

Firm Dynamics within an Urban Economy*

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Abstract

We develop a new dynamic general equilibrium model to explain firm entry, exit, and relocation decisions in an urban economy with multiple locations. We characterize the stationary distribution of firms that arises in equilibrium. The parameters of the model can be estimated using a two-step estimator. We implement the estimator using unique panel data collected by Dun and Bradstreet for the Pittsburgh metropolitan area. Firms located in the central business district are older and larger than firms located outside the urban core. They use more land and labor in the production process. However, they face higher rental rates for office space which implies that they operate with a higher employee per land ratio. Firms that enter in the city center are typically larger than those that enter in the urban fringe. Our estimates imply that agglomeration externalities increase the productivity of firms by one to two percent. Economic policies that subsidize firm relocations can potentially have large effects on economic growth and firm concentration in central business districts.

1 Introduction

Cities play an important role in the economy since economic proximity makes for more efficient production and trade. These efficiency gains typically arise because of agglomeration externalities.¹ Firms that operate in locations with high externalities have a competitive advantage over firms that are located in less efficient locations. Since firms will bid for the right to locate in areas with high agglomeration externalities, these locations have higher rental prices for land than locations that are less efficient.² As a consequence, firms with different productivity levels will sort in equilibrium with high productivity firms locating in areas with high agglomeration effects and high rental prices for land. Low productivity firms are forced to exit the economy or operate in cheaper locations.

Firm sorting arises for three different reasons. First, entry pattern depend on location specific characteristics which gives rise to an initial sorting of new firms. Second, the productivities of firms change over time which creates incentives to relocate within a city to exploit a better match with the agglomeration externalities.³

¹The idea of geographic returns to scale was first introduced by von Thünen (1825). Marshall (1890) suggested that agglomeration effects may exist within industries. Firms may benefit from lower transaction costs or sharing of a common labor pool. Alternatively, efficiencies may arise due to positive diversity externalities and synergies between different industries (Jacobs, 1969).

²Krugman (1991) provides theoretical foundations for a two-location model of agglomeration. Ellison and Glaeser (1997) argued that agglomeration externalities are important to understand geographic concentration of manufacturing in the U.S. The literature of agglomeration theory is reviewed in Fujita and Thisse (2002) and Duranton and Puga (2004). Anas and Kim (1996) and Lucas and Rossi-Hansberg (2002) have developed equilibrium models of mono- and poly-centric urban land use with endogenous congestion and job agglomeration. Rossi-Hansberg (2004) studies optimal land use policies in a similar framework.

³There is some evidence that shows that agglomeration effects are important to understand firm dynamics. Henderson, Kunkoro, and Turner (1995) show that agglomeration effects for mature industries are related to Marshall scale economies, while newer industries benefit from diversity akin to Jacobs economies. This work is important because it points to agglomeration as part of a dynamic process. Other research has continued to study the relevance of agglomeration in firm life-cycle dynamics. Duranton and Puga (2001) study the effect of agglomeration externalities in innovation and the development of production processes, while Dumais, Ellison, and Glaeser (2002) examine the effect of firm dynamics (entry, exit, expansion, and contraction) on the concentration

Finally, exit rates may depend on location as well since the continuation value for a firm is location specific.

We develop a new dynamic general equilibrium model of firm location choice that can explain the sorting of firms by productivity as well as entry, exit, and relocation decisions of firms in an urban economy with multiple locations. We consider a model with two distinct locations which can be interpreted as inside and outside of a central business district (CBD). In equilibrium these locations differ in the magnitude of their agglomeration externalities which increase with employment density.⁴ Firms are heterogeneous in their productivities. We model firm dynamics and industry equilibrium following Hopenhayn (1992). Firms enter our urban economy with an initial productivity and must pay an entry cost. Productivity then evolves according to a stochastic first order Markov process. Each period firms compete in the product market, must pay a fixed cost of operating, and realize a profit.⁵

We characterize the optimal decision rules for firms in each location as well as those for potential entrants. Low productivity firms exit from the economy, while high productivity firms continue to operate. Relocation choices are driven by the interaction of agglomeration effects and firm productivity shocks. If fixed costs of operation are larger in the city center, large firms with higher productivity shocks

of economic activity.

⁴Deckle and Eaton (1999) find that geographic scale of agglomeration is mostly at the national level, while the financial sector is concentrated in specific metropolitan areas. Other work finds that agglomeration can occur on a much more local scale. In particular, Rosenthal and Strange (2001, 2003) establish the level and type of agglomeration at different geographic scales, and also the measure the attenuation of these externalities within metropolitan areas. Holmes and Stevens (2002) finds evidence of differences in plant scale in areas of high concentration, suggesting production externalities act on individual establishments. A review of empirical evidence of agglomeration economies is found in (Rosenthal and Strange, 2004).

⁵We abstract from innovation which is discussed in detail in Klette and Kortum (2004). Our work is also related to Ericson and Pakes (1995) who consider the implications of oligopolistic competition on market structure. That framework is more appropriate when there are few competitors in the industry. Pakes and McGuire (1994, 2001) discuss how to solve models with oligopolistic competition. Doraszelski and Pakes (2006) provide a survey of that literature.

prefer locations with high agglomeration externalities relative to smaller, less productive firms. As a consequence, a high productivity firm that is located outside the central business district may have strong incentives to relocate to the city center. Entry and exit rules are also location specific and the threshold for entry is higher for firms in the city center. Moreover, the combination of fixed operating costs and high rents in the city center implies, that the exit threshold is also higher in the city than outside the city center.⁶ Low productivity firms leave and move to a location outside the central business district. This process gives rise to a stationary equilibrium in which firms located in the CBD are on average larger and older than firms located outside the CBD.⁷

We then develop a two stage algorithm that can be used to estimate the parameters of our model. We show that a subset of the structural parameters of the model can be estimated using the observed input and output choices without solving for the equilibrium of the model. The remaining parameters of the model which include the cost parameters affect the equilibrium selection rules and are estimated using a nested fixed point algorithm. The inner loop computes the equilibrium for each parameter value, while the outer loop searches over feasible parameter values. The second stage estimator is a simulated method of moments estimator which matches selected moments characterizing entry, exit and relocation of firms within the metropolitan area.⁸

⁶There are some similarities with the literature that studies the sorting pattern of household in urban areas which starts with the classic papers by Alonso (1964), Mills (1967), and Muth (1969).

⁷Related to our research is also work by Melitz (2003) who studies the impact of trade on intra-industry relocations. Rossi-Hansberg and Wright (2007) examine the relationship of establishment scale and entry and exit dynamics. Finally, Combes, Duranton, Gobillon, Puga, and Roux (2010) distinguish between selection effects and productivity externalities by estimating productivity distributions across cities.

⁸In related work, Davis et al. (2009) develop a growth model in which the total factor productivity of cities depends on the density of economic activity. They estimate the magnitude of this external effect and evaluate its importance for the growth rate of consumption per capita in the U.S. Our paper is thus also related to a growing literature in industrial organization that estimates dynamic models of oligopolistic competition. See, for example, Benkard (2004), Bajari, L. Benkard, and Levin (2007), Aguirregabiria and Mira (2007).

We implement the estimator using unique panel data collected by Dun and Bradstreet for the Pittsburgh metropolitan area. U.S. cities often act as a hub for services for a larger region. We, therefore, focus on locational choices within the service sector excluding industries in which proximity to the consumer is a key factor for firm location. The data suggest that firms located in the city are older and larger than firms located in the rest of the metro area. As a consequence they use more land and labor in the production process. However, they face higher rental rates for land and office space. Thus, they operate with a higher employee per land ratio. We find that our model explains these observed features of the data reasonably well. Our estimates imply that agglomeration externalities increase the productivity of firms by one to two percent. Economic policies that subsidize firm relocations can potentially have large effects on economic growth and firm concentration in central business districts.

The rest of the paper is organized as follows. Section 2 describes the data set used in our application and characterizes firm sorting within one metropolitan area. Section 3 develops our stochastic, dynamic equilibrium model and discusses its properties. Section 4 describes the estimation of the parameters of our model. Section 5 presents the empirical results and discusses the policy experiments. Section 6 offers some conclusions that can be drawn from the analysis.

2 Data

Our empirical application focuses on firm location choices in the City of Pittsburgh and Allegheny County.⁹ We are interested in characterizing the observed sorting of establishments as well as entry and exit rates by age, output, employment, and facility

⁹In Appendix B of the paper we show that most other large metropolitan areas in the U.S. show sorting patterns of firms that are similar to the one we find for Pittsburgh. The comparison is based on aggregate Census data while the estimation of our model uses micro level data from Dun and Bradstreet.

size.¹⁰ We focus on service industries, given that there is strong evidence that large U.S. cities have undergone a transformation during the past decades moving from centers of individual manufacturing sectors toward becoming hubs for service industries. Duranton and Puga (2005), for example, show evidence that cities have become more functionally specialized, with larger cities, in particular, emerging as centers for headquarters and business services. They posit that this change is primarily related to industrial structure, and a decrease in remote management costs in particular. Davis and Henderson (2008) provide further evidence that services and headquarters are indeed more concentrated in large cities relative to the entire economy, and that headquarter concentration is linked to availability of diverse services.

We exclude wholesale and retail businesses from our analysis of services since locational decisions of these businesses are primarily driven by proximity to consumers (Hotelling, 1929).¹¹ For similar reasons, we also do not consider businesses in the entertainment sector. Finally, we omit businesses related to agriculture, forestry, mining and fishing for fairly obvious reasons. We thus define the service sector as consisting of businesses that operate in information, finance, real estate, professional services, management, administrative support, education, health care and related sectors.

Figure 1 plots the employment concentration in Allegheny County using data from the U.S. Census. Over 20 percent of employment is concentrated in three zip codes in the center of Pittsburgh which include the downtown central business district and the business district in Oakland which are the two significant dense commercial areas of Pittsburgh.¹² We treat these locations as the CBD in estimation while all remaining

¹⁰While we use the terms "firm" and "establishment" interchangeably, our unit of analysis in the empirical section is an establishment.

¹¹Following Bresnahan and Reiss (1991), there is a large literature that explains entry and exit into markets with a small number of potential entrants. Holmes (2010) has estimated a dynamic model of market penetration of Walmart.

¹²The zip codes are 15222, 15219, and 15213.

places of Allegheny County are treated as the alternate location (NCBD). None of the results reported in this paper rely on this definition of the alternate location. We can, for example, omit those parts of Allegheny county that have little economic development and obtain similar results regarding firm sorting.

We use firm level data from Dun and Bradstreet's Million Dollar Database to estimate our model.¹³ This database covers establishments in Allegheny county in two years, 2008 and 2011. The database provides detailed information on establishments. The coverage is near universal compared to Census counts of establishments in the county. The database provides data on location, facility size, total employment, industry, and year established.¹⁴

We analyze the employment and facility size characteristics for different industries in the Pittsburgh area. Table 1 reports the total employment, the average employment and the facility space per employee for firms in and outside the CBD for selected service industries in 2008.

Table 1 shows that 28.66 percent of all employment in the service industry is located in the CBD or 13.43 percent of all firms in the sample. However, the three zip codes that comprise the city account for a less than one percent of all the land in Allegheny county. We find that finance, education, and professional services are the industries that are most heavily concentrated in the CBD.

Comparing firms that are located inside the CBD with firms that are outside the CBD, we find some important patterns that hold for all service industries. The average employment size of establishments is larger in the central business district. The average establishment in the CBD employs 27 persons while the average firm

¹³Information on Dun and Bradstreet data is available on-line at <http://www.dnbmdd.com/>

¹⁴While most of the data are complete, the year established field, which is used to determine age of establishments, is only available for 52.5 percent of the observations. However, we find little evidence that the missing data field is systematically correlated with other observable data. We, therefore, treat observations without age of establishment as missing at random.

Figure 1: Allegheny County and the City of Pittsburgh

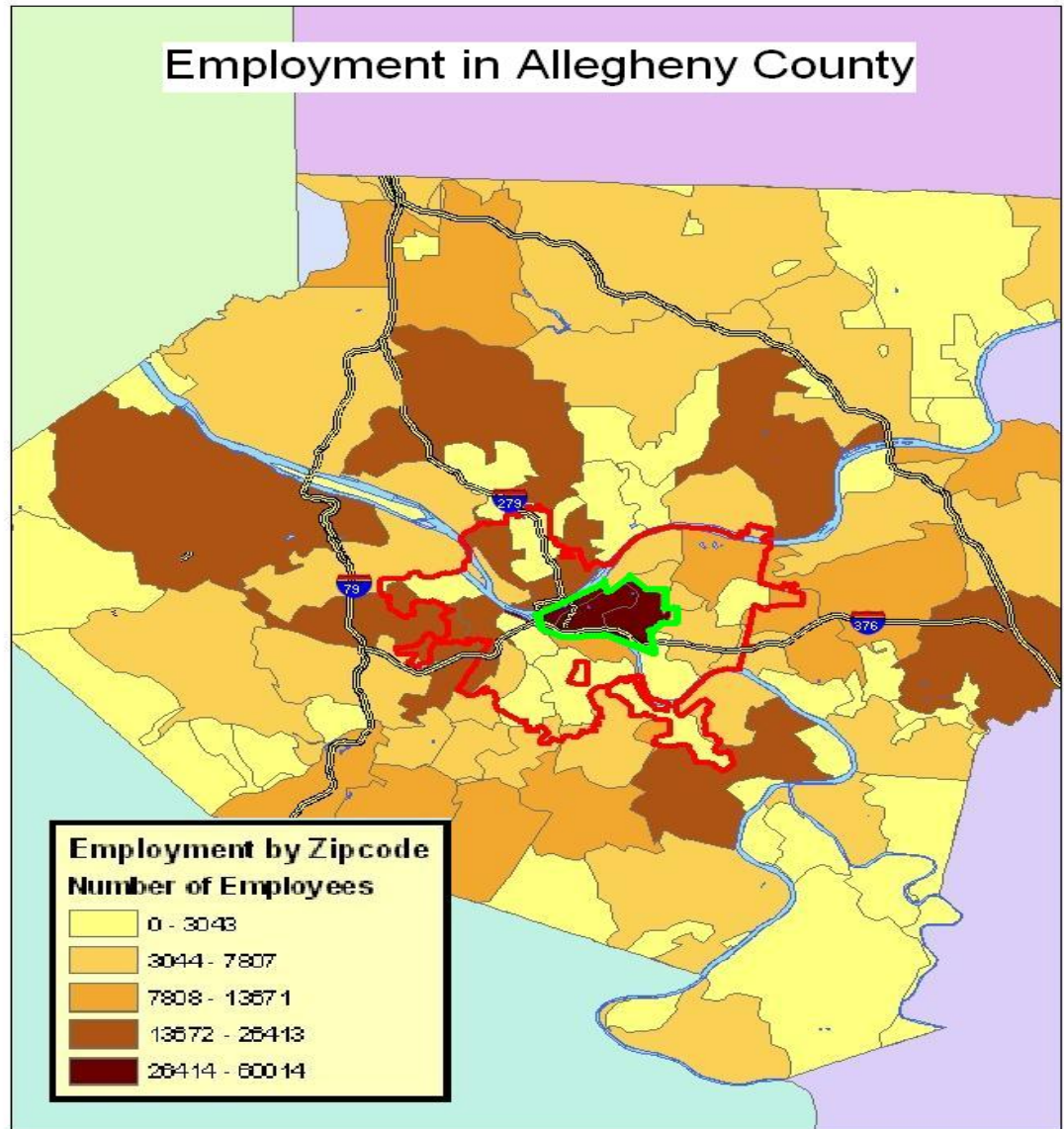


Table 1: Employment and facility size by industry in 2008

| NAICS | Total Emp. | % Emp CBD | Size NCBD | Size CBD | Facility NCBD | Facility CBD |
|---|------------|--------------|--------------|-------------|------------------|-----------------|
| Information | 16,975 | 25.15% | 13.52 | 31.16 | 336.88 | 214.44 |
| Finance | 42,960 | 53.51% | 8.55 | 55.66 | 318.28 | 193.59 |
| Real Estate | 18,459 | 17.97% | 7.51 | 12.43 | 743.36 | 1190.21 |
| Professional Services | 64,076 | 32.85% | 6.99 | 13.29 | 334.83 | 309.60 |
| Management | 2,062 | 11.30% | 19.46 | 14.56 | 272.88 | 360.52 |
| Administrative Support | 41,830 | 14.97% | 11.01 | 20.67 | 240.89 | 352.14 |
| Education | 52,995 | 42.69% | 30.46 | 205.66 | 316.70 | 121.27 |
| Health Care | 115,048 | 18.12% | 16.53 | 24.01 | 293.39 | 291.46 |
| Total | 354,405 | 28.66% | 11.78 | 27.47 | 326.81 | 265.19 |
| Note: Size is average employment. Facility is average facility size measured in square foot per employee. | | | | | | |

outside the CBD has only 12 employees. However, rents for office space are higher in the CBD. As a consequence firms located in the CBD only use 265 square foot per employee while firms outside the CBD use 327.

To get some additional insights into the firm sorting process, we need analyze the full distribution of firms by location. Table 2 reports a number of percentiles of the age, facility size, and employment size distribution by location. Moreover, Dun and Bradstreet also report revenue estimates for each firm in the sample. Table 2 reveals a number of important facts that characterize sorting of firms across locations. Firms in the CBD not only employ more workers and operate in larger facilities as we have seen above. They are also older and have a higher output per employee. The later fact is consistent with the notion that firms in the CBD may have higher productivity levels than firms located outside the CBD. Table 2 also shows that there are significant differences among firms in the right tail of the distribution. Looking at the 90th and higher percentiles, we find large differences between firms inside and outside of the CBD.

One potential concern of the analysis above is that differences between firms lo-

Table 2: Sorting of Firms by Location and Industry in 2008

| All Service Industries | | | | | | | | |
|------------------------|-----|--------|----------|------------|-------------|--------|----------|--------------|
| | CBD | | | | Outside CBD | | | |
| percentile | age | employ | facility | rev/emp | age | employ | facility | rev/emp |
| 10th | 6 | 1 | 890 | 42,500 | 4 | 1 | 780 | 28,000 |
| 25th | 12 | 2 | 1,400 | 60,000 | 10 | 1 | 1,000 | 44,444 |
| 50th | 20 | 5 | 2,500 | 75,000 | 18 | 3 | 1,700 | 67,647 |
| 75th | 34 | 14 | 4,700 | 93,750 | 28 | 7 | 3,000 | 85,000 |
| 90th | 53 | 40 | 11,100 | 185,714 | 41 | 20 | 5,800 | 127,500 |
| 95th | 72 | 78 | 23,000 | 309,524 | 53 | 39 | 10,000 | 210,000 |
| 99th | 119 | 460 | 230,000 | 2,000,000 | 96 | 150 | 50,000 | 892,870 |
| Information Technology | | | | | | | | |
| | CBD | | | | Outside CBD | | | |
| percentile | age | employ | facility | rev/emp | age | employ | facility | rev/emp |
| 10th | 2 | 1 | 790 | 33,333 | 4 | 1 | 760 | 25,714 |
| 25th | 6 | 3 | 1,600 | 60,000 | 7 | 1 | 990 | 52,000 |
| 50th | 14 | 7 | 3,400 | 80,000 | 13 | 2 | 1,500 | 70,000 |
| 75th | 28 | 20 | 4,800 | 110,000 | 23 | 6 | 3,000 | 90,000 |
| 90th | 55 | 60 | 13,000 | 160,000 | 37 | 17 | 6,300 | 117,143 |
| 95th | 90 | 158 | 20,000 | 250,000 | 56 | 33 | 11,000 | 182,442 |
| 99th | 156 | 218 | 172,000 | 4,200,000 | 109 | 100 | 35,000 | 538,462 |
| Financial Services | | | | | | | | |
| | CBD | | | | Outside CBD | | | |
| percentile | age | employ | facility | rev/emp | age | employ | facility | revenues/emp |
| 10th | 6 | 2 | 910 | 78,333 | 4 | 1 | 750 | 76,923 |
| 25th | 12 | 3 | 1,500 | 93,750 | 9 | 2 | 1,200 | 85,000 |
| 50th | 23 | 7 | 2,800 | 116,666 | 17 | 3 | 1,600 | 100,000 |
| 75th | 39 | 17 | 5,500 | 190,000 | 31 | 6 | 2,300 | 127,500 |
| 90th | 70 | 49 | 14,000 | 383,333 | 53 | 14 | 3,900 | 203,333 |
| 95th | 79 | 200 | 34,900 | 725,163 | 72 | 25 | 6,500 | 400,000 |
| 99th | 145 | 2,034 | 344,600 | 22,612,458 | 107 | 60 | 20,000 | 2,500,000 |

cated inside and outside the CBD may be due to aggregation among different industries. In particular, the observed differences among locations may just reflect the fact that differences in industrial composition among locations. In the lower panels of Table 2, we, therefore, report the same statistics for two industries. The middle panel reports the results for the information technology sector which is an "average" service industry in terms of its concentration of employment in the CBD. The lower panel reports the statistics for the finance industry which is the most heavily concentrated industry in our sample. We find that the qualitative differences between firms located inside and outside the CBD are not driven by aggregation across firms in the different service industries. In anything, the differences in the financial service industry are more pronounced than the differences in sample of all service industries.

We observe firms at two points of time, 2008 and 2011. Table 3 reports firm level characteristics for 2008 as well as 2011. We find that the overall firm characteristics are similar in both subsamples.

Table 3: Establishment Characteristics

| | 2008 | 2011 |
|---------------------|----------|----------|
| Establishments CBD | 5,832 | 5,856 |
| Establishments NCBD | 48,755 | 49,734 |
| Avg Employment CBD | 26.63 | 26.00 |
| Avg Employment NCBD | 11.20 | 10.93 |
| Avg Facility CBD | 7,709.27 | 7,663.48 |
| Avg Facility NCBD | 4,543.08 | 5,138.43 |

Finally we characterize differences in entry, exit, and relocation rates by location. Here we exploit the panel structure of our data set. Dun and Bradstreet collect detailed information about names, addresses and age of the firm. In addition, establishments are matched across time periods using the D-U-N-S number, which remains assigned to an establishment regardless of location.¹⁵ Entrants were determined as

¹⁵See Neumark, Zhang and Wall (2005) for discussion of this matching method, as well as a more

establishments that were present in 2011, but not in 2008. Exits were calculated similarly. Also shown are measures of total movement (where the zip code of the establishment changed) as well as movement to and from the city. Table 2 summarizes the main empirical regularities.

Table 4: Dynamics 2008-2011

| | Count | Percent | Avg. Emp. | Avg. Fac. |
|--------------------|--------|---------|-----------|-----------|
| Total Reolcation* | 1,053 | 1.93 | 30.18 | 7,610.06 |
| Move CBD to NCBD** | 135 | 0.25 | 11.91 | 9,827.33 |
| Move NCBD to CBD** | 92 | 0.17 | 18.67 | 4,180.65 |
| Entry CBD | 1,696 | 3.05 | 8.19 | 3,436.80 |
| Entry NCBD | 14,566 | 26.20 | 5.11 | 3,151.09 |
| Exit CBD | 1,629 | 2.98 | 16.04 | 4,299.49 |
| Exit NCBD | 13,630 | 24.97 | 6.47 | 2,838.57 |

*all establishments that changed zip code

**size characteristics before move

We find that firms that enter in the CBD are on average larger than firms that enter outside the CBD. They hire on average 8.19 employees while new entrants outside the CBD employ 5.11 workers. Facility sizes are only slightly larger for firms that enter in the city. The difference in characteristics of firms that exit is even more pronounced. The average employment of existing firms in the CBD is 16.04 compared to 6.47 for firms outside the CBD. Moreover, the fraction of firms that enter and exit over the three year period is substantial. We thus conclude that there are large differences in entry and exit rates between the CBD and the rest of the metropolitan area.

We also observe a fair amount of relocation within the metropolitan area. Measured at the zip code level, approximately two percent of all firms relocate during the three year period. At the more aggregate level of location that is used in the estimation below, we find that 0.42 percent of all firms relocate during the three year period.

general discussion of the Dun and Bradstreet dataset.

3 A Dynamic Model of Firm Location Choices

We develop a new dynamic general equilibrium model of firm location choice that can explain the type of sorting of firms by productivity as well as entry, exit, and relocation decisions of firms that we have characterized in the previous section. We consider a model with two distinct locations, denoted by $j = 1, 2$, which can be interpreted as inside and outside of a central business district (CBD). In equilibrium these locations differ in the magnitude of their agglomeration externalities which increase with employment density. We first specify technologies and markets and then characterize equilibria.

3.1 Technologies and Markets

There is a continuum of firms that produce a single output good and compete in a product market. In each period a firm chooses to stay where it is, relocate to the other location, or shutdown. Firms are heterogeneous and productivity evolves according to a stochastic law of motion.

Assumption 1 *In each period a firm is subject to an exogenous probability of exiting. We denote by ξ the complement probability of a firm surviving into the next period. If the firm survives, it draws a new productivity shock, φ' each time period. The productivity shock evolves over time according to a Markov process with a conditional distribution $F(\varphi'|\varphi)$. A firm located in j also faces an exogenous probability λ_j of relocating to the other location.*

In our parametrized model, we assume that the logarithm of the productivity shock follows an AR(1) process, i.e. $\log(\varphi)' = \rho \log(\varphi) + \varepsilon'$, where ρ is the correlation coefficient and ε is a normally distributed random variable with mean μ_ε and variance σ_ε^2 .

Each firm produces a single output good using labor and land as input factors. The technology that is available to the firms in the economy satisfies the following assumption.

Assumption 2 *The production function of a firm in location j can then be written as:*

$$q = f(\varphi, n, l; e_j) \quad (1)$$

where q is output, n is labor, l is land, and e_j is the agglomeration externality in location j . The production function satisfies standard regularity conditions.

Rosenthal and Strange (2003) suggest that the externality acts as a multiplier on the production function. We use a standard Cobb-Douglas function with parameters α and γ in our computational model, $q = \varphi e_j n^\alpha l^\gamma$.

The agglomeration externality arise due to a high concentration of firms operating in the same location.

Assumption 3 *The agglomeration externality can be written as*

$$e_j = \Theta(L_j, N_j) \quad (2)$$

where N_j and L_j are aggregate measures of labor and land respectively. The function Θ is such that $\Theta_L < 0$ and $\Theta_N > 0$.

Following Lucas and Rossi-Hansberg (2002), we use the following parameterization:

$$e_j = \left(\frac{N_j}{L_j} \right)^\theta \quad (3)$$

If $\theta > 0$, the externality is an increasing function of a measure of concentration of economic activity in a location j . This measure is represented by the ratio of the

total number of workers and the amount of land used in production.

The urban economy is part of a larger economic system which determines output prices and wages.¹⁶

Assumption 4 *Output prices, p , and wages, w , are constant and determined exogenously.*

Rental prices, r_j , however, are equilibrium outcomes. The supply of land is determined by an inverse land supply function in each location.

Assumption 5 *The inverse land supply function is given by:*

$$r_j = r_j(L_j), \quad j = 1, 2 \tag{4}$$

The inverse supply function is increasing in the amount of land denoted by L_j .

In the computational analysis, we adopt an iso-elastic functional form: $r_j = A_j L_j^\delta$, $j = 1, 2$, where A_j and δ are parameters. Since rental prices for land must be higher in equilibrium in locations with high externalities, the agglomeration externality is, at least, partially capitalized in land rents.

We can break down the decision problem of firms into a static and a dynamic problem. First, consider the static part of the decision problem that an existing firm has to solve each period. This problem arises because firms compete in the product market each period.

Assumption 6 *The product market is competitive and firms behave as price takers. Firms make decisions on land and labor usage after they have observed their productivity shock, φ , for that period.*

¹⁶It is straight forward to endogenize wages and output prices by adding a local labor market and an aggregate demand for output to our model.

Let π_j denote a firm's one period profit in location j . The static profit maximization problem can be written as:

$$\{n, l\} = \arg \max_{\{n, l\}} \pi_j(n, l; \varphi), \quad (5)$$

where the profit function is given by:

$$\pi_j(n, l; \varphi) = pf(\varphi, n, l; e_j) - wn - r_j l - c_{fj}. \quad (6)$$

The parameter c_{fj} denotes a fixed cost of operation which depends on location. Solving this problem we obtain the demand for inputs as a function of φ , denoted by $n_j(\varphi)$ and $l_j(\varphi)$, as well as an indirect profit function, denoted by $\pi_j(\varphi)$.¹⁷

Let μ_j denote the measure of firms located in j . The mass of firms located in j , denoted by S_j , is given by the following expression:

$$S_j = \int \mu_j(d\varphi) \quad (7)$$

Given the static choices for land and labor use for each firm, we can also calculate the aggregate levels of land and labor:

$$L_j = \int l_j(\varphi) \mu_j(d\varphi), \quad (8)$$

$$N_j = \int n_j(\varphi) \mu_j(d\varphi) \quad (9)$$

After choosing labor and land inputs, each firm faces the (dynamic) decision of whether to stay in its current location, move to the other location, or shut down. The following Bellman equations formalize the decision problem of a firm that begins the

¹⁷Note that the sub-index j summarizes the dependence of the profit and input demand functions on location j 's rent and externality.

period in location j with a productivity shock φ :

$$\begin{aligned}
V_1(\varphi) &= \pi_1(\varphi) + \beta\xi(1 - \lambda_1) \max \left\{ 0, \int V_1(\varphi') F(d\varphi'|\varphi), \int V_2(\varphi') F(d\varphi'|\varphi) - c_r(\varphi) \right\} + \\
&\quad \beta\xi\lambda_1 \left\{ \int V_2(\varphi') F(d\varphi'|\varphi) - c_r(\varphi) \right\} \\
V_2(\varphi) &= \pi_2(\varphi) + \beta\xi(1 - \lambda_2) \max \left\{ 0, \int V_2(\varphi') F(d\varphi'|\varphi), \int V_1(\varphi') F(d\varphi'|\varphi) - c_r(\varphi) \right\} + \\
&\quad \beta\xi\lambda_2 \left\{ \int V_1(\varphi') F(d\varphi'|\varphi) - c_r(\varphi) \right\}
\end{aligned} \tag{10}$$

where β is the discount factor, $c_r(\varphi)$ is the cost of relocating from one location to another. We explore different specifications in the quantitative analysis. One specification assumes that relocation costs depend on the size of the firm, $q(\varphi)$.

Solving the dynamic decision problem above implies decision rules of the following form for firms currently in location j :

$$x_j(\varphi) = \begin{cases} 0 & \text{if firm exits in next period} \\ 1 & \text{if firm chooses location 1 in next period} \\ 2 & \text{if firm chooses location 2 in next period} \end{cases} \tag{11}$$

To close the model, we need to specify the process of entry.

Assumption 7 *Firms can enter into both locations. All prospective entrants are ex-ante identical. Upon entering a new firm incurs a cost c_{ej} and draws a productivity shock φ from a distribution $\nu(\varphi)$. The firm can then decide whether to begin production or instead shut down immediately in which case nothing is ever produced.*

Note that we allow the entry cost to vary by location. In our parametrized model the entrant distribution is assumed to be log-normal with parameters μ_{ent} and σ_{ent}^2 . These assumptions guarantee that the expected discounted profits of a prospective

firm are always less or equal than the entry cost:

$$c_{ej} \geq \int \max \{V_j(\varphi), 0\} \nu(d\varphi) \quad , j = 1, 2 \quad (12)$$

If there is positive entry of firms, then this condition holds with equality. Let the decision rule for initiating production in j , upon observing the initial shock φ , be denoted by $y_j(\varphi)$ with:

$$y_j(\varphi) = \begin{cases} 1 & \text{if a firm with initial shock } \varphi \text{ begins production in } j \\ 0 & \text{else} \end{cases} . \quad (13)$$

3.2 Equilibrium

We are now in a position to define a stationary equilibrium to our economy.

Definition 1 *A stationary equilibrium for this economy consists of rents, r_j^* , masses of entrants, M_j^* , stationary distributions of firms, $\mu_j^*(\varphi)$, externalities, e_j^* , land demand functions, $l_j^*(\varphi)$, labor demand functions, $n_j^*(\varphi)$, value functions, $V_j^*(\varphi)$, and decision rules, $x_j^*(\varphi)$ and $y_j^*(\varphi)$, for each location $j = 1, 2$, such that:*

1. *The decision rule (11) for a firm's location is optimal, in the sense that it maximizes the right-hand side of equations (10).*
2. *The decision rule (13) for a firm's production decision is optimal, in the sense that $y_j^*(\varphi) = 1$ if $V_j^*(\varphi) \geq 0$ and $y_j^*(\varphi) = 0$ else.*
3. *The decision rules for labor and land inputs solve the firm's static problem in (5).*
4. *The free entry conditions (12) are satisfied in each location, with equality if $M_j^* > 0$.*

5. *The market for land clears in each location consistent with equation (4).*
6. *The mass of firms in each location is given by equation (7).*
7. *The externalities are consistent with (2)*
8. *The distributions of firms μ_j^* are stationary in each location and consistent with firms' decision rules.*

Any stationary equilibrium of our model can be characterized by vector of equilibrium values for rents, mass of entrants, externalities in each location, and cut-offs for beginning production $(r_1, r_2, M_1, M_2, e_1, e_2)$. Finding an equilibrium for this model is equivalent to the problem of finding the root of a nonlinear system of equations with six equations. For any vector $(r_1, r_2, M_1, M_2, e_1, e_2)$, we can

1. solve the firms' static profit maximization problem and obtain land demand, labor demand, and the indirect profit functions for each location;
2. solve the dynamic programming problem in equations (10) and obtain the optimal decision rules for exit, entry, and relocation;
3. use the initial mass of entrants in each location and simulate the economy forward until the distribution of firms, μ_j , converges to a stationary distribution;
4. calculate the aggregates land and labor demands, as well as the land supply in the economy;
5. check whether market clearing conditions and the equations that define the mass of firms and the externalities in each location are satisfied.

If the equilibrium conditions are not satisfied, we update the vector of scalars and repeat the process until all of the conditions for equilibrium are satisfied. If this algorithm converges, we have computed an equilibrium of the model.

Note that the mapping described above is not a simple contraction mapping. As a consequence we cannot apply standard techniques and proof of existence of equilibrium. In Appendix B of the paper, we provide a proof of existence of equilibrium for a simplified version of our model in which firm productivity is constant across time. Moreover, we have computed equilibria for a large number of different specifications of our general model. We, therefore, conclude that equilibria exist for reasonable parameterizations of the model.

The task of computing an equilibrium can be simplified by exploiting some properties of the parametrization used in our computational model. The static first order condition that determines that ratio of land and labor inputs is given by:

$$\frac{n}{l} = \frac{\alpha r_j}{\gamma w} \quad (14)$$

Notice that the ratio in this equation is the same for all firms in the same location j . Aggregating over all firms in such location, we obtain that:

$$\frac{N_j}{L_j} = \frac{\alpha r_j}{\gamma w} \quad (15)$$

Equation (15) then implies an expression linking the externality, e_j in each location to that location's rent, r_j . We can, therefore, solve the Bellman equations without knowing the aggregate levels of land and labor. As a consequence we can characterize equilibrium rent values solely based on the free entry conditions expected values functions, which can be written as:

$$\begin{aligned} EV_1(r_1, r_2) &= \int_0^\infty \max \{V_1(\varphi), 0\} d\nu(\varphi) \\ EV_2(r_1, r_2) &= \int_0^\infty \max \{V_2(\varphi), 0\} d\nu(\varphi) \end{aligned}$$

The entry condition for location one then defines a mapping $r_1 = \Gamma_1(r_2)$, i.e. for given r_2 , $\Gamma_1(r_2)$ is the value of r_1 such that $EV_1(r_1, r_2) = c_e$. Similarly, we can define a mapping $r_1 = \Gamma_2(r_2)$ for location two. These two mappings then effectively define the set of rent pairs $\{r_1, r_2\}$, such that the two free entry conditions are satisfied with equality.

The non-linearity of the model implies that $\Gamma_1(r_2)$ and $\Gamma_2(r_2)$ can intersect multiple times. As a consequence, there may be more than one possible candidate values for equilibria with entry in both locations.¹⁸

Next, define the ratio of entrants in the two locations as $m = M_1/M_2$ and the distribution of firms standardized by the mass of entrants in locations 2 as,

$$\hat{\mu}_j = \frac{\mu_j}{M_2}.$$

The standardized stationary distributions satisfy

$$\begin{aligned} \int_0^{\varphi'} \hat{\mu}_1(dz) &= \xi(1 - \lambda_1) \int F(\varphi'|z) 1\{x_1(z) = 1\} \hat{\mu}_1(dz) \\ &\quad + \xi(1 - \lambda_2) \int F(\varphi'|z) 1\{x_2(z) = 1\} \hat{\mu}_2(dz) \\ &\quad + \xi\lambda_2 \int F(\varphi'|z) \hat{\mu}_2(dz) + m \int_0^{\varphi'} 1\{y_1(z) = 1\} \nu(dz) \end{aligned}$$

¹⁸In addition to equilibria with entry in both locations, it is also possible to have equilibria in which entry only occurs in one of the two locations.

and

$$\begin{aligned} \int_0^{\varphi'} \hat{\mu}_2(dz) &= \xi(1 - \lambda_1) \int F(\varphi'|z) 1\{x_1(z) = 2\} \hat{\mu}_1(dz) \\ &\quad + \xi(1 - \lambda_2) \int F(\varphi'|z) 1\{x_2(z) = 2\} \hat{\mu}_2(dz) \\ &\quad + \xi\lambda_1 \int F(\varphi'|z) \hat{\mu}_1(dz) + \int_0^{\varphi'} 1\{y_2(z) = 1\} \nu(dz) \end{aligned}$$

where $1\{x_j(\varphi) = j\}$ is an indicator function equal to 1 if x_j equals j and 0 otherwise and $1\{y_j(\varphi) = j\}$ is similarly defined.

Given a value for m , forward iteration on these two equations yields the equilibrium standardized stationary distributions $\hat{\mu}_j$, $j = 1, 2$. To find the equilibrium value of m , substitute the aggregate demands for land in the two locations into the inverse land supply functions and take their ratios. Given that the inverse elasticity, denoted by δ , is the same in both locations we then obtain:

$$\frac{r_1}{r_2} = \frac{A_1}{A_2} \left[\frac{\int l_1(\varphi) \hat{\mu}_1(d\varphi)}{\int l_2(\varphi) \hat{\mu}_2(d\varphi)} \right]^\delta. \quad (16)$$

Let $r_1 = r_m(r_2; m)$ be the value of r_1 that clears the relative land markets given r_2 and m , keeping in mind that both the labor demand functions $l_j(\varphi)$ and the masses of firms $\hat{\mu}_j$ depend on r_1 and r_2 .

We can thus conclude that all rent pairs $\{r_1, r_2\}$ that are consistent with entry in both locations are characterized by the intersection of the two functions $\Gamma^j(r_2)$. In addition, we have characterized the set of rent pairs consistent with land market clearing condition, $r_m(r_2; m)$, corresponding to different values of m . By analyzing these functions all together, we can completely characterize the set of triplets $\{r_1, r_2, m\}$ consistent with equilibrium in the economy.

Finally, the mass of entrants in location 2, M_2 , is determined by the market clearing condition for land:

$$\left(\frac{r_2}{A_2}\right)^{\frac{1}{\delta}} = M_2 \int l_2(\varphi) \hat{\mu}_2(d\varphi), \quad (17)$$

Note that M_2 can be solved for analytically.

Characterizing additional properties of the equilibrium requires additional assumptions. In Appendix B of the paper we consider a simplified version of our model in which the productivity of firms does not change over time. Under this simplifying assumption we can analytically characterize the resulting stationary distribution of firms in equilibrium. For the general version of the model, additional insights can be gained using numerical methods.

With respect to uniqueness of stationary equilibrium, there are four issues. First, an equilibrium typically exists with communities that are ex post identical if land supply are the same ($A_1 = A_2$). These “non-sorting” equilibria are not interesting and easily rejected empirically. We analyze sorting equilibria here. Second, the non-convexities in the model associated with community choice preclude the use of standard techniques to establish uniqueness of sorting equilibria. Third, entry conditions may not hold with equality which can give rise to equilibria with entry in only one of the two locations. In the quantitative analysis, we only focus on equilibria with entry in both locations. Last, the endogeneity of the firm productivity distribution in stationary equilibrium may not be unique.

While there are several sources of potential multiplicity, we find in our computational analysis that stationary (sorting) equilibria are robust. When we perturb an equilibrium that we have computed, the algorithm converges back to the original equilibrium. These computational experiments suggest that equilibrium is at least locally unique. We do not have a formal proof of local uniqueness of the sorting

equilibrium.

4 Identification and Estimation

It is useful to adopt a two-step procedure for estimating the parameters of the model, primarily because the parameters that characterize the technology available to firms can be identified and estimated without computing the equilibrium of the model.

4.1 First Stage

Recall that our parametric assumptions imply that the production function is given by:

$$y = \varphi e_j n^\alpha l^\gamma \quad (18)$$

The demand for land can then be written as:

$$l_j(\varphi) = \Gamma_l \varphi^{\frac{1}{1-\gamma-\alpha}} (r_j)^{\frac{\theta+\alpha-1}{1-\gamma-\alpha}} \quad (19)$$

where

$$\Gamma_l = \left[\left(\frac{\alpha}{w} \right)^{\theta+\alpha} \gamma^{1-\theta-\alpha} \right]^{\frac{1}{1-\gamma-\alpha}}$$

Similarly, the demand for labor is given by

$$n_j(\varphi) = \Gamma_n \varphi^{\frac{1}{1-\gamma-\alpha}} (r_j)^{\frac{\theta-\gamma}{1-\gamma-\alpha}} . \quad (20)$$

where

$$\Gamma_n \equiv \frac{\alpha}{w\gamma} \Gamma_l.$$

We make the following assumption about variables that are observed by the econometrician.

Assumption 8

1. w , r_1 and r_2 are observed without error.
2. We observe for a large random sample of firms output, labor and land inputs as well as locational choices.
3. We observe entry, exit and relocation decisions of all firms in the sample.
4. Output is measured with error, i.e. we observe

$$\tilde{y}_{ij} = y_{ij} - u_{ij}^y \tag{21}$$

where y_{ij} denote true output of firm i in location j , \tilde{y}_{ij} denotes observed output, and the error term satisfies $E[u_{ij}^y] = 0$.

The first order conditions for optimal labor demand imply the standard factor share results:

$$\alpha \tilde{y}_{ij} - wn_{ij} = \alpha u_{ij}^y \tag{22}$$

The assumption that $E[u_{ij}^y] = 0$ then identifies α . Similarly, land inputs satisfy:

$$\gamma \tilde{y}_{ij} - r_j l_{ij} = \gamma u_{ij}^y \quad j = 1, 2 \tag{23}$$

and γ is, therefore, identified as well.

To show that the parameters of the distribution of productivity of entrants are identified, it is useful to treat $\bar{\varphi}_1$ and $\bar{\varphi}_2$ as nuisance parameters. We can rewrite the labor demand equation (20) for entrants as

$$\ln(n_{ij}^e) = c_j(\theta) + \frac{1}{1 - \alpha - \gamma} \ln(\varphi_{ij}) \quad (24)$$

where the intercept, $c_j(\theta)$, is known up to the parameter θ . We have

$$\min\{\ln(n_{ij}^e)\} = c_j(\theta) + \frac{1}{1 - \alpha - \gamma} \ln(\bar{\varphi}_j) \quad j = 1, 2 \quad (25)$$

and

$$E[\ln(n_{ij}^e)] = c_j(\theta) + \frac{1}{1 - \alpha - \gamma} E[\ln(\varphi_{ij}) | \ln(\varphi_{ij}) \geq \ln(\bar{\varphi}_j)] \quad j = 1, 2 \quad (26)$$

These four equations identify the two cut-off points $\bar{\varphi}_j$, μ_{ent} and θ . Note that θ is for all practical purposes identified of the differences in mean labor inputs among the two locations since it operates as a shifter of the intercept in the labor demand equation. Moreover, we have:

$$V[\ln(n_{ij}^e)] = \left(\frac{1}{1 - \alpha - \gamma}\right)^2 V[\ln(\varphi_{ij}) | \ln(\varphi_{ij}) \geq \ln(\bar{\varphi}_j)] \quad (27)$$

These two equations over-identify σ_{ent} .

Finally, consider identification of the law of motion of productivity. Assumption 8 implies that we observe labor inputs for all firms that active in period t including those that will exit at the end of period t . The law of motion for labor inputs can be written as

$$\ln(n_{ijt}) - c_j - \rho (\ln(n_{ikt-1}) - c_k) = \frac{1}{1 - \alpha - \gamma} \epsilon_{jt} \quad (28)$$

By assumption exit occurs at the end of the period, and there is no attrition problem at this stage. Hence, we can identify μ_ϵ , σ_ϵ and ρ based on the equation above.

All parameters above are identified of the observed data and can, therefore, be consistently estimated without computing the equilibrium of the model. Using the orthogonality conditions above, we can define a GMM estimator that can be used to estimate the structural parameters $\alpha, \gamma, \theta, \mu_{ent}, \sigma_{ent}, \mu_e, \sigma_e, \rho$ as well as the nuisance parameters $\ln(\bar{\varphi}_j)$. (See Appendix C for a list of the sample moments used for the first stage GMM estimator.)

4.2 Second Stage

We can adopt a simulated method of moments approach to estimate the remaining parameters of our model.¹⁹ The estimation strategy relies on the idea that the structural model should replicate the observed moments. We match the fraction of firms in each location, the fraction of entrants in each location, the fraction of firms relocating from 1 to 2 and from 2 to 1, and the fraction of firms exiting in each location. Moreover we measure per period profits before fixed operating costs as follows:

$$\tilde{\pi}_{ij} = \tilde{y}_{ij} - w n_{ij} - r_j l_{ij} \tag{29}$$

We observe this measure for all firms. We can match moments of profits conditional on firms' entry, exit and relocation decisions .

While we do not have a formal argument for identification, we can provide an intuitive explanation for identification. The cost parameters largely influence the selection rules. Holding all parameters constant, the cost of relocation determines

¹⁹Alternatively, we could estimate an auxiliary model using semi-nonparametric estimation as developed by Gallant and Nychka (1987) and then match the scoring functions of the auxiliary and the structural model (Gallant and Tauchen, 1989).

the extent of relocation in equilibrium. That parameter is thus identified of the observed relocation behavior in the data. Similarly the cost of operation determines exit. That parameter should be identified of the observed exit behavior in the data. The entry cost should be identified of the zero expected life-time profit conditions. It is identified of the observed profits of incumbents. It is hard to formalize the arguments since we do not have closed form solutions of the equilibrium selection rules (i.e. decision rules).

Combine all moments used in the estimation procedure into one vector m_N and denote with $m_S(\theta)$ their simulate counterparts where S denotes the number of simulations. The orthogonality conditions are then given by

$$g_{N,S}(\theta) = m_N - m_S(\theta) \quad (30)$$

Following Hansen (1982), θ can be estimated using the following moments estimator:

$$\theta_{S,N} = \arg \min_{\theta \in \Theta} g_{S,N}(\theta)' A_N g_{S,N}(\theta) \quad (31)$$

for some positive semi-definite matrix A_N which converges in probability to A_0 . Since we can make the simulation error arbitrarily small, we suppress the dependence of our estimator on S . The estimator θ_N is a consistent estimator of θ_0 and the asymptotic covariance matrix of the two stage estimator is given in McFadden and Newey (1994). Furthermore, standard J-statistics can be used for hypothesis and specification tests.²⁰

We can impose the efficiency of the estimator by imposing the constraints that the equilibrium values of $\underline{\varphi}_j$ and r_j are equal to the ones we estimated in the first stage.

²⁰Strictly speaking, one would need to correct for the sampling error induced into the estimation procedure by the simulations. However, if the number of simulations is large, these errors will be negligible. For a review see Gourieroux and Monfort (1993).

5 Some Preliminary Estimation Results

Table 5 reports the parameter estimates and the estimated standard errors for a variety of model specifications. Wages in our model are equal to \$48,661 which corresponds the average yearly income in the financial/service sectors, based on census business patterns data. We also set α , the labor share of to be equal to 0.65.²¹ The facility supply elasticity, denoted by δ , is set equal to 0.2.²² Last, we set the exogenous exit probability to 0 (or $\xi = 1$) and the discount factor equal to 0.95, taking a year as the relevant unit of time.

We consider three different model specifications. The first model reported in column I of Table 5 assumes that relocation costs are constant and equal \$200,000. The second specification reported in column II assumes that relocation costs are proportional to output. This specification accounts for the fact that large firms face higher relocation costs than small firms. The third specification uses a concave relocation cost function.

First, consider the version of the model with relocation costs equal to \$200,000. We find that the estimate of the land share parameter is 0.0884. The parameter estimate of the agglomeration externality is 0.102. We have seen before that the restriction that $\theta > \gamma$ is necessary to get an equilibrium sorting pattern in which high productivity firms prefer locations with high agglomeration externalities. These parameters are estimated with high precision.

The fixed costs of operation are \$52,000, or a quarter of the costs of relocating to a different community. Entry costs differ by location and are estimated to be \$764,000 and \$786,000, respectively. The minimum land requirement is approximately 1210

²¹Estimates about the land share are reported in Deckle and Eaton (1993), Adsera (2000), and Caselli and Coleman (2001).

²²Estimates vary for this rent elasticity of supply for office space, but are generally accepted to be significantly greater than unity. See Wheaton (1999) and Henderschott et al (1999) for estimates.

Table 5: Parameter Estimates

| | I | II | III |
|-------------------|-------------------|-----------|-----------|
| c_r | 200,000 | .01 * q | $q^{.73}$ |
| ρ | 0.9773 (0.001) | 0.9772 | 0.9770 |
| σ_ϵ | 0.137 (0.001) | 0.132 | 0.137 |
| μ_ϵ | 0.239 (0.001) | 0.240 | 0.243 |
| μ_{ent} | 11.67 (0.001) | 11.63 | 11.62 |
| σ_{ent} | 0.230 (0.001) | 0.306 | 0.343 |
| l_{min} | 1210 (344) | 1058 | 1120 |
| θ | 0.102 (0.001) | 0.104 | 0.110 |
| γ | 0.088 (0.001) | 0.090 | 0.093 |
| c_f | 51900 (2880) | 52590 | 59831 |
| ce_1 | 764000 (12900) | 780000 | 844000 |
| ce_2 | 786000 (13000) | 785000 | 849000 |
| A_1/A_2 | 1.6327 (0.189) | 1.8411 | 1.7830 |

square foot with an estimate standard error of 344. The productivity shocks are highly correlated across time. The point estimate of 0.977 is consistent with previous estimates in the literature (Hopenhayn and Rogerson, 1993).

The parameter estimates are similar for the other two specifications of the model reported in columns II and III. Fixed costs and relocation costs are higher in these specifications while minimum land requirement and thus the location specific fixed costs are lower. However, the three specifications imply equilibria that differ in some qualitative and quantitative features. Table 6 reports rental price ratios and relocation patterns implied by the three model specifications.

Table 6: Rental Price Ratios and Relocation

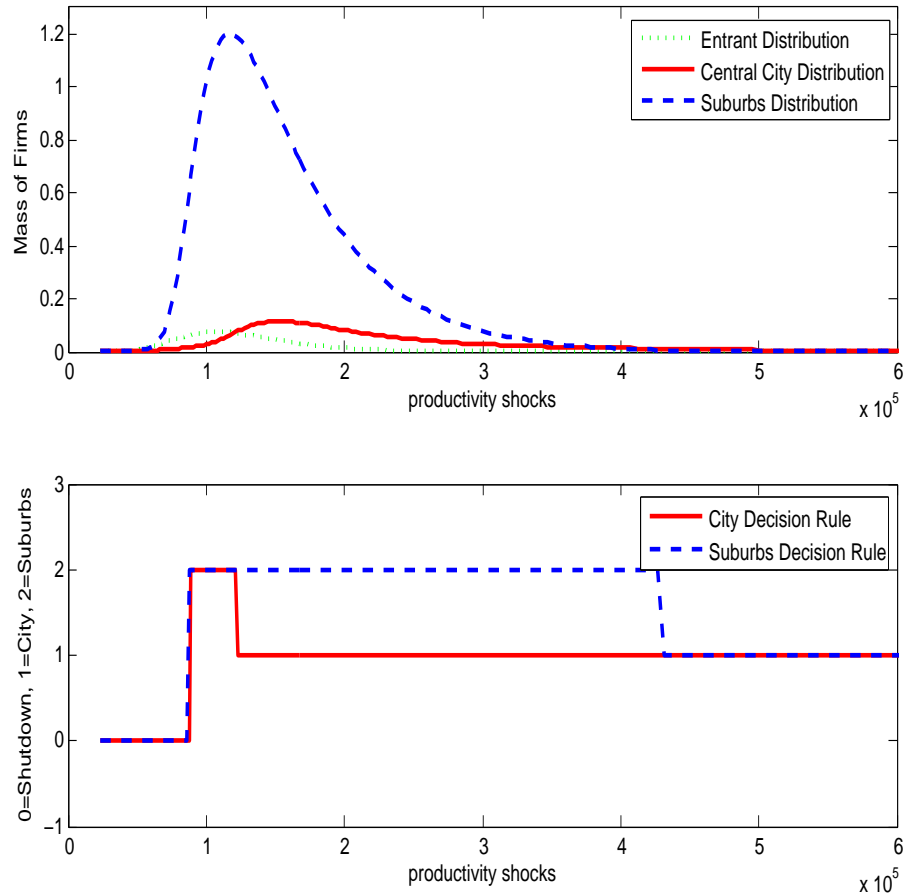
| | I | II | III |
|---------------------------|---------|-----------|-----------|
| c_r | 200,000 | $.01 * q$ | $q^{.73}$ |
| ratio of rents | 1.216 | 1.228 | 1.114 |
| Reloc. cost per emp. (\$) | 732.18 | 813.67 | 1214.12 |
| % est. Move (2 to 1) | 0.03 % | 0.09 % | 0.07 % |
| % emp Move (2 to 1) | 1.02 % | 1.21 % | 1.36 % |
| %t est. Move (1 to 2) | 0.00 % | 1.68 % | 1.11 % |
| % emp. Move (1 to 2) | 0.00 % | 0.53 % | 0.21 % |

The equilibrium associated with the first model specification implies that the rental rate for office in the CBD is approximately 21 percent higher than the rate outside the CBD. This estimated price ratio is similar to the one reported by the Building Owners and Managers Association.²³ The price ratio along with the estimate of the externality parameter, θ , implies that firms located in the CBD receive a 2.02 percent productivity gain over firms located elsewhere due to the local agglomeration externality. For the third model specification the price ratio is smaller and we obtain a 1.2 percent productivity gain. Overall, the magnitude of these productivity gains

²³The Building Owners and Managers Association collects information on expenses and income for office space throughout North America. They report a rent ratio of 1.22 for suburban and CBD office space for the United States. See www.boma.org for more information.

are small, but not irrelevant.

Figure 2: Stationary Distributions and Decision Rules



The upper panel of Figure 2 plots the stationary distribution of firms in both locations as well as the distribution of entrants for specification III. Note that neither of these distributions must integrate to one since the mass of firms and entrants are equilibrium outcomes. The lower panel of Figure 2 plots the optimal decision rules. The equilibrium implies that firms with high productivity shocks relocate to the CBD while low productivity firms leave the CBD to operate in cheaper locations.

Table 6 quantifies the impact of relocation. Specification 1 of our model implies that there is only relocation of large firms from outside the CBD into the CBD. While only a small fraction of firms move to the CBD, they account for approximately one percent of employment in the metro area. In specifications II and III, the equilibrium implies that small firms will relocate from the CBD to locations outside the CBD. Approximately one percent of all small firms leave the CBD per year.

Table 7: Age-Employment distributions of establishments by location, as a percentage of total establishments in the entire county (computational moments in parenthesis)

| Inside CBD | | | | | |
|-------------|---------|------------------|------------------|----------------|-----------------|
| | | Age | | | |
| | | 1 to 10 | 11 to 20 | 21 to 30 | > 30 |
| Employment | 1 to 3 | 1.48 (2.42) | 1.71 (1.13) | 0.96 (0.36) | 1.17 (1.68) |
| | 4 to 8 | 0.59 (0.94) | 1.10 (0.56) | 0.56 (0.36) | 0.93 (1.07) |
| | 9 to 16 | 0.33 (0.27) | 0.56 (0.25) | 0.48 (0.18) | 0.64 (0.62) |
| | > 16 | 0.41 (0.12) | 0.64 (0.22) | 0.63 (0.25) | 1.24 (1.18) |
| Outside CBD | | | | | |
| | | Age | | | |
| | | 1 to 10 | 11 to 20 | 21 to 30 | > 30 |
| Employment | 1 to 3 | 16.25 (20.99) | 16.00 (10.25) | 9.48 (5.97) | 8.17 (11.77) |
| | 4 to 8 | 3.59 (7.86) | 5.89 (4.82) | 4.64 (3.11) | 4.44 (6.56) |
| | 9 to 16 | 1.44 (2.25) | 2.69 (2.07) | 1.81 (1.50) | 2.31 (3.38) |
| | > 16 | 1.44 (0.95) | 2.84 (1.61) | 2.04 (1.42) | 3.50 (3.57) |

Finally, we evaluate the within sample fit of our model. Table 7 reports the distribution of firms by age and employment size for our sample and the one predicted by our model for specification I of the model. We have seen before that the data

suggest that firms located in the city are older and larger than firms located in the rest of the metro area. Moreover, the firms in the city have more employees holding age constant. Overall, our model fits this feature of the data reasonably well.

Table 8: Age-Facility Size establishment distributions by location as percentage of total establishments in the county, (computational moments in parenthesis)

| Inside CBD | | | | | |
|------------------|--------------|-----------------|----------------|----------------|----------------|
| | | Age | | | |
| | | 1 to 10 | 11 to 20 | 21 to 30 | > 30 |
| Facility (sq ft) | 1 to 1250 | 0.82 (2.06) | 0.73 (0.95) | 0.44 (0.54) | 0.65 (1.38) |
| | 1251 to 2150 | 0.64 (0.82) | 1.18 (0.43) | 0.59 (0.26) | 0.88 (0.43) |
| | 2151 to 3850 | 0.75 (0.48) | 1.13 (0.32) | 0.70 (0.21) | 0.82 (0.64) |
| | > 3850 | 0.60 (0.38) | 0.98 (0.47) | 0.91 (0.43) | 1.63 (1.81) |
| Outside CBD | | | | | |
| | | Age | | | |
| | | 1 to 10 | 11 to 20 | 21 to 30 | > 30 |
| Facility (sq ft) | 1 to 1250 | 9.57 (15.39) | 9.76 (7.42) | 5.51 (4.26) | 4.20 (8.31) |
| | 1251 to 2150 | 7.08 (6.99) | 7.56 (3.58) | 4.86 (2.17) | 4.74 (3.40) |
| | 2151 to 3850 | 3.81 (5.53) | 6.01 (3.38) | 4.14 (2.18) | 4.18 (4.58) |
| | > 3850 | 2.30 (4.14) | 4.09 (4.38) | 3.45 (3.40) | 5.13 (7.97) |

Table 8 focuses on the age-facility size establishment distributions. Employment and facility size are highly correlated in the data as predicted by our model. As a consequence, Table 8 reinforces our previous findings about the model fit. Firms in the city face higher rental rates for land and office space. As a consequence they operate with a higher employee per land ratios. Our model explains these features of the data well.

Our analysis has some important policy implications. Relocation costs prevent establishments from moving because the gains for the individual firm are not worth the moving cost. However, this decision may not be efficient since firms ignore the external benefits of density and agglomeration to other firms when making locational decisions. A relocation subsidy may help firms to sort more efficiently in the urban economy. However, it is also possible that relocation policies are not desirable. In large metropolitan areas, there are often many independent communities that compete among each other to attract business using targeted subsidies. It is not obvious that this type of tax and subsidy competition among communities increases economic welfare.

To evaluate these types of policies, we need to measure economic welfare. In our model, establishments have zero expected profits. Hence, the most useful measure of welfare in this economy is surplus that arises to land owners in equilibrium. This surplus can be measured as the area between the rent and the land supply curve:

$$Surplus = \int_0^{L_1^*} (r_1^* - A_1 L^\delta) dL + \int_0^{L_2^*} (r_2^* - A_2 L^\delta) dL. \quad (32)$$

– to be continued –

6 Conclusions

We have developed a new dynamic general equilibrium model to explain firm entry, exit, and relocation decisions in an urban economy with multiple locations. We have characterized the stationary distribution of firms that arises in equilibrium. The parameters of the model can be estimated using a nested fixed point algorithm by matching the observed distribution of firms by location and the one implied by our model. We have implemented the estimator using unique data collected by Dun

and Bradstreet for the Pittsburgh metropolitan area. Firms located in the central business district are older and larger than firms located outside the urban core. They use more land and labor in the production process. However, they face higher rental rates for office space which implies that they operate with a higher employee per land ratio. Our estimates imply that agglomeration externalities increase the productivity of firms by one to two percent. Economic policies that subsidize firm relocations can potentially have large effects on economic growth and firm concentration in central business districts. We view the findings of this paper as promising for future research. Our model can also be used to study relocations of firms across metropolitan area. Moreover, we extend the modeling framework in a number of useful directions to analyze investment and innovation decisions.

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A Firm Location Choices in a Sample of Large U.S. Cities

To get some additional quantitative insights into firms sorting behavior, we collected Census data for a number of metro areas. We define a business district within the metropolitan area as those zip codes within a city that have a high density of firms signifying local agglomeration. To make this concept operational, we use an employment density of at least 10,000 employees per square mile. These locations need not be contiguous, as some metropolitan areas exhibit multiple dense business districts.

Table 9: Concentration of employment in dense business districts

| MSA | Total Emp. Outside CBD | Total Emp. in CBD | Avg. Emp. outside CBD | Avg. Emp. in CBD | % Services outside CBD* | % Ser- vices in CBD** |
|--------------|------------------------|-------------------|-----------------------|------------------|-------------------------|-----------------------|
| Atlanta | 1,115,398 | 229,002 | 15.79 | 29.25 | 45.24% | 63.31% |
| Boston | 1,728,075 | 531,349 | 15.66 | 39.01 | 41.99% | 59.90% |
| Chicago | 3,070,387 | 528,529 | 15.86 | 24.47 | 41.85% | 66.50% |
| Columbus | 705,534 | 63,278 | 18.69 | 23.73 | 42.88% | 58.64% |
| Hartford | 499,718 | 18,783 | 17.26 | 26.95 | 40.31% | 61.41% |
| Houston | 1,720,625 | 286,574 | 16.38 | 28.47 | 42.86% | 65.51% |
| Jacksonville | 491,959 | 24,315 | 15.24 | 25.38 | 43.09% | 66.28% |
| Los Angeles | 4,257,269 | 974,693 | 15.02 | 19.39 | 44.16% | 52.39% |
| Philadelphia | 1,921,626 | 196,428 | 15.91 | 27.66 | 43.99% | 55.74% |
| Phoenix | 1,551,921 | 64,793 | 18.31 | 27.78 | 47.79% | 71.01% |
| Pittsburgh | 822,013 | 157,009 | 14.58 | 40.04 | 39.16% | 60.90% |
| Salt Lake | 440,239 | 53,086 | 15.22 | 21.08 | 45.64% | 58.90% |
| San Antonio | 655,740 | 26,572 | 17.21 | 20.49 | 43.22% | 56.59% |
| Seattle | 1,260,335 | 179,230 | 14.55 | 20.33 | 42.07% | 58.97% |
| St Louis | 1,253,959 | 84,034 | 16.38 | 42.57 | 41.41% | 52.43% |
| Wash. DC | 1,930,848 | 303,770 | 15.42 | 21.68 | 49.96% | 60.05% |

Source: 2006 Zip Code Business Patterns, U.S. Census

*Percentage of establishments outside the CBD that are in the service industries.

**Percentage of establishments in the CBD that are in the service industries.

Table 9 shows the concentration of employment in dense business districts for a

sample of U.S. cities. First, we report statistics using all firms that are located in the metro area. We find that there is a significant amount of heterogeneity among the cities in our sample. There are some cities such as Phoenix and Hartford where employment is not concentrated in dense business districts. Most larger cities in the U.S. such as Los Angeles, Chicago, Boston, Washington, Philadelphia, and Houston have a significant fraction of firms located in high density central business districts. This finding is also true for a variety of mid-sized cities such as Pittsburgh and Seattle. Focusing on the differences between firms located in and out of the CBD, we find firms in the CBD are larger than the MSA average. This indicates that they have higher levels of productivity. This finding is common among all cities in our sample. In addition, firms in the service sector are more concentrated in the CBD compared to firms in general, suggesting that service oriented firms benefit more from local agglomeration than other sectors.

B Analytical Properties of Equilibrium

To get some additional insights into the properties of our model it is useful to simplify the structure of the model and shut down the future productivity shocks. We can then characterize the equilibrium of the model almost in closed-form.²⁴ Let us impose the following additional assumptions.

Assumption 9

1. *The shock is drawn upon entry once and for all from a uniform distribution in $[0, 1]$:*

$$\nu(\varphi) = 1 \text{ for } \varphi \in [0, 1]. \quad (33)$$

²⁴The model cannot be entirely solved in closed form because the equilibrium r_2 has to satisfy a highly non-linear equation. Sufficient conditions on the model's parameters for r_2 to exist and be unique are imposed instead. Conditional on r_2 , everything else can be solved for analytically.

2. *There are no fixed cost of operation: $c_f = 0$.*

3. *Importance of externality: $\theta = 1 - \alpha > \gamma$*

Let 1 denote the high rent location and 2 the low rent one (1=city, 2=suburb). We show how to construct a unique equilibrium in which $r_1 > r_2$ and firms move from location 2 to location 1, but not vice versa. Firms who enter in location 1 stay there all the time or exit.

First note that under assumptions 2 and 3 above the indirect profit functions can be written as:

$$\pi_j(\varphi) = r_j (\Delta \varphi^\eta - \bar{l}), \quad j = 1, 2, \quad (34)$$

where $\Delta > 0$ and $\eta > 1$ are known functions of the parameters of the model. Consider location in the city. We have the following result.

Proposition 1 *If $r_1 > r_2$,*

a) *then firms in location 1 follow a simple cut-off rule. Firms below a threshold φ_l exit while firms above the threshold stay in location 1 forever. The cut-off is defined as:*

$$\varphi_l = \left(\frac{\bar{l}}{\Delta} \right)^{\frac{1}{\eta}}. \quad (35)$$

b) *then firms in location 2 follow a simple cut-off rule. Firms below the threshold φ_l exit, firms with shocks between φ_l and φ_h stay in location 2, and firms with shocks larger than φ_h move to location 1. The cut-off φ_h is defined as:*

$$\varphi_h = \left(\frac{\bar{l}}{\Delta} + \frac{c_r(1 - \beta\xi)}{\Delta(r_1 - r_2)} \right)^{\frac{1}{\eta}}. \quad (36)$$

Proof:

a) Note that static firm profits are monotonically increasing in φ . Define φ_l such that

$\pi_1(\varphi_l) = 0$. Then firms with $\varphi < \varphi_l$ exit immediately. It is straight forward to show that

$$V_1(\varphi_l) = \pi_1(\varphi_l) + \beta\xi \max\{0, V_1(\varphi_l), V_2(\varphi_l) - c_r\} = \pi_1(\varphi_l) = 0 \quad (37)$$

where the second equality follows from the fact that the productivity cut-off for switching to location 2 is less than cut-off for exit if $r_1 > r_2$ as assumed. Firms with $\varphi > \varphi_l$ stay in location 1 as long as they survive the exogenous destruction shock ξ . Their payoffs are:

$$V_1(\varphi) = \frac{\pi_1(\varphi)}{1 - \beta\xi} > 0 \quad (38)$$

b) Next consider the decision rule of firms located in the suburb. Firms with $\varphi < \varphi_l$ exit immediately:²⁵

$$V_2(\varphi_l) = 0 \quad (39)$$

Firms with shocks in (φ_l, φ_h) stay in 2 forever (as long as they survive the exogenous destruction shock). Firms with high shock move to 1. The indifference condition for staying vs moving is:

$$\frac{\pi_2(\varphi_h)}{1 - \beta\xi} = \pi_2(\varphi_h) + \beta\xi (V_1(\varphi_h) - c_r). \quad (40)$$

This equation defines the cut-off value φ_h . The lemma then follows from the result that benefits of switching to location 1 monotonically increase with φ . Q.E.D.

Next we consider the free entry conditions and show that these conditions determine the rents in both locations. We have the following result:

Proposition 2 *There is at most one set of rental rates (r_1, r_2) that are consistent with the entry in both locations. Conditions on the parameter values guarantee existence of (r_1, r_2) .*

²⁵Note that equation (34) implies that z_l does not depend on the location.

Proof (of uniqueness):

First consider the free entry condition in location 1 which is given by

$$\int V_1(\varphi) \nu(\varphi) d\varphi = c_e. \quad (41)$$

Substituting in our optimal decision rule and simplifying we obtain the equilibrium rent in location 1:

$$r_1 = \frac{c_e(1 - \beta\xi)(\eta + 1)}{\Delta(1 - \beta\xi\varphi_l^{\eta+1}) - \bar{l}(1 - \beta\xi\varphi_l)(\eta + 1)}. \quad (42)$$

Free entry in location 2 requires:

$$\int V_2(\varphi) \nu(\varphi) d\varphi = c_e. \quad (43)$$

Replacing the value function in location 2 and taking into account the definition of φ_h in (36) this equation simplifies to:

$$\eta\varphi_h^{1+\eta} - (1 + \eta)\varphi_h^\eta - K = 0, \quad (44)$$

where K represents a non-positive combination of the parameters and is defined in the appendix. The left hand side of this equation is positive when $K = 0$ and has a negative first derivative. Thus, if a solution for φ_h exists it must be unique. In turn, φ_h is monotonically related to r_2 by equation (36):

$$r_2 = r_1 - \frac{c_r(1 - \beta\xi)}{\Delta\varphi_h^\eta - \bar{l}}. \quad (45)$$

Thus, if the solution φ_h to equation (44) is unique, the equilibrium value of r_2 is also unique. The appendix provides sufficient conditions on the parameters for this solution to exist. Q.E.D.

Next we characterize the equilibrium distribution of firms in each location.

Proposition 3 *For each value of M_2 , there exists a unique stationary equilibrium distributions of firms in each location.*

Proof:

Without loss of generality, let us normalize the model so that entry in location 2 is always equal to $M_2 = 1$. This implies a specific choice of A_2 . Given this the mass of firms in location 2 is $\hat{\mu}_2(\varphi)$:

$$\hat{\mu}_2(\varphi) = \begin{cases} z_l & \text{if } \varphi < z_l \\ \frac{1}{1-\xi}(z_h - z_l) & \text{if } \varphi \in [z_l, z_h] \\ 1 - z_h & \text{if } \varphi > z_h \end{cases} . \quad (46)$$

Note that firms in location 2 with $\varphi < z_l$ exit and there is a measure z_l of them. Firms with $\varphi > z_h$ move to 1, and there is a measure $1 - z_h$ of them. Firms in the middle group $\varphi \in [z_l, z_h]$ remain in 2 forever subject to surviving the death shock ξ .

Let m denote entry in location 1. The mass of firms in location 1 is:

$$\hat{\mu}_1(\varphi) = \begin{cases} m z_l & \text{if } \varphi < z_l \\ \frac{m}{1-\xi}(z_h - z_l) & \text{if } \varphi \in [z_l, z_h] \\ \frac{(\xi+m)}{1-\xi}(1 - z_h) & \text{if } \varphi > z_h \end{cases} . \quad (47)$$

Firms in the first group exit immediately. Firms in the middle group stay in 1 forever. Firms with $\varphi > z_h$ come from 2 sources: 1. firms who entered in 1 and stayed there forever subject to death shock $m(1 - z_h)/(1 - \xi)$ plus firms who entered in location 2 last period, survived the shock and moved to 1 where they remain forever: $\xi(1 - z_h)/(1 - \xi)$. Q.E.D.

Finally, we have the following result:

Proposition 4 *There is at most one value of m such that the relative demand for land equals the relative supply of land. Under conditions on the parameters, m is shown to exist.*

Proof:

Given the equilibrium distributions, we can solve for equilibrium value for entry, denoted by m . Note that given the assumptions the demand for labor is:

$$l_j(\varphi) = \bar{l} + \left(\frac{\alpha}{w}\right)^\eta \varphi^{\frac{1}{1-\alpha-\gamma}} r_j^{\frac{1-\alpha}{1-\alpha-\gamma}}. \quad (48)$$

The equilibrium value of m is such that it solves the relative land equilibrium condition which can be written as

$$\int l_1(\varphi) \hat{\mu}_1(d\varphi) = \frac{A_2 r_1}{A_1 r_2} \int l_2(\varphi) \hat{\mu}_2(d\varphi) \quad (49)$$

where the right hand side does not depend on m . The left-hand side depends linearly in m through the mass $\hat{\mu}_1(\varphi)$ in an increasing way. This means that if m exists it is unique.

For $m \rightarrow \infty$ the left hand side of (49) goes to infinity. For $m \rightarrow 0$ the left hand side is strictly positive. To show that it is less than the right hand side A_1 must be sufficiently small. Since the rest of the equilibrium is independent of A_1 one can always choose A_1 small enough in order to guarantee existence. Thus, there exists a unique value of m . Q.E.D.

In what follows we present the equilibrium of the model in a numerical example.

Result 1 *Consider the following parameter values: $\beta = 0.5$, $\alpha = 0.65$, $\theta = 0.35$, $\xi = 0.9$, $\bar{l} = 0.01$, $\gamma = 0.01$, $\eta = 2.94$, $w = 1$, $\Delta = 0.0367$, $A_1 = 0.5$, $A_2 = 1.0$, $c_e = 0.1$, $c_r = 0.01$, $\delta = 1$. Then, the unique equilibrium of the model is characterized by the following: $\varphi_l = 0.64$, $\varphi_h = 0.69$, $r_1 = 37.18$, $r_2 = 34.98$, $m = 0.21$.*

The analysis of this section shows that there exists a unique (up to scale) equilibrium with entry in both locations. Our analysis in the previous section reinforces the notion that equilibria with entry in both locations are often locally unique.

C First Stage Estimation: Sample Moments

The first set of moments are based on the input shares:

$$\frac{1}{N_j} \sum_{i=1}^{N_j} \alpha \tilde{y}_{ij} - w n_{ij} \quad (50)$$

$$\frac{1}{N_j} \sum_{i=1}^{N_j} \gamma \tilde{y}_{ij} - r_j l_{ij} \quad (51)$$

Using the 0.1 percentile to estimate $\underline{\varphi}_j$, we therefore obtain the following two moment conditions:

$$\frac{1}{N_j^e} \sum_{i=1}^{N_j^e} 1\{(1 - \alpha - \gamma)(\ln(n_{ij}^e) - c_j(\theta)) \leq \ln \underline{\varphi}\} - 0.001 \quad j = 1, 2 \quad (52)$$

Moreover, we have the following moments for the mean and the variance of the distribution of the entrant's productivity:

$$\frac{1}{N_j^e} \sum_{i=1}^{N_j^e} \ln(n_{ij}^e) - c_j(\theta) - \frac{1}{1 - \alpha - \gamma} E[\ln(\varphi_{ij}) | \ln(\varphi_{ij}) \geq \ln(\bar{\varphi}_j)] \quad (53)$$

$$\frac{1}{N_j^e} \sum_{i=1}^{N_j^e} [\ln(n_{ij}^e) - \overline{\ln(n_j^e)}]^2 - \left(\frac{1}{1 - \alpha - \gamma}\right)^2 V[\ln(\varphi_{ij}) | \ln(\varphi_{ij}) \geq \ln(\bar{\varphi}_j)] \quad (54)$$

Finally, we have the following moment conditions based on the law of motion of labor inputs

$$\frac{1}{N_j} \sum_{i=1}^{N_j} \ln(n_{ijt}) - c_j - \rho (\ln(n_{ikt-1}) - c_k) - \frac{1}{1 - \alpha - \gamma} \mu_\epsilon \quad (55)$$

$$\frac{1}{N_j} \sum_{i=1}^{N_j} n_{ijt-1} (\ln(n_{ijt}) - c_j - \rho (\ln(n_{ikt-1}) - c_k) - \frac{1}{1 - \alpha - \gamma} \mu_\epsilon) \quad (56)$$

$$\frac{1}{N_j} \sum_{i=1}^{N_j} (\ln(n_{ijt}) - c_j - \rho (\ln(n_{ikt-1}) - c_k) - \frac{1}{1 - \alpha - \gamma} \mu_\epsilon)^2 - \sigma_\epsilon^2 \quad (57)$$

For the entrants let's assume that that $\ln(\varphi)$ is normally distributed.²⁶

²⁶We can therefore use the following results for truncated normal distributions. The distribution of entrants' productivities follows a truncated normal distribution with mean equal $\mu_e + \lambda(\varphi)\sigma_e$ and variance $\sigma_e^2(1 - \delta(\varphi))$, where

$$\lambda = \phi\left(\frac{\varphi - \mu}{\sigma}\right) / [1 - \Phi\left(\frac{\varphi - \mu}{\sigma}\right)] \quad (58)$$

$$\delta = \lambda \left(\lambda - \frac{\varphi - \mu}{\sigma} \right) \quad (59)$$