)\left(\theta - E\theta\right)\right] > E\theta g_X\left(X\left(E\theta\right)\right) \end{aligned}$$

because $X_\theta > 0$ and $g_{XX}\left(X\right) < 0$. ■

However, when there are multiple inputs ($J > 1$), despite the possibility that estimated marginal products may be biased by endogenous inputs, there are interesting cases where estimated ratios of marginal products are consistent. This occurs because the unobserved factor enters multiplicatively and cancels when comparing marginal products.

Theorem 2 *If either X is exogenous or $g(\cdot)$ is homothetic in X, then*

$$plim \frac{\widehat{g}_j(X)}{\widehat{g}_k(X)} = \frac{g_j(X)}{g_k(X)} \ \forall j, k.$$

Proof. If X is exogenous, then $\widehat{g}_X(X)$ is consistent which implies the ratio is consistent. If $g(\cdot)$ is homothetic in X, then the asymptotic bias in $\widehat{g}_X(X)$ can be written as

$$E[g_X(X)(\theta - E\theta)] = g_X(X(E\theta)) E\left[\frac{g_X(X)}{g_X(X(E\theta))}(\theta - E\theta)\right]$$

with

$$\frac{g_X(X)}{g_X(X(E\theta))} = b(\theta) 1$$

where $b(\theta)$ is a scalar and 1 is a vector of 1's. Thus the bias is proportional to $g_X(X(E\theta))$ and cancels in a ratio. ■

Theorem 2 implies that, if either X is exogenous or $g(\cdot)$ is homothetic in X, then we can construct a consistent estimate of the ratio of marginal product of two inputs, even if the estimates of the marginal product of each input is biased. We rely upon this result in the later analysis, focusing on relative marginal products of the two anesthesia providers.

4 Data

We used two different data sources to estimate our model. First, we used data on surgeries and inputs aggregated at a county level from the 1999 Area Resources File. There are aggregation problems and data smoothing problems associated with using this data. However, we still thought it was useful. Second, we collected data on surgeries and inputs from a sample of hospitals across the United States and used this data to estimate the model. For each data set, we describe the collection mechanism and the characteristics of the data. Throughout the analysis, we aggregate surgeries of different types into one variable called "surgeries." We do this because the data is not readily available and would require extensive programming and conceptual development to create a meaningful "surgical case mix" variable.

4.1 County Data

We collected information on surgeries and each input disaggregated by county from the 1999 Area Resources File. Throughout, we assume that there is a single production function for each county and ignore any effects due to linear aggregation of nonlinear functions across different hospitals in the same county.[9]

---

[9] A sufficient condition for aggregation to be meaningful is that the production function is homothetic as in Theorem 2 and that hospitals within a county face similar environments and differ only in size.

Data were smoothed using equation (16) in a later section to allow for consumers and input providers crossing county lines in the market for surgeries. Moments of the data are displayed in Table 1. Smoothing has no effect on means,[10] but it significantly reduces standard deviations because some of the large variables in populated counties are attributed to people in nearby less populated counties, decreasing the numbers in populated counties and increasing them in less populated counties.

Table 1
Unsmoothed Moments
of County Data

Variable	Mean	Std. Dev.
Surgeries	8142.0	25497.7
Surgeons	46.5	192.5
RNs	683.1	2043.5
MDs	10.9	47.4
CRNAs	8.7	23.8
# Obs	3114	

Univariate nonparametric regression results in Figure 1 and Figure 2 show a strong positive relationship between the availability of CRNAs and surgeries and a weaker one between the availability of MDs and surgeries. On the vertical axis is smoothed log surgeries, and on the horizontal axis is smoothed log CRNAs and smoothed log MDs. The deviant behavior at small values of the inputs is mostly a "small sample size" problem that disappears in the later empirical analysis.

4.2 Hospital Data

A survey was conducted of hospitals to determine their utilization of anesthesia providers. A list of hospitals was obtained from Medical Marketing Service (2003).[11] A stratified random sample was used. The number of hospitals in each state was identified. States were stratified into three groups:

Group 1	< 61 hospitals
Group 2	$61 - 199$ hospitals
Group 3	≥ 200 hospitals

All hospitals in Group 1 were included in the sample. The hospitals in Groups 2 and 3 were divided into subgroups reflecting those with < 100 and ≥ 100 beds. A random sample of 25 hospitals from each subgroup of Group 2 and of 50 hospitals from each subgroup of Group 3 was included in the sample. This sampling plan was necessary to facilitate the development of state-level vacancy rates from the survey responses. Surveys were mailed to hospital administrators. After the first mailing, it was apparent that the mailing list contained some ineligible

---

[10] See Theorem 3 below.

[11] They are the list provider believed to have the most comprehensive list available, and they include small facilities.

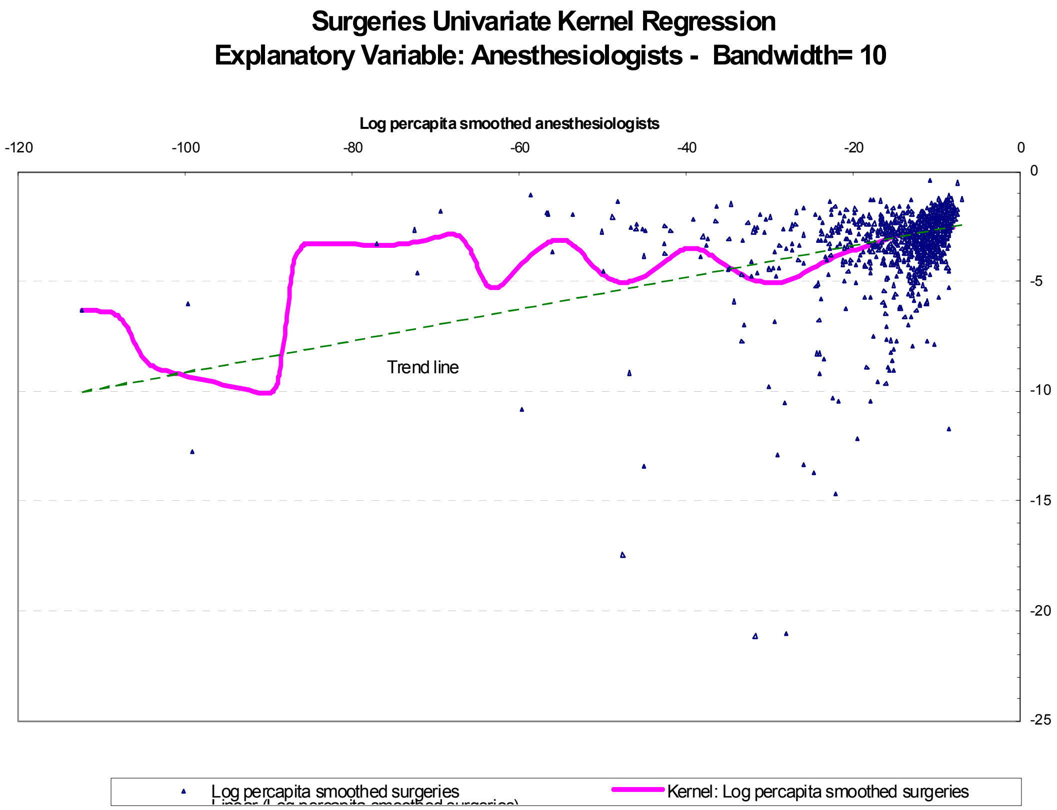
Surgeries Univariate Kernel Regression
Explanatory Variable: Anesthesiologists - Bandwidth= 10
Log percapita smoothed anesthesiologists
-120
-100
-80
-60
-40
-20
0
0
-5
-10
-15
-20
-25
Trend line
Log percapita smoothed surgeries
Kernel: Log percapita smoothed surgeries

Figure 1:

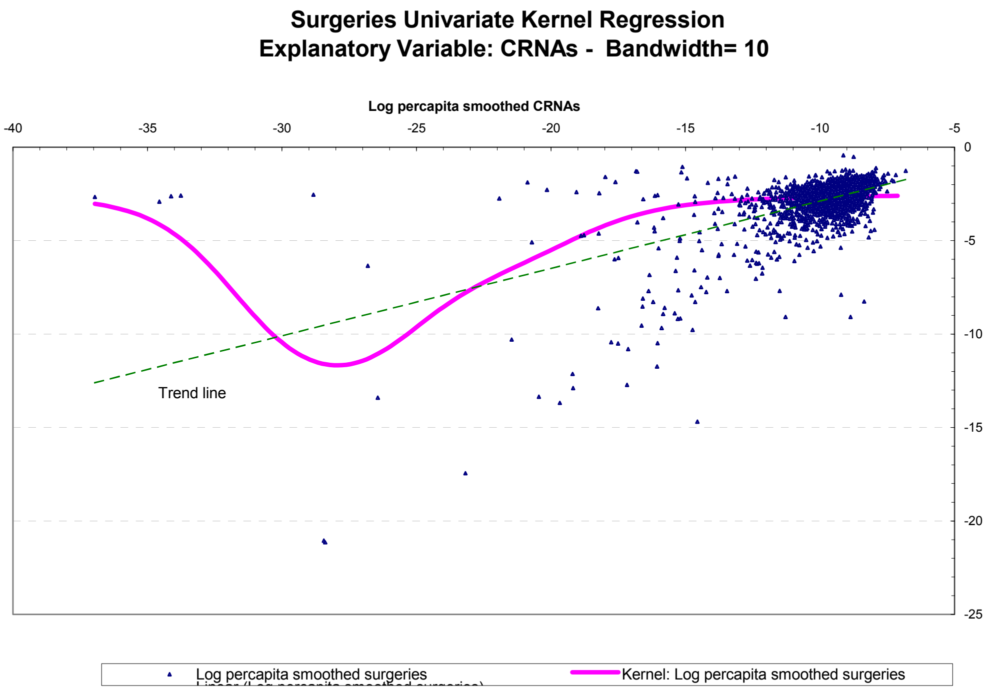
Surgeries Univariate Kernel Regression
Explanatory Variable: CRNAs - Bandwidth= 10
Log percapita smoothed CRNAs
-40
-35
-30
-25
-20
-15
-10
-5
0
-5
-10
-15
-20
-25
Trend line
Log percapita smoothed surgeries
Kernel: Log percapita smoothed surgeries

Figure 2:

hospitals such as psychiatric and rehabilitation hospitals. These hospitals were dropped from the sample. Two additional mailings were sent. The first and second mailings were sent to the hospital administrator. The third and final mailing was sent the Director of Nursing. There were 2661 hospitals sampled. Responses were received from 41.6% for a sample of 1107 hospitals.

There were an average of 68 surgical providers in surveyed hospitals. These providers conducted an average of 5690 surgeries or procedures requiring anesthesia providers. The hospitals had an average of 7 operating rooms in 2001. For facilities with any anesthesia providers, there were an average of 8.5 (SD 28.9) providers. There were an average of 7.0 (SD 28.3) CRNAs working regularly in these settings at the time of the survey in the fall of 2002 with 5.65 (SD 23) full-time equivalent (FTE) positions for CRNAs. On average, there were 6.2 (SD 25.4) FTE anesthesiologists in 2001. With such large standard deviations, medians provide useful information. The median number of anesthesia providers in the survey facilities was 4. Twenty-one percent of facilities used CRNAs only, and 21% used MDs only. The rest used a combination of the two. Of those using both types of providers, 21% had less than 50% CRNAs and 37% had over 50% CRNAs. The mean and median number of anesthesia providers in hospitals surveyed is 10 and 4 respectively. On average, 57% of the providers were CRNAs with a median value of 63%.

5 Econometric Methodology

A significant problem in estimation is that, in general, $g(X)$ in equation (1) has a large dimension. This makes nonparametric estimation too costly. Instead, we specify a production function

$$g(X_i) = \widetilde{g}(Z_i, \alpha)\, \psi(A_i, N_i) \tag{11}$$

where Z_i is the vector of the amount of each input excluding MDs and CRNAs, A_i is the amount of MDs, and N_i is the amount of CRNAs. We give $\widetilde{g}(\cdot,\cdot)$ a parsimonious flexible functional form (e.g., a second order Taylor series approximation) depending on a small set of parameters α and specify $\psi(\cdot,\cdot)$ nonparametrically. We take logs to get a generalization of Robinson's (1988) partial linear model[12] and then use a nonparametric method of moments (Ichimura and Lee, 1991) estimation procedure by solving[13]

$$\min_{\alpha} \sum_i \left[\log y_i - \log \widetilde{g}(Z_i, \alpha) - \log \widehat{\psi}(A_i, N_i)\right]^2 \tag{12}$$

where

$$\log \widehat{\psi}(A_i, N_i) = \frac{\sum_{k \neq i} \left[\log y_i - \log \widetilde{g}(Z_i, \alpha)\right] K(A_i - A_k, N_i - N_k)}{\sum_{k \neq i} K(A_i - A_k, N_i - N_k)}.$$

---

[12] This is a generalization only in the sense that the variable entering nonparametrically is a 2-element vector.

[13] Li et al. (2002) consider a related but different model where the variable entering nonparametrically is a scalar but it can interact with α nonparametrically.

To construct the covariance matrix of $\widehat{\alpha}$, define

$$\widetilde{Z}_i = \frac{\partial \log \widetilde{g}\left(Z_i, \alpha\right)}{\partial \alpha} + \frac{\partial \log \widehat{\psi}\left(A_i, N_i\right)}{\partial \alpha}$$

and

$$\varphi_i\left(\alpha\right) = \log y_i - \log \widetilde{g}\left(Z_i, \alpha\right) - \log \widehat{\psi}\left(A_i, N_i\right).$$

Consider a Taylor series approximation of the condition that $\widetilde{Z}_i$ should be orthogonal to $\varphi_i\left(\alpha\right)$:

$$\frac{1}{n}\sum_i \left[\widetilde{Z}_i'\varphi_i\left(\widehat{\alpha}\right) - \widetilde{Z}_i'\varphi_{i\alpha}\left(\widehat{\alpha}\right)\left(\widehat{\alpha} - \alpha\right)\right].$$

Then

$$\widehat{\alpha} - \alpha = \left[\frac{1}{n}\sum_i -\widetilde{Z}_i'\varphi_{i\alpha}\left(\widehat{\alpha}\right)\right]^{-1}\left[\frac{1}{n}\sum_i \widetilde{Z}_i'\varphi_i\left(\widehat{\alpha}\right)\right], \tag{13}$$

and

$$Cov\left(\widehat{\alpha}\right) = \left[\frac{1}{n}\sum_i \widetilde{Z}_i'\varphi_{i\alpha}\left(\widehat{\alpha}\right)\right]^{-1}\left[\frac{1}{n}\sum_i \widetilde{Z}_i'\varphi_i\left(\widehat{\alpha}\right)\varphi_i'\left(\widehat{\alpha}\right)\widetilde{Z}_i\right]\left[\frac{1}{n}\sum_i \varphi_{i\alpha}\left(\widehat{\alpha}\right)'\widetilde{Z}_i\right]^{-1}.$$

5.1 Allowing for Effects Across Nearby Counties

Both surgical input providers and consumers of surgery cross county lines to meet in the market for surgeries. However, our county data measures surgeries performed in each county and surgical input providers who live in each county. Thus, without smoothing, we would conclude that people living in counties without hospitals receive no surgical services, and surgical input providers who live in counties with no hospitals are not productive. Below, we suggest a method to smooth our county data to allow for crossing county lines. Other researchers such as Bolduc et al. (1992) and Conley (1999) allow for spatial covariance terms. But they do not consider how deviations between where people live, work, and receive services affect estimation. This problem is more like a measurement error problem[14] than a error correlation problem though error correlation is a relevant issue.

Let d_{ik} be the geographical distance between two counties i and k. Let $\phi\left(d_{ik}\right)$ be a function (somewhat like a kernel function) such that

$$\begin{aligned} \arg\max_d \phi\left(d\right) &= 0, \\ \frac{\partial \phi\left(d\right)}{\partial d} &\leq 0, \\ \phi\left(d\right) &= 0 \ \ \forall d : \left|d\right| \geq d_{\max}. \end{aligned} \tag{14}$$

---

[14] Conley and Topa (2004) consider a similar measurement error problem. However, their problem is specialized enough so that their results do not apply here.

We generalize equation (8) to

$$\widehat{g}^{*}\left(X^{*}\right)=\frac{\sum y_{i}^{*} K\left(X_{i}^{*}-X^{*}\right)}{\sum K\left(X_{i}^{*}-X^{*}\right)} \tag{15}$$

where

$$\begin{aligned} y_{i}^{*} &= \frac{\sum_{k} \phi\left(d_{ik}\right) y_{k}}{\sum_{k} \phi\left(d_{ik}\right)}; \\ X_{i}^{*} &= \frac{\sum_{k} \phi\left(d_{ik}\right) X_{k}}{\sum_{k} \phi\left(d_{ik}\right)}. \end{aligned} \tag{16}$$

Note that $\left(y_{i}^{*}, X_{i}^{*}\right)$ satisfy a resource constraint (inputs used in one county can't be used in another).

Theorem 3 $\sum_{i} X_{i}^{*}=\sum_{i} X_{i}$.

Proof.

$$\begin{aligned} \sum_{i} X_{i}^{*} &= \sum_{i} \frac{\sum_{k} \phi\left(d_{ik}\right) X_{k}}{\sum_{k} \phi\left(d_{ik}\right)}=\frac{\sum_{i} \sum_{k} \phi\left(d_{ik}\right) X_{k}}{\sum_{k} \phi\left(d_{ik}\right)} \\ &= \frac{\sum_{k} \sum_{i} \phi\left(d_{ik}\right) X_{k}}{\sum_{k} \phi\left(d_{ik}\right)}=\frac{\left(\sum_{i} \phi\left(d_{ik}\right)\right)\left(\sum_{k} X_{k}\right)}{\sum_{k} \phi\left(d_{ik}\right)} \\ &= \sum_{k} X_{k}=\sum_{i} X_{i}. \end{aligned}$$

■

The effect of smoothing on the density of each variable is to replace zeros with small numbers and very large numbers with smaller numbers. Figure 3 shows how the density of log surgeries changes when smoothing with a $N(0,500)$ density function is used.[15]

If we parameterize $\phi(\cdot)$ subject to the conditions in equation (14), we can estimate the parameters by using a objective function, such as

$$\min \sum_{i}\left[y_{i}^{*}-\widehat{g}^{*}\left(X_{i}^{*}\right)\right]^{2}$$

as in Ichimura and Lee (1991). It is straightforward to generalize equation (12) to allow for cross county effects by replacing $\left(y_{i}, Z_{i}, A_{i}, N_{i}\right)$ with corresponding $\left(y_{i}^{*}, Z_{i}^{*}, A_{i}^{*}, N_{i}^{*}\right)$. If we assume that $\left(y_{i}^{*}, Z_{i}^{*}, A_{i}^{*}, N_{i}^{*}\right)$ are the "true" outputs and inputs, the consistency follows from Theorems 1 and 2. This is equivalent to treating $\left(y_{i}, Z_{i}, A_{i}, N_{i}\right)$ as measured without error and $\phi(\cdot)$ as known. Alternatively, we can estimate the parameters of $\phi(\cdot)$ and treat $\left(y_{i}, Z_{i}, A_{i}, N_{i}\right)$ as measured without error to get consistent estimates. The covariance matrix of $\widehat{\psi}\left(A_{i}, N_{i}\right)$ and the parameter estimates in $\widetilde{g}\left(Z_{i}, \alpha\right)$ requires an adjustment explained in Appendices 1 and 2.

---

[15] The normal density function is truncated at ±4 standard deviations.

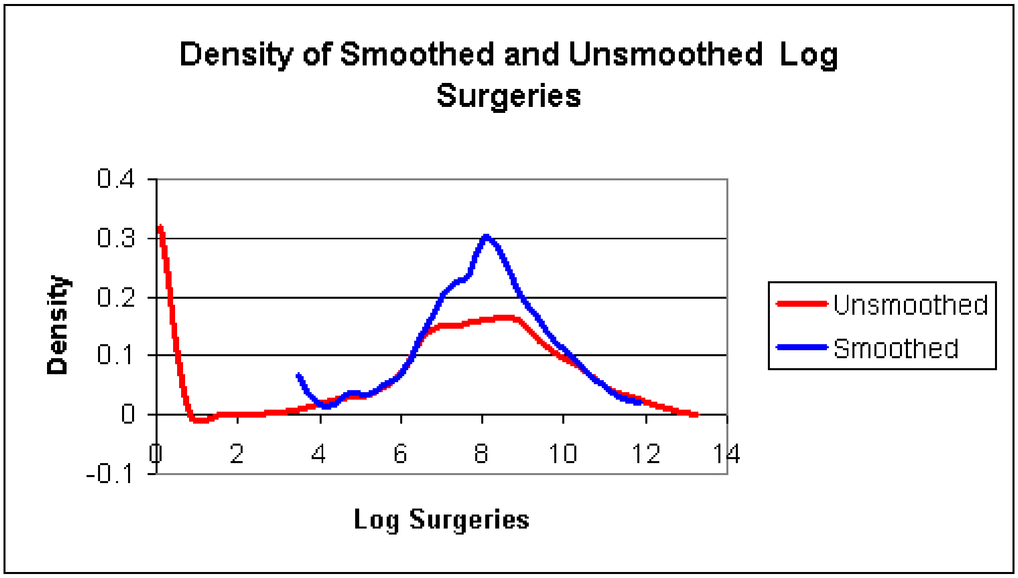

Figure 3:

5.2 Alternative Estimators

An alternative to the kernel based estimation used here is to use flexible functional forms originally suggested by Diewert (1971) and Christensen, Jorgenson, and Lau (1971, 1973). However, to some degree, these flexible functional forms have fallen out of favor for reasons summarized in Diewert and Wales (1987).[16] More recent work such as Olley and Pakes (1996) and Li et al. (2002) use kernel based methods.

Many researchers including Barnett and Lee (1985), Diewert and Wales (1987), and Olley and Pakes (1996) invert the cost function as we discuss above and then estimate input demand functions using instruments such as input prices to control for endogenous regressors. While there might some philosophical differences of opinion about this approach (e.g., see Heckman, 2000), they are irrelevant here because the obvious instrument, input prices, is not observable. While we have access to good data on CRNA wages from American Association or Nurse Anesthetists (AANA) data and we can construct reasonable estimates of RN wages from Census data (given the large number of RNs in each Census PUMA), constructing meaningful estimates of surgeon and MD wages is not feasible because MDs are not distinguished from other physicians in Census data. Thus, we do not pursue this approach.

Some researchers focus on estimating the "frontier" or envelope of the cost

---

[16] Thurston and Libby (2002) use Diewert's (1971) methodology to estimate a production function in the health services industry.

function or production function. Sickles (2003) provides a nice survey of this literature. It is somewhat focused on learning about firm specific "efficiency effects" or the distribution of those effects. In these models, estimation of production function curvature parameters is quite similar to the flexible functional form literature cited above.

6 Results

The production function was estimated in two ways: using a linear model and using a semiparametric, nonlinear model. In both cases, smoothed log surgeries for each county or hospital was specified as a function of smoothed log surgeons, smoothed log RNs, smoothed log MDs, and smoothed log CRNAs. Both were estimated using each of the two data sets, one county based and one hospital based.

6.1 County Data

6.1.1 Ordinary Least Squares Estimates

The ordinary least squares (OLS) results are[17]

$$\begin{aligned}\log Surgeries_i &= \underset{(0.52)}{5.28^*} + \underset{(0.014)}{0.396^*} \log Surgeons_i + \underset{(0.012)}{0.298^*} \log RNs_i \\ &\quad + \underset{(0.005)}{0.005} \log MDs_i + \underset{(0.010)}{0.146^*} \log CRNAs_i + u_i\end{aligned}$$

with an $R^2 = 0.882$. The standard errors are biased downward because the correlation across observations caused by smoothing is not accounted for. However, even after correlation is corrected for using a Newey-West (1987) standard error correction procedure, the results are qualitatively similar.

Note that:

1. The estimated coefficient on log MDs is small and insignificant. Taken literally, this means that MDs have no significant effect on the production of surgeries once one controls for the other inputs. This result is not credible. However, it is a very robust result. In particular, it is true for all other specifications of the model as the amount of smoothing is varied and some nonlinear spline terms are added. This result could be caused by nonlinearities not captured by this equation. The usage of nonparametric, nonlinear methods discussed below addresses this concern. Anticipating results, it is found that allowing for such nonlinearities causes the coefficient on MDs to significantly increase.

2. The sum of estimated coefficients (excluding the constant) is 0.845. This implies that a 10% increase in each of the inputs results in an 8.45%

---

[17] A sample of 3114 smoothed counties was used. Standard errors are in parentheses, and starred items are significant at the 1% level.

increase in surgeries.[18] This sounds quite reasonable given the existence of other important inputs not included in this analysis such as operating rooms.

6.1.2 Semiparametric Estimates

The semiparametric specification of the model is

$$\begin{aligned}\log Surgeries_i &= \beta_1 \log Surgeons_i + \beta_2 \log RNs_i + \beta_3 \left[\log Surgeons_i\right]^2 \\ &\quad + \beta_4 \left[\log RNs_i\right]^2 + \beta_5 \left[\log Surgeons_i\right]\left[\log RNs_i\right] \\ &\quad + g\left[\log MDs_i, \log CRNAs_i\right] + u_i. \end{aligned} \tag{17}$$

The flexibility of this specification allows the data to show how hospitals substitute MDs and CRNAs to produce the surgeries that are demanded. It ignores possible interactions of MDs and/or CRNAs with surgeons and/or RNs. But a) such interactions are probably small in the process of producing surgeries, b) such interactions are not really the focus of this study, and c) the exclusion of such interactions from consideration has a very large advantageous impact on the statistical precision of the model estimates.

The estimates of the linear coefficients are reported in Table 2.

Table 2
Semiparametric Parameter Estimates

Variable	Estimate	Std. Error
$\log Surgeons$	0.523*	0.061
$\log RNs$	0.127*	0.034
$[\log Surgeons]^2$	0.019*	0.005
$[\log RNs]^2$	0.017*	0.006
$[\log Surgeons]\,[\log RNs]$	-0.034*	0.012

The results for surgeons and RNs suggest that:

1. A 10% increase in surgeons, holding all other inputs fixed, results in a 5.23% increase in surgeries with a small, positive (0.019) quadratic effect and small interaction effect with RNs.
2. A 10% increase in RNs, holding all other inputs fixed, results in a 1.27% increase in surgeries with a small, positive (0.017) quadratic effect and small interaction effect with surgeons.

Figure 4 shows the contribution to log surgeries of log surgeons and log RNs graphically; i.e, it plots the quadratic part. It shows that, as either log surgeons or log RNs increase, log surgeries increase. The boundaries of the different regions in the figure map out sequences of isoquants. The surprising feature of the picture is the isoquants' concavity. This occurs because the estimate of the

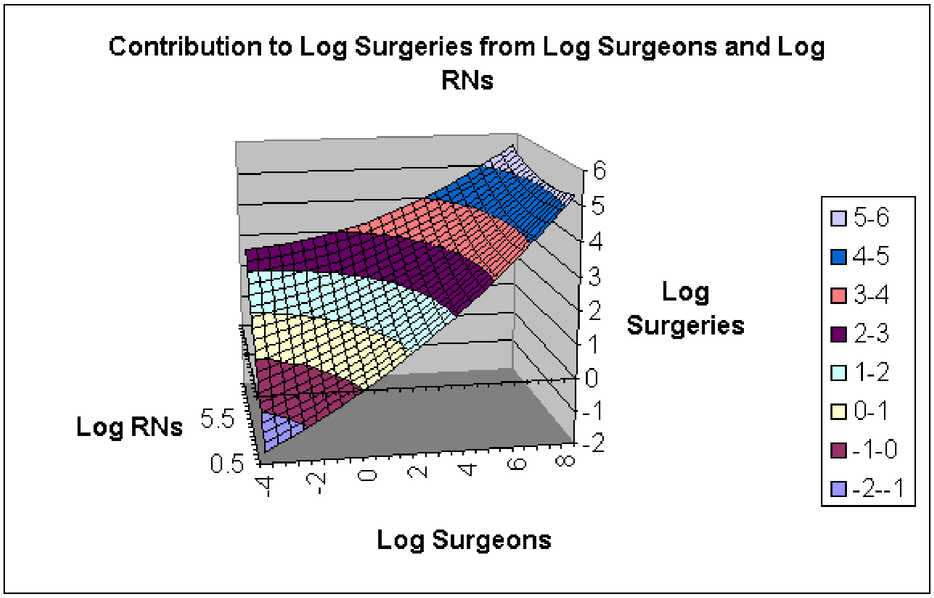

Figure 4:

coefficient on the interaction term is negative. This suggests that, to a small degree, surgeons and RNs get in each other's way in producing surgeries.

The important issue here is how MDs and CRNAs affect surgeries. This is captured by the estimate of the $g[\cdot,\cdot]$ function from equation (17) in Figure 5. It shows how the contribution to log surgeries changes with log MDs and log CRNAs. Each point in the figure is $\widehat{g}$ at a particular combination of log MDs and log CRNAs evaluated using equation (15). The slope of the curve measures the effect of an additional input on the number of surgeries to be performed. Note that surgeries are increasing in both MDs and CRNAs, suggesting that both are productive inputs in the production of surgeries. As in Figure 4, the boundaries of the different regions are estimated isoquants. Note how well behaved the isoquants are despite the lack of restrictions on their shape imposed by the estimation procedure.

A more informative picture might focus on partial slopes of $\widehat{g}$ (estimated marginal products) rather than $\widehat{g}$ itself. First, the partial slopes are more important for economic analysis. Second, given the results of Theorems 1 and 2, the ratio of slopes is less likely sensitive to bias caused by unobserved heterogeneity in θ. Figure 6 shows how the ratio of the slope for CRNAs to the slope for MDs is changing with the mix of CRNAs and MDs. This is the negative of the slopes of the isoquants in Figure 5. Note that CRNAs allow more

---

[18] However, given the results of Theorem 1, it may be the case that all of the estimates are biased upwards.

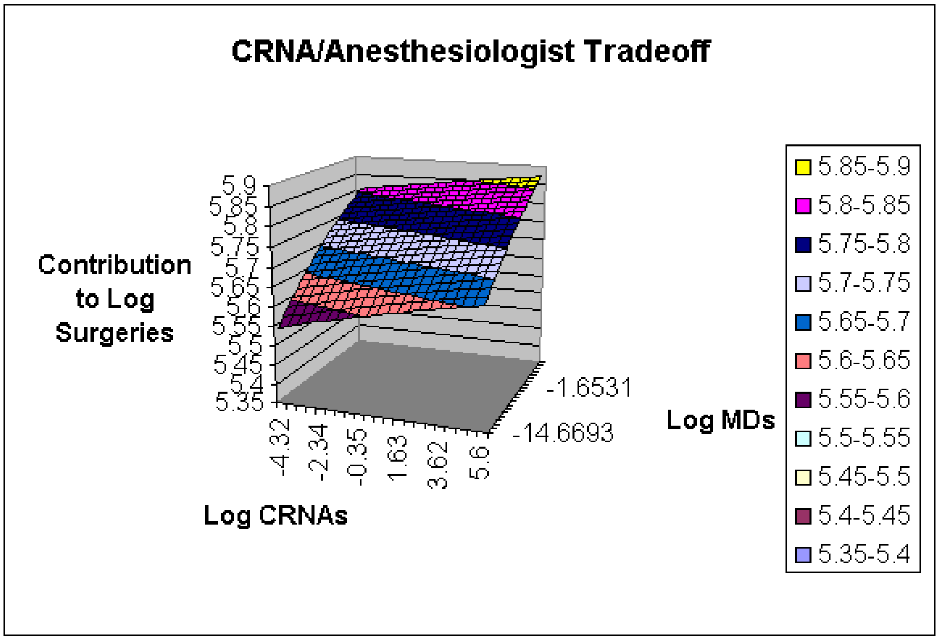

Figure 5:

surgeries to occur relative to MDs as the ratio of CRNAs to MDs decreases, and MDs allow more surgeries to occur relative to CRNAs as the ratio of CRNAs to MDs increases. This result is a standard assumption made in most production function analyses.

The mean log MDs is -1.1, and the mean log CRNAs is 0.74.[19] In this range, the ratio of slopes $(\frac{\partial \hat{g}}{\partial CRNA}/\frac{\partial \hat{g}}{\partial MD})$ is about 0.11. At the modal county, with 5.5 MDs and 5.5 CRNAs, the ratio is about 0.58. In more populated counties, where there are, for example, 55 MDs and 55 CRNAs, the ratio is approximately 0.43. The implication of these figures is that MDs and CRNAs are substitutes for each other but with decreasing effect on the number of surgeries as their relative numbers increase. The weighted average slope for MDs is 1.58 with a standard error of 0.07, and the weighted average slope for CRNAs is 0.99 with a standard error of 0.07. The weighted average log ratio of slopes for MDs to slopes for CRNAs is 0.20 with a standard error of 0.81. A log ratio of 0.20 translates into a ratio of 1.22. This means that the last MD hired causes, on average, 22% more surgeries to be performed than the last CRNA hired. The standard errors of the estimates of log ratio of slopes are relatively large. So it is inappropriate to put much weight on any particular point estimate. For example, a 90% confidence interval for the weighted average log ratio of slopes

---

[19] This corresponds to MDs equal to 0.36 per county and CRNAs equal to 2.1 per county.

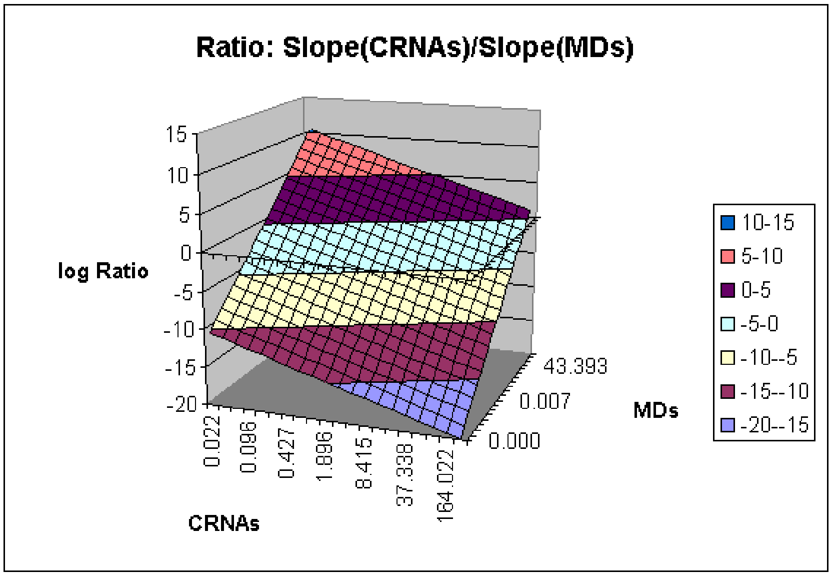

Figure 6:

is from -1.38 to 1.81.[20] However, it is very clear that MDs and CRNAs exhibit substitution characteristics with decreasing returns holding constant the other input. In particular, in those counties with many CRNAs and few MDs, the ratio is very large; one extra MD would significantly increase the number of surgeries. This is consistent with published goals to encourage MDs to move to rural areas. Also, in those counties with many MDs and few CRNAs, the ratio is very small; one extra CRNA would significantly increase the number of surgeries. Overall, the results strongly imply that MDs and CRNAs are able to produce more surgeries in a setting in which MDs and CRNAs are both utilized.

6.2 Hospital Data

6.2.1 Ordinary Least Squares Estimates

The production function also was estimated with data from the survey of hospitals using OLS and a semiparametric model. The hospital data did not need smoothing because both the number of surgical procedures and the quantity of inputs at each hospital were observed directly; there was no cross-hospital movement of patients or medical health professionals as was the case with the

---

[20] This corresponds to a range for the ratio of slopes from 0.25 to 6.11.

county data.[21] The OLS equation estimates were[22]

$$\begin{aligned}\log Surgeries_i \quad = \quad & \underset{(0.175)}{6.41^*} + \underset{(0.058)}{0.250^*} \log Surgeons_i + \underset{(0.083)}{0.492^*} \log MDs_i \\ & + \underset{(0.067)}{0.264^*} \log CRNAs_i + \underset{(0.008)}{0.014^*} \left[\log Surgeons_i\right]^2 \\ & + \underset{(0.008)}{0.031^*} \left[\log MDs_i\right]^2 + \underset{(0.007)}{0.007} \left[\log CRNAs_i\right]^2 \\ & - \underset{(0.010)}{0.025^*} \left[\log Surgeons_i\right] \left[\log MDs_i\right] \\ & - \underset{(0.008)}{0.009} \left[\log Surgeons_i\right] \left[\log CRNAs_i\right] \\ & - \underset{(0.004)}{0.078^*} \left[\log MDs_i\right] \left[\log CRNAs_i\right] + u_i\end{aligned}$$

with an R^2 of 0.87. As was true in the county data, surgeons have a significant, positive effect on the number of surgeries. Now, however, both MDs and CRNAs also have a significant positive effect. The coefficient estimates on the surgeon variables here are smaller than they were when county data was used. This is surprising in that a) in this specification, surgeons should be picking up some of the effect of RNs, which was excluded before, and b) there is probably less measurement error in the surgeon variable in the hospital data than in the smoothed county data.

6.2.2 Semiparametric Estimates

The estimates of the linear terms are

$$\begin{aligned}\log Surgeries_i \quad = \quad & \underset{(0.082)}{0.395^{**}} \log Surgeons_i + \underset{(0.013)}{0.004} \left[\log Surgeons_i\right]^2 \\ & + g\left[\log MDs_i, \log CRNAs_i\right] + u_i.\end{aligned}$$

As was true in Table 2, the returns to surgeons is positive but not as large as in the county data.

The results of greatest concern are the estimates of the relative slopes of MDs and CRNAs, the $g[\cdot,\cdot]$ function in equation (17). As before, both the estimated slopes and the log ratio of slopes can be plotted. Figure 7 shows the contribution of MDs and CRNAs to surgeries; it is directly comparable to Figure 5. As before, the two horizontal axes correspond to the numbers of MDs and CRNAs, and the vertical axis is the contribution to surgeries. The spike down in the front is the only significant part of the picture; all of the other variation in the function is second order relative to the spike. This spike corresponds to hospitals with no MDs or CRNAs. The picture shows that such hospitals produce significantly fewer surgeries (holding constant the number of surgeons) than hospitals with some MDs and/or CRNAs.

---

[21] RNs are not included in this equation because information about RNs was not collected as part of the survey.

[22] Numbers in parentheses are standard errors, and starred items are significant at the 5% level.

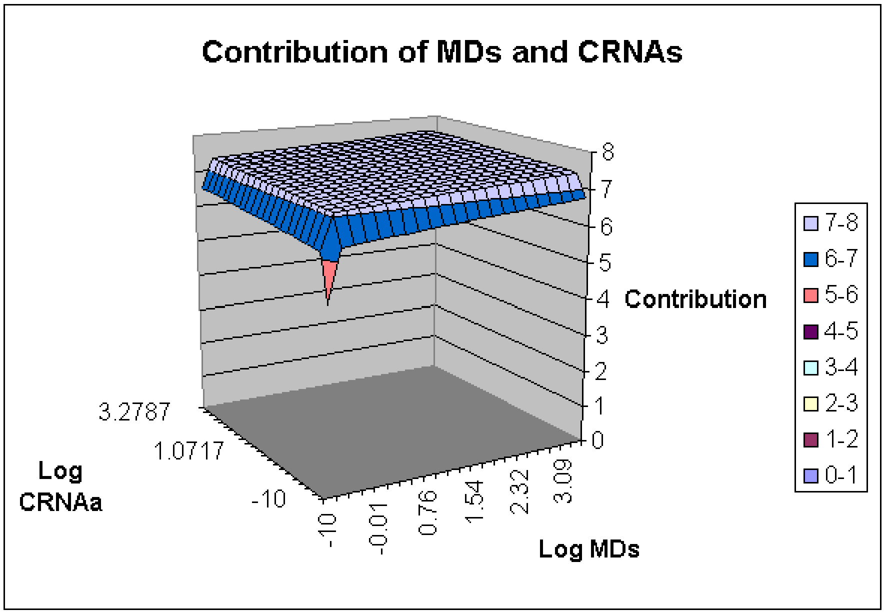

Figure 7:

Figure 8 shows the same picture conditional on having a positive number of MDs and CRNAs so that it is apparent how MDs and CRNAs affect surgeries at all but the smallest hospitals. Now it becomes clear that again both MDs and CRNAs contribute to the production of surgeries (because the surface is increasing in height as the number of either MDs or CRNAs increases). Again, the lines separating the different colors are estimated isoquants.

The slopes over different combinations of MDs and CRNAs are compared in Figure 9 (which is directly comparable to Figure 6). Again, both inputs exhibit decreasing returns to scale and have their greatest contribution when both are utilized. This is seen by the fact that, in hospitals with a large ratio of MDs to CRNAs, the slope for CRNAs Figure 9 is much greater than the slope for MDs, and, in hospitals with a small ratio of MDs to CRNAs, the slope for CRNAs is much smaller than the slope for MDs. The average slope for MDs is 24.7, and the average slope for CRNAs is 29.5. Both are estimated with small standard errors. This suggests that CRNAs allow 19% more surgeries to occur than MDs. While this result differs from the result using county data (where MDs allowed 22% more surgeries to occur than CRNAs), together they suggest that MDs and CRNAs have relatively similar effects on the production of surgeries.[23]

---

[23] Note that in Figure 9 the log ratio of slopes shoots up for hospitals with a large number of CRNAs and shoots down for hospitals with a large number of MDs. More than likely, this is just a feature of semiparametric estimation near the boundaries of the data, and not much significance should be placed on it.

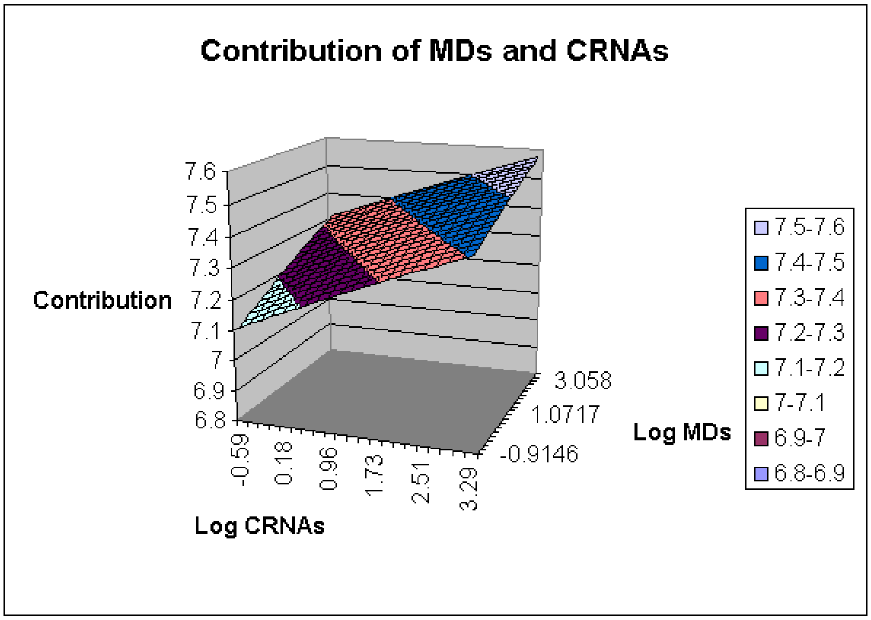

Figure 8:

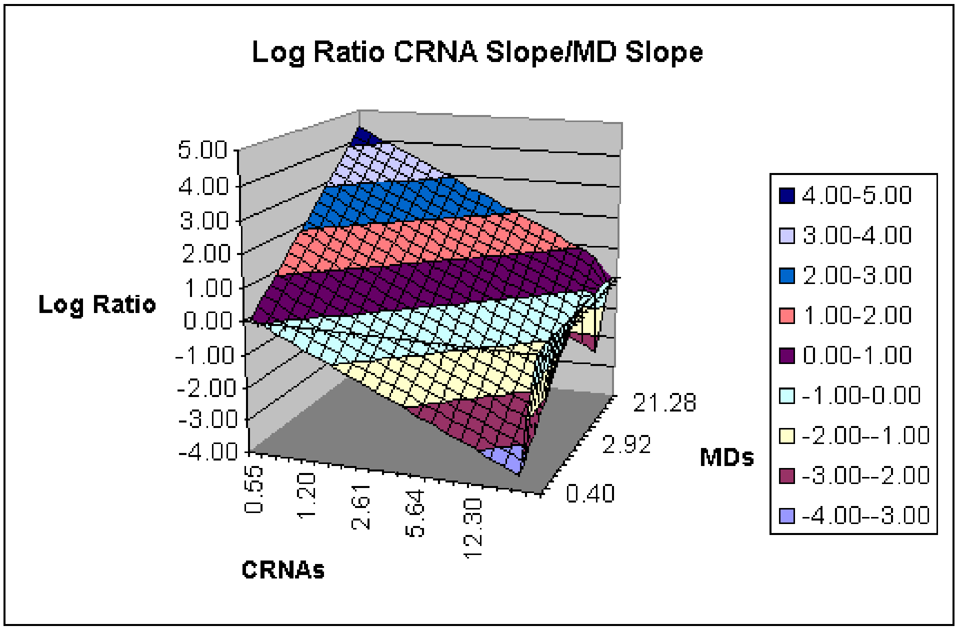

Figure 9:

7 Conclusions

The best use of MDs and CRNAs depends upon the cost of each. If the pricing of MDs and CRNAs were as simple as the wage or salary (plus fringe benefits) paid to them, then the results would suggest that MDs and CRNAs should be present in each organization as is the case in many of the hospitals surveyed. However, the issue is greatly complicated by Medicare and other insurance reimbursement rules. If, for example, hospitals are allowed to pass through to Medicare the whole cost of the MD but only a portion of the cost of the CRNA, then hospitals will behave as if MDs are free and CRNAs are expensive. This will cause hospitals to use an inefficient mix of the two inputs that is disproportionately weighted towards the "free" input, MDs.

From a societal viewpoint, the results suggest an inefficient allocation of scarce medical resources. In particular, the nation has two inputs (CRNAs and MDs) that estimates suggest are approximately equally effective at producing more surgeries. There is no literature that suggests one is safer than the other. Yet, the cost of MDs to society is more than twice that of CRNAs.[24] At an organizational level, this occurs because of inefficient reimbursement rules that favor MDs. The estimation results strongly suggest that reimbursement rules should be changed to encourage more efficient usage of available anesthesia providers. Note that the results do not suggest that MDs should play no role in anesthesia provision. In fact, the results strongly point to decreasing returns to both inputs, implying that both should be used. However, the nation should move toward more usage of CRNAs because they are the more cost efficient providers at this time.

---

[24] In 2000, the median salary for MDs was $250,000 (American Medical Association, 2003), and the median salary for CRNAs was $102,000.

8 Appendices

8.1 Adjustment of the Covariance Matrix for Nearby County Correlations

Squaring equation (13) results in

$$\begin{aligned}
Cov\left(\widehat{\alpha}-\alpha\right) &= plim\left[\frac{1}{n}\sum_i -\widetilde{Z}_i'\varphi_{i\alpha}\left(\widehat{\alpha}\right)\right]^{-1}\cdot \\
&\quad plim\left[\frac{1}{n}\sum_i\sum_j \widetilde{Z}_i'\varphi_i\left(\widehat{\alpha}\right)\varphi_j\left(\widehat{\alpha}\right)\widetilde{Z}_j'\right] plim\left[\frac{1}{n}\sum_i \varphi_{i\alpha}\left(\widehat{\alpha}\right)'\widetilde{Z}_i\right]^{-1} \\
&= plim\left[\frac{1}{n}\sum_i -\widetilde{Z}_i'\varphi_{i\alpha}\left(\widehat{\alpha}\right)\right]^{-1}\cdot \\
&\quad plim\left[\frac{1}{n}\sum_i\sum_j \sigma_{ij}\widetilde{Z}_i'\widetilde{Z}_j'\right] plim\left[\frac{1}{n}\sum_i \varphi_{i\alpha}\left(\widehat{\alpha}\right)'\widetilde{Z}_i\right]^{-1}
\end{aligned}$$

where $\sigma_{ij} = E\varphi_i\left(\widehat{\alpha}\right)\varphi_j\left(\widehat{\alpha}\right)$. In the earlier analysis, we assume that $\sigma_{ij} = 0$ if $i \neq j$. But, σ_{ij} may be nonzero because a) our smoothing methodology causes correlation among geographically nearby counties and b) the errors in $\varphi\left(\widehat{\alpha}\right)$ may have been correlated naturally because of geographical proximity. The possibility of (b) suggests that a straightforward correction controlling for the correlation induced by smoothing would not be sufficient. An alternative, following the lead of Newey and West (1987), is to estimate

$$\widehat{\sigma}_{ij} = \begin{cases} \varphi_i\left(\widehat{\alpha}\right)\varphi_j\left(\widehat{\alpha}\right) & \text{if } \phi\left(d_{ik}\right) > 0 \\ 0 & \text{if } \phi\left(d_{ik}\right) = 0 \end{cases}.$$

While $\widehat{\sigma}_{ij}$ is not a consistent estimate of σ_{ij}, as in Newey and West (1987), $\left[\frac{1}{n}\sum_i\sum_j \widehat{\sigma}_{ij}\widetilde{Z}_i'\widetilde{Z}_j'\right]$ is a consistent estimator of $plim\left[\frac{1}{n}\sum_i\sum_j \sigma_{ij}\widetilde{Z}_i'\widetilde{Z}_j'\right]$ for the same reasons.

8.2 Covariance Matrix for $\widehat{\psi}$

Let $x = (A, N)$. Starting from Pagan and Ullah (1999), Theorem 3.5,

$$(nh)^{1/2}\left[\widehat{\psi}\left(x\right) - E\widehat{\psi}\left(x\right)\right] = \widehat{f}^{-1}\left(0\right)(nh)^{-1/2}\sum_i K_i\left(x\right)u_i.$$

Therefore,

$$\begin{aligned}
& Cov\left[\widehat{\psi}\left(x_1\right), \widehat{\psi}\left(x_2\right)\right] \\
= \quad & \widehat{f}^{-2}\left(0\right) plim\left[\left(nh\right)^{-1}\sum_i\sum_j K_i\left(x_1\right)K_j\left(x_2\right)u_iu_j\right] \\
= \quad & \widehat{f}^{-2}\left(0\right)\left[\left(nh\right)^{-1}\sum_i\sum_j K_i\left(x_1\right)K_j\left(x_2\right)\sigma_{ij}\right].
\end{aligned}$$

Note that, if $\sigma_{ij} = 0$ for all $i \neq j$, then $Cov\left[\widehat{\psi}\left(x_1\right), \widehat{\psi}\left(x_2\right)\right] = 0$ for all $x_1 \neq x_2$ because $K_i\left(x_1\right)K_i\left(x_2\right) = 0$.

Define

$$\psi^*\left(x\right) = \begin{pmatrix} \widehat{\psi}\left(x_1, x_2\right) \\ \widehat{\psi}\left(x_1+\delta_1, x_2\right) \\ \widehat{\psi}\left(x_1, x_2+\delta_2\right) \end{pmatrix}$$

Consider

$$r\left(\psi^*\left(x\right), x\right) = \log\left[\frac{\frac{\exp\{\psi(x_1+\delta_1,x_2)\}-\exp\{\psi(x_1,x_2)\}}{\exp\{x_1+\delta_1\}-\exp\{x_1\}}}{\frac{\exp\{\psi(x_1,x_2+\delta_2)\}-\exp\{\psi(x_1,x_2)\}}{\exp\{x_2+\delta_2\}-\exp\{x_2\}}}\right]$$

with estimator

$$\widehat{r}\left(\psi^*\left(x\right), x\right) = \log\left[\frac{\frac{\exp\left\{\widehat{\psi}(x_1+\delta_1,x_2)\right\}-\exp\left\{\widehat{\psi}(x_1,x_2)\right\}}{\exp\{x_1+\delta_1\}-\exp\{x_1\}}}{\frac{\exp\left\{\widehat{\psi}(x_1,x_2+\delta_2)\right\}-\exp\left\{\widehat{\psi}(x_1,x_2)\right\}}{\exp\{x_2+\delta_2\}-\exp\{x_2\}}}\right].$$

Then

$$Var\left[\widehat{r}\left(\psi^*\left(x\right), x\right)\right] = r_{\psi}^{'}\left(x\right)Cov\left[\psi^*\left(x\right)\right]r_{\psi}\left(x\right)$$

where

$$\begin{aligned}
r_{\psi}\left(x\right) \quad &= \quad \frac{\partial r\left(x\right)}{\partial\psi^*\left(x\right)} = \\
&= \quad \begin{pmatrix} \frac{-[1-r(x)]}{\exp\left\{\widehat{\psi}(x_1+\delta_1,x_2)-\widehat{\psi}(x_1,x_2)\right\}-1} \\ \frac{\exp\left\{\widehat{\psi}(x_1+\delta_1,x_2)-\widehat{\psi}(x_1,x_2)\right\}}{\exp\left\{\widehat{\psi}(x_1+\delta_1,x_2)-\widehat{\psi}(x_1,x_2)\right\}-1} \\ -\frac{\exp\left\{\widehat{\psi}(x_1,x_2+\delta_2)-\widehat{\psi}(x_1,x_2)\right\}}{\exp\left\{\widehat{\psi}(x_1,x_2+\delta_2)-\widehat{\psi}(x_1,x_2)\right\}-1} \end{pmatrix}
\end{aligned}$$

and $Cov\left[\psi^*\left(x\right)\right]$ is given above.

References

[1] American Medical Association. (2003). *Physician Socioeconomic Statistics.*

[2] Barnett, William and Jane Binner (2004). *Functional Structure and Approximation in Econometrics.* Forthcoming.

[3] Barnett, William and Yul Lee (1985). "The Global Properties of the Minflex Laurent, Generalized Leontief, and Translog Flexible Functional Forms." *Econometrica.* 53(6): 1421-1438.

[4] Bolduc, D., R. Laferrière, and G. Santarossa (1992). "Spacial Autoregressive Error Components in Travel Flow Models." *Regional Sciences and Urban Economics.* 22: 371-385.

[5] Christensen, L., D. Jorgenson, and L. Lau (1971). "Conjugate Duality and the Transcendental Logarithmic Production Function." *Econometrica.* 39: 255-256.

[6] Christensen, L., D. Jorgenson, and L. Lau (1973). "Transcendental Logarithmic Production Frontiers." *Review of Economics and Statistics.* 55: 28-45.

[7] Conley, Timothy (1999). "GMM Estimation with Cross-Sectional Dependence." *Journal of Econometrics.* 92(1): 1-45.

[8] Conley, Timothy and Giorgio Topa (2004). "Identification in Local Interaction Models with Imperfect Location Data." *Journal of Applied Econometrics.* Forthcoming.

[9] Diewert, W. (1971) "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function." *Journal of Political Economy.* 79: 481-507.

[10] Diewert, W., and T. Wales (1987). "Flexible Functional Forms and Global Curvature Conditions." *Econometrica.* 55: 43-68.

[11] Heckman, James (2000). "Causal Parameters and Policy Analysis in Economics: A Twentieth Century Retrospective." *Quarterly Journal of Economics.* 115: 45-97.

[12] Ichimura, Hidehiko and Lung Fei Lee (1991). "Semiparametric Least Squares Estimation of Multiple Index Models: Single Equation Estimation." in *Nonparametric and Semiparametric Methods in Econometrics and Statistics.* (eds.) William Barnett, James Powell, and George Tauchen. Cambridge: Cambridge University Press.

[13] Li Q., C. Huang, D. Li, and T. Fu (2002). "Semiparametric Smooth Coefficient Models." *Journal of Business and Economic Statistics.* 20(3): 412-422.

[14] Medical Marketing Service (2003). www.mmslists.com.

[15] Merwin, Elizabeth, Steven Stern, and Lorraine Jordan (2004). *Supply and Demand of Nurse Anesthetists: Present and Future Projections.* Report to the American Association of Nurse Anesthetists.

[16] Mundlak, Yair (1978). "On the Pooling of Time Series and Cross Section Data." *Econometrica.* 46(1): 69-85.

[17] Newey, Whitney and Kenneth West (1987). "A Simple, Positive Semidefinite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica.* 55(3): 703-708.

[18] NHS, Centre for Reviews and Dissemination (1996). "Concentration and Choice in the Provision of Hospital Services." CDR Report 8. York: University of York.

[19] Olley, G. and A. Pakes (1996). "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica.* 64(6): 1263-1297.

[20] Pagan, Adrian, and Aman Ullah (1999). *Nonparametric Econometrics.* Cambridge: Cambridge University Press.

[21] Robinson, Peter (1988). "Root- N-Consistent Semiparametric Regression." *Econometrica.* 56(4): 931-954.

[22] Sickles, Robin (2003). "Panel Estimators and the Identification of Firm-Specific Efficiency Levels in Parametric, Semiparametric and Nonparametric Settings." Rice University manuscript.

[23] Smet, Mike (2002). "Cost Characteristics of Hospitals." *Social Science & Medicine.* 55(6): 895-906.

[24] Thurston, N. and A. Libby (2002). "A Production Function for Physician Services Revisited." *Review of Economics and Statistics.* 84(1): 184-191.