A Method for Modeling the Intrinsic Dynamics of Intraindividual Variability: Recovering the Parameters of Simulated Oscillators in Multi–Wave Panel Data

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Abstract

A simple method for fitting differential equations to multi–wave panel data performs remarkably well in recovering parameters from underlying continuous models with as few as three waves of data. Two techniques for fitting models of intrinsic dynamics to intraindividual variability data are examined by testing these techniques' behavior in recovering the parameters from data generated by two simulated systems of differential equations. Each simulated data set contains 100 "subjects" each of whom are measured at only three points in time. A local linear approximation of the first and second derivatives of the subject's data accurately recovers the true parameters of each simulation. A state–space embedding technique for estimating the first and second derivatives does not recover the parameters as well. An optimum sampling interval can be estimated for this model as that interval at which multiple R^2 first nears its asymptotic value.

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Introduction

Although investigators of behavior and behavior change mainly have been oriented toward equilibrium and stability in both the articulation of theory and the conduct of empirical research, since the time of Wundt, Ebbinghaus, and other pioneers there has been a somewhat more subtle emphasis on identifying and studying fluctuations and other kinds of changes that represent more or less reversible phenomena (Helson & Stewart, 1994; Nesselroade & Boker, 1994). In the past couple of decades three powerful influences have combined to fan relatively small flames of concern with intraindividual variability into what is now looming on the horizon as a fire that bids fair to alter substantially the landscape of behavioral science.

One source of influence is the theoretical and empirical work that has helped establish the concept of state alongside that of trait in the lingua franca of behavioral science (Cattell & Scheier, 1961; Singer & Singer, 1972). This has forced attention to the fact that differences among individuals at a given point in time are due, in part, to asynchronous (across persons) intraindividual variability. Simply put, the idea is that "because I am somewhat unlike my self today, I am also somewhat unlike everyone else." A second influence is the recognition that the history of successful scientific disciplines follows a spiraling course that moves from static to dynamic concepts and definitions of phenomena (Cattell, 1966; West, 1985). This evolutionary process of change is stimulated in part, no doubt, by dissatisfaction with the level of predictability and explanation attainable with static representations of interesting phenomena. The signs that it is happening in psychology are difficult to ignore. A third influence is the growing realization by behavioral and social scientists that analytic methods and modeling techniques abound for representing more complicated dynamical structures and that they look promising for our field (see e.g. Arminger, 1986; Coleman, 1968; Hamagami, 1994; Kelso, 1995; McArdle, 1988; Molenaar, 1985; Smith & Thelen, 1993; Tuma & Hannan, 1984; Vallacher & Nowak, 1994).

Transition to more dynamical representations of interesting behavioral phenomena, even for those investigators who favor it, is hampered not only by the attachment psychologists have for the designs and analytic methods in which they were originally trained but also by what seems at first blush to be the necessity for enormous amounts of data if dynamical model parameters of any consequence are to be estimated. It is to this latter problem that we attend as we explore a method for recovering the parameters of a process from limited amounts of data: a few occasions of measurement on a moderately large sample of individuals. Data of this type represents what can be reasonably expected to obtain in many experimental and epidemiological studies that might posit dynamical models.

Self–Regulation

One area of application of dynamical systems theory to psychology is self–regulating systems. Self–regulation involves a mechanism in which the level and rate of change of some quantity are used to adjust future levels and rates of change in that quantity. Thus, information about a variable is used to effect change in that variable. Self–regulation is a commonly used construct in gustatory and emotion research, but many other psychological systems may exhibit some form of self–regulation.

In order to model self–regulation, it is helpful to examine simple physical systems that exhibit self–regulating behavior. For instance, consider a very simple thermostat that can regulate temperature in a house during the winter. If the temperature as sensed by the thermostat falls below a lower threshold, the thermostat turns on the furnace which warms the air by some gradient — say 10 degrees Celsius per hour. But soon the temperature is above some upper threshold, and so the thermostat turns off the furnace. The insulation in the house leaks heat into the winter air outside at some gradient — say 5 degrees Celsius per hour. Thus eventually the temperature falls again below the lower threshold of the thermostat and the cycle begins again.

It is apparent that this thermostat will never stabilize the temperature at the single fixed value that is set by the owner of the house. The temperature will always be cycling, overshooting the ideal temperature while the furnace is running or undershooting the ideal temperature while the furnace is off. This simple thermostat can sense when the temperature is different than the desired equilibrium temperature and when this difference is too large, it effects a change in the way that the temperature is changing; If the temperature were plotted over time, it would look something like Figure 1–a. At the points where the temperature is either a maximum or minimum is where the curvature (the change in the change) is greatest. This plot bears a non-trivial resemblance to the motion of a frictionless pendulum over time.

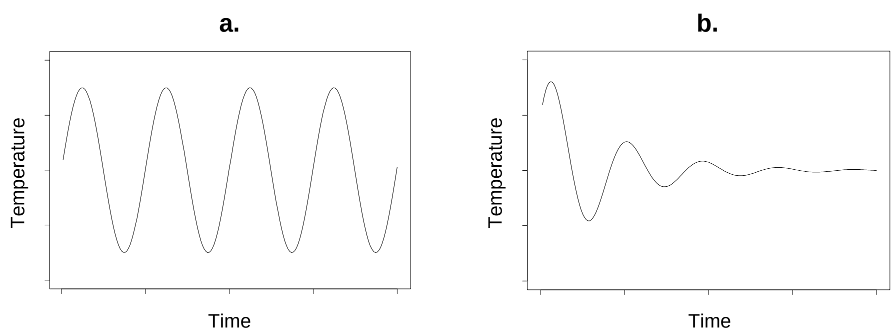

Figure 1. Idealized plot of temperature over time for two simple thermostats. (a) A thermostat that only responds to difference from the desired equilibrium temperature. (b) A thermostat that responds to both difference from equilibrium and rate of change in temperature.

Suppose we now modify this simple thermostat so that it responds not only to an upper and lower threshold, but also to the rate of change in the temperature. Now, when the rate of change is highest, the thermostat can either turn on the furnace or turn off the furnace for short bursts. In this case, the thermostat can effect some curvature in the temperature when its rate of change is the highest: when the temperature is near the desired equilibrium. This type of thermostat will be able to dampen the oscillations in temperature to finally balance the furnace input to the loss from insulation and achieve a stable equilibrium temperature. The plotted temperature over time as shown in Figure 1–b also describes the motion of a pendulum with friction, swinging back and forth but finally coming to rest at a stable equilibrium.

These simple thermostats are plausible models of self–regulation in psychological phenomena. For instance, mood self–regulation or hunger cycles might be usefully modeled using a self–regulating system in which excessive distance from equilibrium is avoided and, at the same time, some sort of dampening or resiliency is in operation. Testable models of perceptions of well–being, anxiety, or daily activity cycles might also be formalized as self–regulating dynamical systems if a way can be found to fit them to data that are commonly used in psychology.

The current simulation work demonstrates that if a self–regulating mechanism similar to the simple thermostats is responsible for generating intraindividual variability, the parameters of that mechanism can be recovered from very short (as few as three occasions of measurement) time series. We first present a short overview of dynamical systems theory and how it resolves some of the problems that have bedeviled researchers concerned with longitudinal data. We then will present the simulation and the results of recovering the parameters in the presence of varying degrees of measurement error. Finally we discuss some recommendations for experimental design when the purpose of an experiment is to test for the presence of self–regulation.

Dynamical Systems Theory

A *system* can be thought of as a set of variables that are logically connected (see Beltrami, 1987, for an introduction). For instance, the position, velocity and acceleration of a pendulum could be thought of as comprising a system. Or, a set of variables measuring the health, cognitive performance and mood of an individual might be considered to be a system. Any system of variables that purports to represent the measurement of psychological constructs is subject to the same considerations that apply in evaluating psychometric measurement, such as construct validity and ecological validity. However, validity and stability are not identical notions. A system of psychological variables may exhibit variability over time yet function as a closely integrated, organized ensemble. With dynamical systems data analytic techniques we intend to test models for this type of variability such that reliable parameters of the structure of underlying self–regulating mechanisms can be estimated.

The *state* of a system is considered to be the values of all of the variables at one moment in time. For instance, the position, velocity and acceleration of a pendulum at some time t may be thought of as the state of that system at time t. Similarly, the measured values for some connected set of variables concerning health, cognitive performance and mood for one individual at one occasion of measurement t can be thought of as the state of that system at time t.

When the state of a system at time t is to some degree dependent on the state of the system at a previous time $t - \tau$ where τ is some interval of time between occasions of measurement, then we call this system a *dynamical system* or a system with *intrinsic dynamics*. Thus if one's cognitive performance at time t is dependent on one's cognitive performance at time $t - \tau$ then we might consider cognitive performance to comprise a dynamical system. Also, if one's cognitive performance at time t is dependent on one's health at time $t - \tau$ then these two variables could be considered to comprise a dynamical system.

If the dependence in a dynamical system can be entirely expressed in terms of a linear combination of the variables of the system with no multiplicative or interaction terms, then we call the system a *linear dynamical system*. Linear systems have a variety of properties that make them amenable to analysis using standard statistical techniques. Those systems that cannot be expressed as simple linear combinations are called *nonlinear dynamical systems* (Thompson & Stewart, 1986; Wiggins, 1990). Nonlinear dynamical systems pose special problems for analysis and will not be treated here. However with some simplifying assumptions, the methods we present can be extended to nonlinear systems.

Underlying Assumptions

Much psychological research today relies on between–subjects, cross–sectional design. Such data are gathered on many individuals, each of whom is measured at only one occasion. Two of the assumptions that underlie such a design are (1) that subjects are in some sense replicates of each other in that there is a lawful and generalizable construct that is being measured and that applies to all subjects and (2) that if subjects were measured at a second occasion and the measurements were aggregated across subjects, the difference in the scores from one occasion to the next would be due to systematic change and deviations from this systematic change would be due to measurement error. This second assumption implies that any reliable intrinsic dynamics within the variables apply in the same way to every individual in the sample in the interval between the two measurements and differences in these changes are due to measurement error.

The standard method for analyzing dynamical systems in the physical sciences has been to pick a single exemplar of the desired dynamical system and measure it intensively, gathering thousands or tens of thousands of observations of that one individual system (Abarbanel, Brown, Sidorowich, & Tsimring, 1993). Two of the assumptions that underlie this type of design are (1) that different instances of the system in question are exact replicates of each other and so only one instance needs to be measured in order to draw generalizable conclusions and (2) that the intrinsic dynamics of the system remain *stationary* over time, in other words the parameters of the dynamical system are not varying during the time of measurement.

Unfortunately, thousands or even hundreds of occasions of measurement are prohibitively expensive or impossible to obtain in most arenas of psychological measurement. Even psychological experiments that are designed to capture meaningful information in intraindividual variability rarely have more than 10 occasions of measurement. The method presented here is intended to exploit such data. It can effectively estimate intrinsic dynamical parameters with as few as three occasions of measurement with a moderate number of subjects. However, an optimal sampling interval must be chosen if the estimates of the dynamic parameters are to be unbiased.

A method for estimating the optimum sampling interval for a set of data is presented. These preconditions match the type of data that psychological researchers typically are able to acquire. For simplicity of presentation, we will make two main assumptions in this article that are a blend of the assumptions from cross–sectional analysis and dynamical systems analysis, (1) that subjects are replicates of each other in the sense that there is an underlying intrinsic dynamical system that generalizes to all subjects and (2) that the intrinsic dynamics of the system remain stationary over time. However, the method as presented here is not constrained to require these assumptions of homogeneity and stationarity. For instance, the method can be extended to allow model parameters to vary over individuals: a random coefficients approach making the assumption that parameters of self–regulation remain stable over time within an individual (Boker & Nesselroade, 2000). A further relaxation of assumptions would allow for nonstationarity, that is the parameters within an individual would be allowed to vary over time. Each of these models could in theory be tested, but to do so would require many more occasions of measurement per individual.

Problems with Analyzing Change Data

The analysis of change poses several challenging problems (Rogosa, 1995). On the one hand, these problems do not necessarily pose difficulties for the analysis of processes that change slowly and for which all individuals show a similar pattern. For instance, it is relatively straightforward to fit a growth curve to height data from children measured annually. With such relatively slowly changing scores, reliability and test–retest correlation tend to have a close correspondence.

On the other hand, when fluctuations in intraindividual variability are cyclic about an overall tendency, the correspondence between reliability and test–retest correlation can break down completely and thereby induce conclusions that a phenomenon is stable or exhibiting growth when no such underlying relationship exists (Nesselroade & Boker, 1994). Two problems that give rise to this and other related epiphenomena that have bedeviled researchers interested in modeling change are the problems we call *the phase problem* and *the measurement interval problem.*

The Phase Problem.

Suppose one is measuring a phenomenon that is simply and lawfully oscillating about a mean value for each individual, similar to the way a pendulum would swing back and forth past its equilibrium point. Suppose further that data are gathered at two occasions of measurement on a large sample of individuals. If people are not synchronized in their oscillations, then on the first occasion of measurement some people will be above their own mean and some people will be below their own mean. This lack of synchronization can be expressed as the sample having a *randomized phase.* If we equate reliability and test–retest correlation, then these oscillations will be considered to be measurement error due to the randomized phase of the oscillations, since the overall test–retest correlation for such a sample will tend toward zero (Nesselroade & Boker, 1994). Thus the intraindividual variability in such a sample can be misrepresented as measurement error even if it were a perfectly deterministic oscillation.

To further illustrate randomized phases, suppose that 100 individuals are measured using a mood scale at two occasions of measurement separated by one hour. During the interval between measurements the subjects sit quietly in a controlled environment. Some individuals come in to the first occasion having a good day and some come in having a bad day. Perhaps one just had a car accident or an argument with a spouse, another may have just gotten married or just became a parent. If the subjects are strangers to one another, then external events in individual subjects' lives may be considered to be independent and thus each individual's intrinsic oscillatory process will have a random phase, so during the controlled hour the subject may be in a phase of his or her cycle that is becoming more positive or more negative with equal probability. Even if there were an underlying dynamic component to mood, the randomized phases of the individuals at the first occasion of measurement would prevent a standard autoregression or growth curve analysis from detecting it.

One of the strengths of the methods proposed in this work is that they are resistant to the phase problem. This is because they attempt to reconstruct the *phase space* of the underlying dynamical system. The phase space of a dynamical system is one way of representing the time-based dependency in a system so that measurements with similar phases are grouped together. Properly reconstructed, data from an experiment in which participants have a randomized phase at the first occasion of measurement can be phase–synchronized so that both change and phase information can contribute to the estimation of the intrinsic dynamics of the system.

To illustrate this problem concretely, consider the two graphs in Figure 2. Each of these two graphs present simulated data from 100 "subjects" at three occasions of measurement. Figure 2–a plots a sample of subjects measured with perfect reliability whose scores obey a self–regulating process: a deterministic system with an intrinsic dynamic. Figure 2–b presents a sample of subjects with no self–regulating process at all, but there was normally distributed measurement error with about the same variability as in the previous graph. We would like to tell the difference between these two systems by asking the following question: Is the variability we observe due to chance or due to a predictable mechanism?

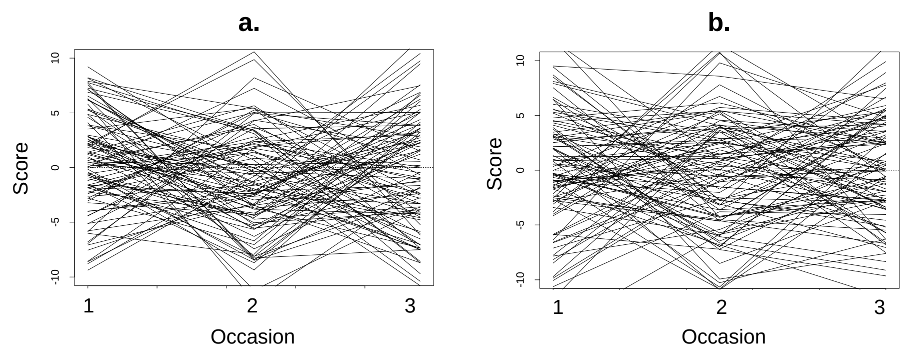

Figure 2. Two samples of 100 simulated subjects measured at three occasions. (a) A sample of scores from 100 subjects whose scores are generated by the same self–regulating process, but with randomized phases and no zero measurement error. (b) A sample of scores from 100 subjects with no self–regulating process, but with normally distributed measurement error with standard deviation $\sigma_e = 5.0$.

The results of a linear model don't distinguish between these two cases. Fitting several well–known models for change (repeated measures ANOVA, linear growth, quadratic growth curve, and random coefficients models) to both of these simulated data sets produces the same answer: no parameters are statistically different than zero and the explained variance is very nearly zero. Thus when the phase of a behavior is not known, variability due to self-regulation may masquerade as measurement error when analyzed using standard methods for longitudinal data.

There seems to be some structure in each of the graphs in Figure 2 such that if a subject's second score is greater than the first score then their third score is likely to be less than their second score. Similarly, if the second score is less than their first score, then the third score is likely to be greater than the second score. But this might be merely regression toward the mean as is the case by design in Figure 2–b. We would like to test whether there is an intrinsic dynamical structure in data such as these. The method by which we simulated the self–regulation data in Figure 2–a and methods for distinguishing between variability due to self–regulation and variability due to measurement error will be described in detail below.

The Measurement Interval Problem.

A second major problem in modeling intraindividual variability is the measurement interval problem. This problem can create a variety of threats to the analysis of change. Perhaps the most critical decision in experimental design of longitudinal studies is selecting the appropriate interval between occasions of measurement. A measurement interval that is very short with respect to the rate of change of the intrinsic dynamic will produce data that are problematically sensitive to measurement error if successive measurements are used in the analysis.

To see why this is so, suppose we use two measurements to estimate a slope and suppose that the true model is simple linear growth but there is some error of measurement. The slope is the difference between the two observed values divided by the interval of time between the two occasions of measurements. If the interval between the two occasion is made to be half as long, the expected value of the difference between the two measurements will also be half as much. However, the error of measurement does not change. Thus as the interval of time between occasions becomes smaller, the proportion of error variance to difference in true scores becomes larger.

This type of oversampling in time can be can be still be extremely revealing of underlying dynamics. However, it requires the estimation of an optimal interval for the calculation of slope and curvature: the first and second derivatives of the variables of interest. The methods proposed here use all the data from an oversampled design, but estimate an interval for analysis that may be different than the interval between occasions of measurement.

A measurement interval that is too long with respect to the rate of change of the intrinsic dynamic will produce data that will very likely underestimate any temporal organization in the process and thereby produce an outcome that is an oversimplification of the true intrinsic dynamic. Undersampling, allowing too much time to elapse between measurements, will tend to underestimate curvature. Undersampling may also entirely miss a full cycle of self–regulation, thus artificially inflating the apparent stability of a process. Since so often reliability is equated with stability, one might be tempted to increase the interval between measurements since it apparently increases the reliability of estimation of slope. But this apparent increased reliability may simply be the effect of a biased estimate of the curvature.

When the intrinsic dynamic is oscillatory such as may be seen in a self–regulatory process, selecting the measurement interval becomes an especially difficult problem. One can produce positive, negative or zero test–retest correlations simply by measuring the same oscillatory process using different intervals between occasions (Nesselroade & Boker, 1994). It is our opinion that the measurement interval problem is a major source of the confusion and conflict that has plagued the analysis of change.

The methods proposed in the present work are no panacea for the measurement interval problem. However, the simulations presented here do suggest that an optimal measurement interval can be empirically determined, and that given such a measurement interval, parameters for dynamical systems can be accurately estimated even in the presence of considerable measurement error. A research design for this method requires an oversampling in time: occasions of measurement more closely spaced than the interval over which one would logically expect the dynamic to operate.

The Dampened Linear Oscillator

In keeping with the emphasis on modeling dynamical systems, we have chosen to illustrate temporally–organized intraindividual variation in a system by the use of the dampened linear oscillator (see e.g. Beltrami, 1987). We will develop and present the results of simulation experiments that assess the ability to recover the parameters for differential equations describing the behavior of dampened linear oscillators from very limited observations of the systems' behavior.

We will use the dampened linear oscillator as a simple and yet general model for the way that intraindividual variability may be patterned over time. A dampened linear oscillator is a dynamical system that behaves similarly to a pendulum with friction and produces a trajectory similar to that shown in Figure 1–b. Substantively, for example, one's mood may vary on a day to day basis about a trait–like overall mean value and thus high values at time 1 may be predictive of low values at time 2. If mood were to behave as a pendulum, an external event that causes a large deviation from the mean value of mood would, with no additional input, result in a trajectory over time that eventually settled into a more stable state.

This system produces a trajectory that is obviously curved, so why do we call it a *linear system*? As was indicated earlier, the definition of a linear system in dynamical systems theory is one in which the equation defining the relationship between the variables in the system consists only of a linear combination, a sum of variables multiplied by constants. A nonlinear system, in dynamical systems terms, is a system for which the equation cannot be reduced to a linear combination, for instance where two variables are multiplied together. The equation for the dampened linear oscillator is a standard multiple regression except that the variables, instead of being observed values for several variables for each individual, are the displacement of a single variable from equilibrium and the values of the first and second derivatives of this displacement. A model that expresses the relationship between a variable and its derivatives is called a differential equation model.

Differential Equation Model for the Dampened Linear Oscillator

In order to simulate a linear dampened oscillator and later estimate its parameters from the resulting data, we use a differential equation. This differential equation can be thought of as a multiple regression equation involving a variable x and its first derivative $\frac{dx}{dt}$ (the rate of change in x) and second derivative $\frac{d^2x}{dt^2}$ (the rate of change in the rate of change in x). For instance, if x were the displacement of a pendulum from its equilibrium point, the first derivative of x would be velocity and the second derivative of x would be acceleration.

The differential equation for a dampened linear oscillator similar to a pendulum with friction can be expressed as (Beltrami, 1987),

$$\frac{d^2x_{(t)}}{dt^2} - \zeta\frac{dx_{(t)}}{dt} - \eta x_{(t)} = 0 \ , \qquad (1)$$

where $x_{(t)}$ represents the value of a variable x at time t, η represents the frequency of oscillation and ζ represents the dampening. This second order differential equation can be rewritten as a system of two first order equations by rearranging so that,

$$\frac{d^2x_{(t)}}{dt^2} \quad = \quad \zeta\frac{dx_{(t)}}{dt} + \eta x_{(t)} \ . \qquad (2)$$

The wily reader may have noticed that Equation 2 looks very much like the familiar multiple regression equation where the second derivative of x is the outcome variable and $\frac{dx}{dt}$ and x are the predictor variables. When a stable interrelationship between a variable and its own derivatives occurs, the variable is said to exhibit intrinsic dynamics.

Note that there is no mathematical reason why the second derivative must be the outcome variable. However, when expressed in this way, the coefficients η and ζ become interpretable in such a way that they map to appealing concepts in psychology. The coefficient η is a function of the frequency of oscillation, how rapidly a self–regulatory cycle occurs. The coefficient ζ is a function of the dampening (when $\zeta < 0$) or amplification (when $\zeta > 0$) of the oscillator, similar to the respective concepts of resiliency or excitability.

When initial conditions $x_{(0)}$ and $\frac{dx_{(0)}}{dt}$ are chosen and equation 2 is numerically integrated, the result is a particular trajectory of the dampened linear oscillator. For instance, a pendulum at its resting state has initial conditions $x_{(0)} = 0$ and $\frac{dx_{(0)}}{dt} = 0$. If we move the position of the pendulum to $x_{(0)} = .1$ and give it a push with a velocity $\frac{dx_{(0)}}{dt} = .2$, the pendulum will describe a predictable trajectory until it comes to rest similar to the that shown in Figure 1–b. Note that many different trajectories could be created simply by starting the pendulum at different initial conditions. However, in each case, the relationship between the displacement, the velocity and acceleration of the pendulum would remain the same. It is this stable relationship that we wish to recover from a relatively few occasions of measurement from a sample of simulated "subjects" as was plotted in Figure 2

Simulating a population of oscillators

A simulation was performed in which 100 time series were generated by numerically integrating equation 2 using a fourth order Runge–Kutta algorithm (Ellis, Johnson, Lodi, & Schwalbe, 1992; Hubbard & West, 1991). The parameters for equation 2 were set so that the frequency parameter $\eta = -2.00$ and the dampening parameter $\zeta = -0.25$. These parameters were chosen so that within 100 time steps several oscillations had taken place and a visible amount of dampening had occurred. Each time series was given independent random initial conditions drawn by a pseudorandom number generator from a uniform distribution on the intervals $-10 < x_{(0)} < 10$ and $-5 < \frac{dx_{(0)}}{dt} < 5$. Runge–Kutta numerical integration was performed for each time series using a time step size $\Delta t = 0.1$ and iterated for 100 time steps.

In order to test estimation procedures against a reference criterion, the first and second derivatives were saved for each measurement in each time series. In this way, we could fit a regression model to the actual derivatives and values to see how well the parameters could be recovered when the derivatives do not need to be estimated. In empirical data the actual derivatives are usually unknown and must be estimated. We examined two methods for estimating the derivatives from the data, and by making comparisons between the outcomes of fitting the model to the actual derivatives versus fitting it to the estimated ones, we can understand to what degree any inaccuracy in the recovered parameters is due to derivative estimation and to what degree it is due to problems associated with the parameter estimation technique.

We can consider this simulation as resulting in time series representing the trajectories of 100 subjects. Each subject has his or her own independent initial conditions, but all subjects have the same underlying dynamical structure of intraindividual variability. Each time series is composed of 100 observations. Thus, in all we have simulated data for 100 individuals, each of whom was measured at 100 occasions.

Selecting a sample of occasions of measurement

The previously described simulation does not match the constraints posed by many experimental designs. 100 occasions of measurement is often prohibitively expensive to obtain. We tested whether the underlying parameters of the intrinsic dynamics of this population can be recovered with as few as three occasions of measurement for each individual participant. The following method was used to select a sample of occasions of measurement.

A first occasion of measurement t was chosen for each time series by selecting an independent pseudorandom number uniformly distributed on the interval $1 \leq t \leq 50$. A second and third occasion of measurement were chosen to be $t + \tau$ and $t + 2\tau$ where τ remained fixed for each sample of occasions of measurement. In this way sets of samples could be selected using a range of values for τ in order to examine the effect of choice of sampling interval on the results of estimating the parameters.

We also examined the effect of adding noise to the measurement samples. For each noise condition, a pseudorandom number drawn from an independent normal distribution with a mean of zero and a standard deviation of σ_e was added to each sample in each time series prior to the selection of occasions of measurement. The standard deviation of this additive measurement noise was fixed within each noise condition as one of the following: $\sigma_e = \{0.01, 0.05, 0.10, 0.50, 1.00\}$. Thus all 100 time series within each noise condition had measurement error from the same distribution.

The above described sampling method was repeated 1000 times for each choice of τ and choice of σ_e in order to estimate the stability of derivative estimation methods, and to generate empirical confidence intervals for the resulting parameter estimates.

Model to be fitted

To estimate the model parameters, a structural multiple regression model was fit to the simulated single dampened linear oscillator data. The same model was fit to three sets of data where three different methods were used to estimate the derivatives from each set of 3 occasions of measurement. The model was fit to the actual derivatives, to local linear approximations of the derivatives, and to derivatives estimated from a state–space embedding reconstruction of the phase space of the data (Parker & Chua, 1989) as described below.

The model had the form shown in Figure 3 which is the expression of equation 2 as a structural model. The manifest variable labeled **X** represents x, the manifest variable labeled **dX** represents $\frac{dx}{dt}$, the manifest variable labeled **d2X** represents $\frac{d^2x}{dt^2}$, and the latent variable labeled **e** represents the additive measurement error. The variance of **X** is the two–headed arrow labeled **VX**, the variance of **dX** is the two–headed arrow labeled **VdX** and the variance of the error is the two–headed arrow labeled **Ve**. The covariance between **X** and **dX** is the two–headed arrow labeled **Cx,dX**. The regression parameters by which **X** and **dX** predict **d2X** are labeled η and ζ respectively. In other words, η represents the frequency with which a variable tends to oscillate and ζ represents how rapidly the variable tends to return to a stable state when it is perturbed by outside forces. The regression parameter of the latent error variable **e** is fixed at the value 1.

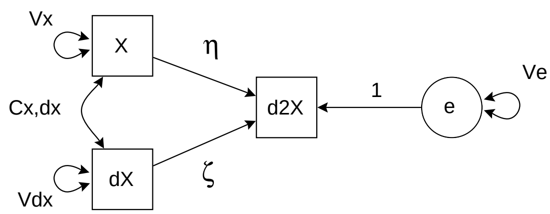

Figure 3. Multiple regression structural model for a single dampened linear oscillator.

Data to which model is fitted

Three methods were used to provide data to which the linear oscillator model was fitted. In each case each simulated individual provided three observations. As a comparison standard, a score and the true derivatives at a single occasion was sampled from each individual. Two methods of estimating derivatives were then compared against the standard, each of which started with three scores from each individual. Thus in each case we obtained a 100×3 data matrix.

True Derivatives.

When fitting the model shown in Figure 3 to the true derivatives, the data for the manifest variables **X**, **dX** and **d2X** were taken from the values of $x_{(t_i)}$, $\frac{dx_{(t_i)}}{dt}$, $\frac{d^2x_{(t_i)}}{dt^2}$ where t_i was the selected first occasion of measurement for each individual i. Thus a 100×3 data matrix was constructed where each row represented one subject i and the columns were the three values $x_{(t_i)}$, $\frac{dx_{(t_i)}}{dt}$ and $\frac{d^2x_{(t_i)}}{dt^2}$. A covariance matrix was calculated for this data matrix and the model shown in Figure 3 was fit using the Mx structural equation modeling software (Neale, 1994).

Local Linear Approximation.

When fitting the model from Figure 3 to the local linear approximations, a 100×3 data matrix must be constructed where each row represented one subject i and the columns were the value $x_{(t_i)}$, and estimated values for $\frac{dx_{(t_i)}}{dt}$ and $\frac{d^2x_{(t_i)}}{dt^2}$. These three values for each individual were estimated from three observations drawn from the individual's time series

X. Figure 4 presents a geometric view of the estimation of the local linear approximation of the first and second derivatives from three points, x_1, x_2 and x_3.

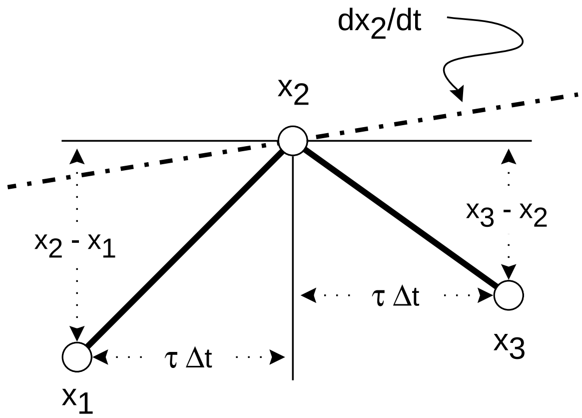

Figure 4. Local linear approximation of $\frac{dx}{dt}$. The local linear approximation of the $\frac{dx}{dt}$ at the point x_2 is the mean of the two surrounding slopes, $(x_2 - x_1)/(\tau\Delta t)$ and $(x_3 - x_2)/(\tau\Delta t)$. The approximation of the second derivative $\frac{d^2x}{dt^2}$ is the difference in the slopes with respect to time, $((x_3 - x_2) - (x_2 - x_1))/(\tau^2\Delta t^2)$.

These estimates for $\frac{dx_{(t_i)}}{dt}$ and $\frac{d^2x_{(t_i)}}{dt^2}$ were calculated as follows. Using one $[x_{(t_i)}, x_{(t_i+\tau)}, x_{(t_i+2\tau)}]$ sample from one time series i, we can estimate the velocity and acceleration at $x_{(t_i+\tau)}$ as

$$\frac{dx_{(t_i+\tau)}}{dt} \approx \frac{(x_{(t_i+\tau)} - x_{(t_i)}) + (x_{(t_i+2\tau)} - x_{(t_i+\tau)})}{2\tau\Delta t} \quad (3)$$

$$\approx \frac{x_{(t_i+2\tau)} - x_{(t_i)}}{2\tau\Delta t}\text{ , and} \quad (4)$$

$$\frac{d^2x_{(t_i+\tau)}}{dt^2} \approx \frac{(x_{(t_i+2\tau)} - x_{(t_i+\tau)}) - (x_{(t_i+\tau)} - x_t)}{\tau^2\Delta t^2} \quad (5)$$

$$\approx \frac{x_{(t_i+2\tau)} - 2x_{(t_i+\tau)} + x_{(t_i)}}{\tau^2\Delta t^2}\ . \quad (6)$$

Thus, when $\tau = 1$ and $\Delta t = 1$, the estimate of the first derivative becomes the average of two successive difference scores. Similarly, the estimate of the second derivative becomes the difference between two successive difference scores.

Here Δt is the time step chosen in the Runge–Kutta integration that created the simulated data. In practice, the value of Δt corresponds to the minimum time interval between measurement occasions in data with multiple occasions of measurement. We have separated this from the number of occasions that are skipped, τ, so that in data with more that 3 occasions of measurement we may search for an optimal sampling interval $\tau\Delta t$.

The values of $\frac{dx_{(t_i)}}{dt}$ and $\frac{d^2x_{(t_i)}}{dt^2}$ calculated for each individual from the three occasions of measurement were then used to fill a 100×3 data matrix, a covariance matrix was calculated and then the model parameters shown in Figure 3 were estimated using Mx.

State–Space Embedding.

State–space embedding is a technique in which the derivatives of a system can be estimated from a lagged data matrix. Due to theorems from Whitney (1936), Takens (1981), and Sauer, Yorke, and Casdagli (1991), a state space can capture all of the dynamics in the time series if the lag τ is chosen properly, and if the number of dimensions of the state space is sufficient. Taken's proof states that no more than $2d + 1$ lags are required when reconstructing a d–dimensional state–space for arbitrary systems. However, d lags are sufficient to fully capture a stationary linear oscillator.

The surprising result of these theorems is that one can reconstruct estimates of the derivatives solely by using the lagged measurements themselves. The accuracy of these estimates of the derivatives is critically dependent on choosing the proper lag when constructing this data matrix. If the system we are estimating is linear, the best τ will be the first zero crossing of the autocorrelation function for the variable (see Abarbanel, 1996, for a discussion). An illustration of the autocorrelation function for one of the simulated oscillators is shown in Figure 6.

Suppose that we can determine the proper value for τ. Then one sample $[x_{(t_i-2\tau)}, x_{(t_i-\tau)}, x_{(t_i)}]$ can be drawn from time series i and the derivatives $\frac{dx_{(t_i)}}{dt}$ and $\frac{d^2x_{(t_i)}}{dt^2}$ can be estimated as,

$$\frac{dx_{(t_i)}}{dt} \approx x_{(t_i-\tau)} \quad (7)$$

$$\frac{d^2x_{(t_i)}}{dt^2} \approx x_{(t_i-2\tau)} \,. \quad (8)$$

Results

Estimating from True Derivatives.

The model in Figure 3 was first fitted to the score and true derivatives from one time point chosen at a random occasion of measurement t_i in the interval $1 \leq t_i \leq 50$ from within each time series. In practice, of course, the true derivatives wouldn't be available. However, this estimation gives us an upper bound on how well the model could be expected to perform as a best–case. The parameter estimation was run once for each of the 5 different levels of added measurement noise.

As shown in Table 1, the parameters of the dampened linear oscillator can be recovered exactly from one time point from each of the time series when the true derivatives are known. As noise is added to the sampled time points, the parameter values are degraded, but do not begin to diverge until the σ_e is nearly 1.00.

Estimating from Local Linear Approximation.

The same model was next used to estimate η and ζ using local linear approximations for the derivatives. A randomized starting point t_i in the interval $1 \leq t_i \leq 50$ was chosen for each time series i. Three values of x were chosen from each time series $[x_{(t_i)}, x_{(t_i+\tau)}, x_{(t_i+2\tau)}]$ where τ was a fixed interval. At this point the randomized measurement noise was added to the three samples. Local linear approximations for the derivatives were then calculated from these three values and used as the data to which the model was fit.

Table 1: True simulated parameters and recovered parameters for the single dampened linear oscillator when the true derivatives are known. The standard deviation of the additive noise is σ_e.

		Noise Conditions				
σ_e		0.01	0.05	0.10	0.50	1.00
σ_x/σ_e		267	53.6	26.7	5.36	2.67
	Simulated	Recovered Parameters				
ζ	-0.25	-0.25	-0.25	-0.25	-0.21	-0.07
η	-2.00	-2.00	-1.98	-2.01	-1.85	-1.42
R^2	1.00	1.00	1.00	.998	.918	.764

Table 2: True simulated parameters and recovered parameters for the single dampened linear oscillator when the derivatives are estimated using local linear approximation with $\tau = 1$, $\tau = 3$ and $\tau = 6$, . The standard deviation of the additive noise is σ_e.

		Noise Conditions				
σ_e		0.01	0.05	0.10	0.50	1.00
σ_x/σ_e	∞	267	53.6	26.7	5.36	2.67
	Simulated	Recovered Parameters				
		$\tau = 1$				
ζ	-0.25	-0.32	-0.68	-0.20	-2.50	-5.25
η	-2.00	-1.95	-1.60	-1.63	-3.38	-23.37
R^2	1.00	.831	.167	.036	.021	.107
		$\tau = 3$				
ζ	-0.25	-0.25	-0.24	-0.16	-0.26	-1.22
η	-2.00	-1.98	-2.07	-2.04	-2.53	-4.41
R^2	1.00	.998	.956	.847	.208	.235
		$\tau = 6$				
ζ	-0.25	-0.25	-0.26	-0.25	-0.27	-0.16
η	-2.00	-1.88	-1.89	-1.89	-2.08	-2.49
R^2	1.00	1.00	.998	.988	.849	.687

The estimation was run once for each of 5 different levels of added measurement noise and the results are presented in Table 2. When $\tau = 1$ the model shows extreme sensitivity to additive noise. Even small amounts of noise cause the parameter estimates to vary considerably, and the R^2 proportion of explained variance in the model quickly approaches zero. The reason for this sensitivity can be understood by considering the local linear approximation equations 4 and 6. These equations have the value $\tau\Delta t$ or $\tau^2\Delta t^2$ in the denominator. When $\tau = 1$ and $\Delta t = .1$, the effect of the noise in the numerator will be amplified by dividing by a number smaller than 1. This effect is reduced as we increase the value of τ.

When τ is increased to a value of 3, as shown in the middle portion of Table 2, the estimates remain stable and close to correct as σ_e is increased to 0.10, but the estimates begin to deviate at higher noise levels. When $\tau = 6$, as shown in the bottom part of Table 2, the magnitude of the parameter η is underestimated for low noise levels, but becomes very close to correct when $\sigma_e = 0.50$. Note that the $R^2 = .847$ when $\tau = 3$ and $\sigma_e = .10$, the largest magnitude of noise that correctly recovers the parameters when $\tau = 3$. When $\tau = 6$ and $\sigma_e = .50$, the point at which the estimates are most nearly correct when $\tau = 6$, the $R^2 = .849$. This correspondence could be an indication of an interaction between noise level and the best choice for τ.

The potential interaction between τ and the level of noise was further explored by fitting models to local linear approximation data sets in which the value of τ was systematically varied in the range $1 \le \tau \le 25$. Figure 5 plots the results of these simulations for a noise level of $\sigma_e = 0.05$ and $\sigma_e = 0.50$. The value for ζ, the dampening parameter in the linear oscillator, is correctly recovered for a wide range of τ. However, the value for η is correctly recovered only for a narrow range of τ, and that range of τ is dependent on the noise level, σ_e added to the simulated data. For both noise levels, the range of τ that best recovers the parameters is when the value of R^2 first approaches its asymptotic value. Since this dataset is entirely deterministic, (i.e. noise is only measurement error, not noise added to the dynamics), we do not expect the predictability (in this case measured by R^2) to decay with larger τ. This will certainly not be the case in empirical data, where we would expect a decline in R^2 with increasing τ, so we expect the best value of τ to be when R^2 is at a maximum.

Estimating from State–Space Embedding.

The same model was next used to estimate ζ and η using state–space embedding to provide an estimate for the derivatives. A randomized starting point t_i in the interval $1 \le t_i \le 50$ was chosen for each time series i. Three values of x were chosen from each time series $[x_{(t_i)}, x_{(t_i+\tau)}, x_{(t_i+2\tau)}]$ where τ was a fixed interval. At this point the randomized measurement noise was added to the three samples. These values were then used as the derivatives ($\frac{d^2x_{(t_i+2\tau)}}{dt^2} \approx x_{(t_i)}$, $\frac{dx_{(t_i+2\tau)}}{dt} \approx x_{(t_i+\tau)}$, and $x_{(t_i+2\tau)} = x_{(t_i+2\tau)}$) and thus became the data to which the model was fit.

When using state–space embedding as an approximation for the derivatives of a system, the choice of τ becomes critical. The best choice of τ is the minimum τ that maximizes the spread of the points in the resulting state–space. For a linear periodic system, τ can be chosen to be the first zero crossing of the autocorrelation (see Figure 6). To pick a τ for state–space embedding, the autocorrelation function for one of the time series was calcu-

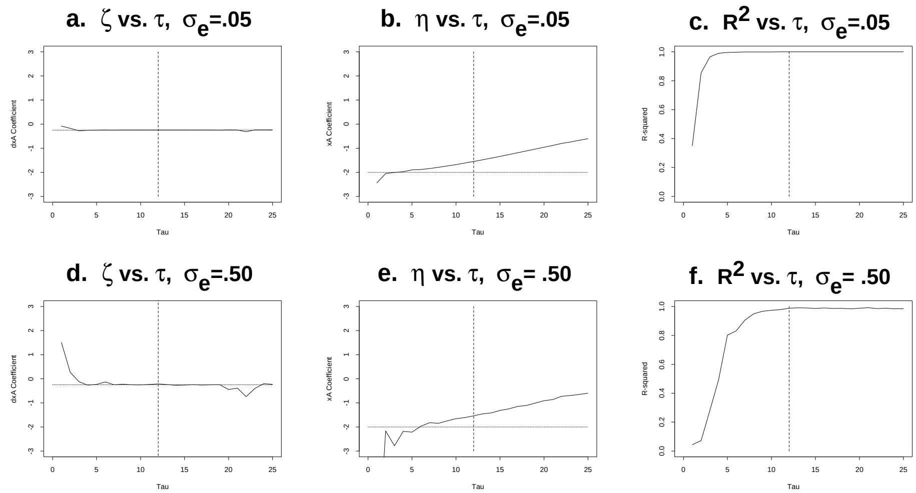

Figure 5. The interdependence of noise, τ and correct estimation of parameters using local linear approximation. (a, b, c) The estimated parameter values and R^2 for τ in the range $1 \leq \tau \leq 25$ and with additive noise with $\sigma_e = 0.05$. The estimates for η and ζ are closest to the correct values (indicated by a horizontal dotted line) when the R^2 is in the range $0.90 \leq R^2 \leq 0.99$, when it first approaches its asymptotic value. (d, e, f) The estimated parameter values and R^2 for τ in the range $1 \leq \tau \leq 25$ and with additive noise with $\sigma_e = 0.50$. The estimates for η and ζ are again closest to the correct values (indicated by a horizontal dotted line) when the R^2 is in the range $0.90 \leq R^2 \leq 0.99$, when it first approaches its asymptotic value.

lated for values of τ in the range $1 \leq \tau \leq 25$ as shown in Figure 6. Making this calculation with empirical data would be problematic in a study that only sampled at 3 fixed–interval time points. The dependence of parameters on analysis interval suggests that when the sampling interval is chosen ad hoc and many occasions of measurement are not obtained, it is unlikely that accurate estimates of dynamic parameters will be obtained using state–space embedding.

Table 3 contains the results of fitting the model from Figure 3 to the data obtained by state–space embedding estimates of the derivatives. The estimates for ζ, the dampening parameter, are almost zero. This is sensible, since we chose τ to minimize the correlation between $x_{(t_i)}$ and $\frac{dx_{(t_i)}}{dt}$. The estimates for η, the frequency parameter, are consistently underestimated. Although the parameter values are not as good as those estimated using the local linear approximation method, the state–space embedding estimation algorithm does appear to be less sensitive to noise.

The dependence of the parameter value estimates on the choice of τ in the state–space embedding method is illustrated in Figure 7. It can be seen that while the estimated value of η is consistently underestimated, it is relatively insensitive to the choice of τ. However, it is evident that the estimated value of ζ follows the autocorrelation function almost exactly. In both cases, the estimated parameter values do not seem to be particularly sensitive to the amount of additive noise in the system.

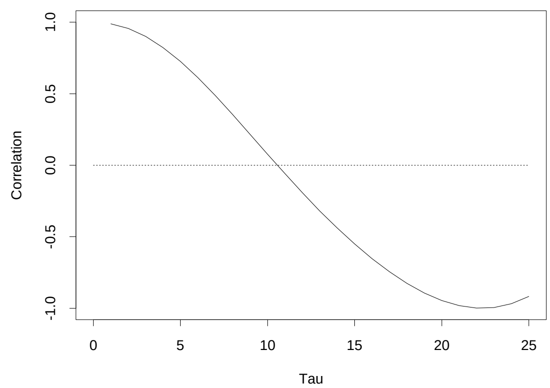

Figure 6. The autocorrelation for the first time series in the population. Note that the autocorrelation crosses 0 between $\tau = 11$ and $\tau = 12$.

Table 3: True simulated parameters and recovered parameters for the single dampened linear oscillator when the derivatives are estimated using state–space embedding with $\tau = 11$. The standard deviation of the additive noise is σ_e.

		Noise Conditions				
σ_e		0.01	0.05	0.10	0.50	1.00
σ_x/σ_e		267	53.6	26.7	5.36	2.67
	Simulated	Recovered Parameters				
ζ	-0.25	0.05	0.06	0.05	0.05	0.08
η	-2.00	-1.32	-1.32	-1.34	-1.25	-1.03
R^2	1.00	1.00	1.00	.996	.941	.729

Results from Only Measurement Error

The results presented above suggest that if a linear oscillator is generating the data, the linear approximation method of estimating derivatives allows the recovery of model parameters from that data using just a few waves of measurement. However, there is an attendant, complementary problem. Suppose all of the intraindividual variability was just noise as is shown in Figure 2–b. Would the linear approximation method create a result when none existed?

To test this, we applied the linear approximation method to a sample of normally distributed random numbers with a standard deviation of $\sigma = 5$, similar to those plotted in Figure 2–b. The results of this analysis are summarized in Figure 8. To begin with, note that although the correlation between x and $\frac{dx}{dt}$ (Figure 8–a) and the correlation between $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ (Figure 8) are near zero for all values of τ, the correlation between x and $\frac{d^2x}{dt^2}$ remains near to -0.8 for all values of τ. There is a simple explanation for this phenomenon

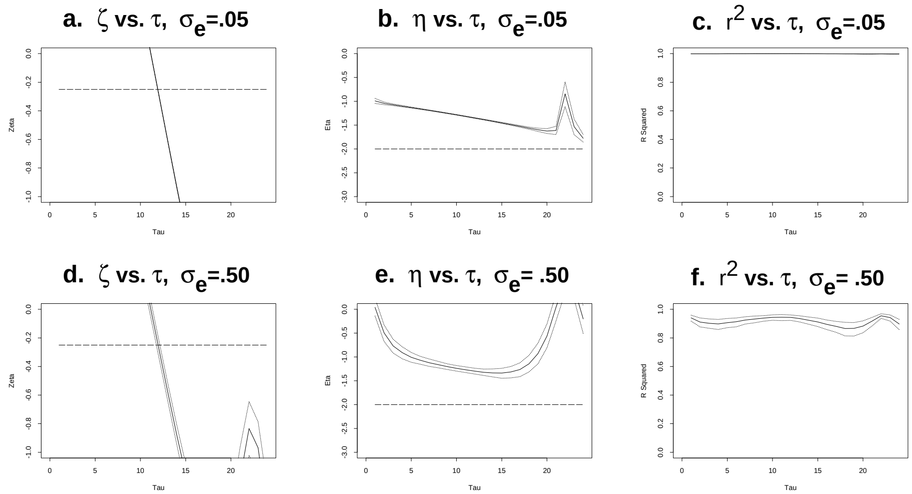

Figure 7. The dependence of the correct estimation of parameters on τ when using state–space embedding. (a, b, c) The estimated parameter values and R^2 for τ in the range $1 \leq \tau \leq 25$ and with additive noise with $\sigma_e = 0.05$. The estimates for η and ζ are closest to the correct values (indicated by a horizontal dotted line) when $\tau = 12$. (d, e, f) The estimated parameter values and R^2 for τ in the range $1 \leq \tau \leq 25$ and with additive noise with $\sigma_e = 0.50$. The estimates for η and ζ are again closest to the correct values (indicated by a horizontal dotted line) when $\tau = 12$. The interaction between τ and noise that was evident in local linear approximation does not appear in state–space embedding.

that is related toward regression to mean. Remember that $\frac{d^2x}{dt^2}$ is calculated for the second time point as the difference between the slope connecting the first and second time point and the slope connecting the second and third time point. If one plots the three scores against time and connects subsequent points by line segments, $\frac{d^2x}{dt^2}$ becomes the amount of "bend" between the two line segments connecting the three scores. If the first and third score are normally distributed random numbers about a mean of zero, and the second score is far from the mean, then there is a greater chance that there will be a large amount of bend than if the second time point is near the mean. Large positive values for x at the second time point are likely to be related to large negative values of $\frac{d^2x}{dt^2}$ and large negative values for x at the second time point are likely to be related to large positive values of $\frac{d^2x}{dt^2}$. Thus, there will be a substantial negative correlation between x and $\frac{d^2x}{dt^2}$. This negative correlation between x and $\frac{d^2x}{dt^2}$ means that there will be a relatively large R^2 when the oscillator model is fit to the random data, and thus x does a good job of predicting $\frac{d^2x}{dt^2}$ in simple measurement error in three wave panel data.

There are two characteristics of the results of the analysis of these data comprised entirely of measurement error that differ from the analysis of data generated by a self–regulating process. First, in these data the R^2 has no dependence on τ as was observed when the model was fit to the simulated oscillator data. Second, the R^2 from these data is unlikely to be as large as that observed when fitting to the simulated oscillator data. After fitting models to 20,000 replications of the random data, for more than 95% of the models

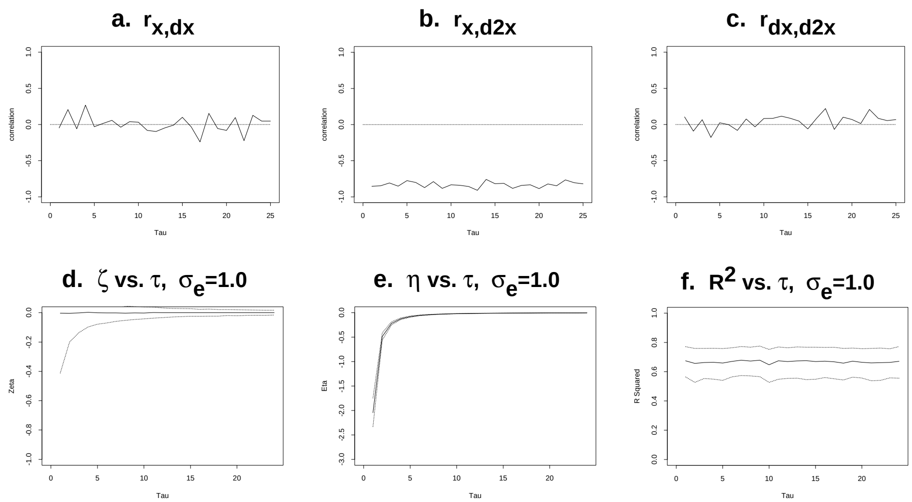

Figure 8. Cross correlations and estimated parameters of the linear oscillator model applied to random data. (a,b,c) Cross correlations between x, $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ for values of τ ranging from 1 to 25. Note that there is no dependence on τ, which would be expected if there were periodic structure to the intraindividual variability. (d,e,f) Estimated parameter values for η and ζ, and R^2 explained variance when the linear oscillator model is fit to the random data. Upper and lower empirical 95% confidence intervals are plotted as dashed lines and were estimated from the results of 1000 replications for each τ.

$R^2 \leq 0.766$ and for more than 99% of the models $R^2 \leq 0.798$. Thus a null hypothesis of no intrinsic dynamics for the linear oscillator model could be tested using these cutoffs for R^2. All of the usual caveats for null hypothesis testing should be born in mind (e.g. Cohen, 1994), but in addition recall that these cutoffs are specific to this particular linear oscillator model. Other models may produce other cutoffs.

The clearest indicator of periodic structure in a time series is the existence of a periodic relationship between autocorrelation or cross correlation and the analysis interval τ. Note that in Figures 8–a, b, and c there is no relation between the correlation between the variables and τ. Contrast the plot of the autocorrelation of the random data as illustrated in Figure 9 with the plot of the autocorrelation of the simulated linear oscillator in Figure 6. The random data show no relationship between the analysis interval τ and the autocorrelation, whereas the simulated oscillator has a strong relationship between τ and the autocorrelation. The most telling diagnostic is that the autocorrelation for the random data is near zero for all values of τ, thus there is no evidence that there is any dynamical process at all in the random data.

A variation on the surrogate data test (Kennel & Isabelle, 1992; Schreiber & Schmitz, 1996; Theiler, Eubank, Longtin, & Galdrikian, 1992) can be performed to test for the presence of a dynamical process in time series data. This test is quite simple and can be applied to multi-wave panel data. This test relies on the methods of statistical resampling (Efron, 1979b, 1979a). First, 20 surrogate data sets are created by scrambling the time ordering of the real data. Then the autocorrelation of the real data is tested to see if it is lies outside the distribution of autocorrelations from all 20 surrogates. If the real

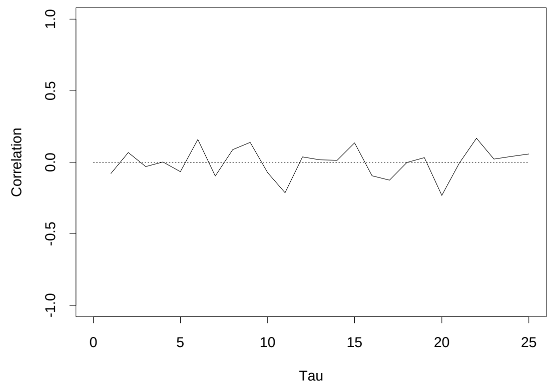

Figure 9. The autocorrelation for the random time series.

autocorrelation can be distinguished from the surrogates, then we can say that it is unlikely ($p < 0.05$) that the observed autocorrelation for this distribution of scores would be due to chance alone.

In the case of multi-wave panel data, this type of surrogate data set can be created by essentially dumping all of the scores in one large bin, then drawing scores from that bin and assigning them to individuals and occasions of measurement at random. If this procedure of random assignment of scores to individuals and occasions of measurement produces autocorrelation functions that cannot be distinguished from the original data, then there is no point in modeling the intraindividual variability as a dynamical process.

General Discussion

One of the main results of the simulations and analyses presented here is a demonstration that a local linear approximation of derivatives can accurately recover the parameters of a simulated linear oscillator with the restrictive assumptions that (a) the oscillator is a homogeneous process in a population and (b) the process is stationary over time. However, we have demonstrated that the parameters of the model can be recovered even when each individual in the population comes to the experiment with different initial conditions (i.e. the data exhibit random phases), only a few observations are available from each individual, and there is measurement error added to each observation. In practical terms, this means that there is reason to believe that if there were a self–regulatory process that led to intraindividual variability for a particular variable, this process could be distinguished from measurement error in data resulting from research designs that are reasonable for use in psychology.

The second main result is showing that the state–space embedding method for approximating derivatives did not accurately recover the parameters of the simulated oscillator and consistently underestimated the contribution of the frequency parameter even though by measures of R^2, the state–space embedding method was explaining just as much of the variance in the second derivative as was the local linear approximation method. In addition, the state–space embedding method was particularly sensitive to the choice of analysis interval τ and produced results for the frequency parameter that were dependent on the chosen τ. These problems are particularly worrisome since the state–space embedding method is essentially a form of autoregression or cross–lag regression. If the underlying system that generated the periodic structure was periodic, autoregression and cross–lag regression carry with them the same problems evidenced in the models estimated from state–space embedding, such as parameter values that can be directly manipulated by choice of analysis interval. It is thus recommended that if there is reason to believe that there is a periodic component within the intraindividual variability of a variable, then state–space embedding, autoregression and cross–lag regression should not be employed unless there are additional compelling reasons for fixing a specific value or values for τ.

The third result of this study is evidence that there are promising methods to deal with the measurement interval problem, although this requires occasion–intensive measurement on a subset of individuals. By analyzing the dependence between the autocorrelation and analysis interval and the dependence between the residual variance and analysis interval we can pick an analysis interval that is most likely to give accurate parameter estimates for the parameters of the intrinsic dynamics of a system. This best interval is the one at which the R^2 is just beginning to reach its asymptote. In practice, this means that a pilot study might be more usefully designed to be occasion–intensive rather than individual–intensive and after an appropriate measurement interval was chosen the full study could be performed with fewer waves of measurement than would otherwise be required.

Specifically, if a process is expected to not have individual differences in parameters and is expected to be stationary over time, we recommend designing a pilot experiment with a minimum of 16 occasions of measurement per individual and with the interval between measurements set so that at least eight measurements are scheduled over the period of one expected cycle of the process. In this way, one may vary the analysis interval τ so as to estimate the interval that will produce the least bias given the relative observed levels of intrinsic dynamics and measurement error. If there is reason to suspect that there may be individual differences in the parameters of the intrinsic dynamic, around 90 observations per individual may be needed to estimate a separate model for each individual. Multilevel modeling using differential equations is currently being explored and may be able to substantially reduce this requirement if assumptions are met concerning the distribution of the intrinsic dynamic parameters in the population.

The present simulation demonstrates that given some assumptions, the intrinsic dynamics of a population of linear oscillators can be recovered from as few as three samples per oscillator. Thus one can reasonably expect that in cases where intrinsic dynamics are responsible for intraindividual variability then the parameters of these dynamics could be estimated from multi–wave panel data. However, there are more than a few bumps and potholes yet remaining in this road to understanding. Real data tend not to be as orderly as the data presented here. For instance, the assumptions of homogeneity and stationarity may be violated in a sample. The underlying process might include dynamical noise or be nonlinear. The model chosen for theoretical reasons may have no relationship to the true underlying process. Any of these problems would interfere with the accurate estimation of parameters and would give rise to difficulties that we have not addressed.

The methods that are presented here are applicable to a wide variety of linear systems and aren't dependent on particular characteristics of the linear oscillator model that was chosen as a target for the simulation. Other candidates include coupled oscillator systems, nonlinear oscillators, exponential growth models, and predator–prey models. Some nonlinear systems may be appropriately analyzed with the local linear approximation method, but for other nonlinear systems this may be problematic. In particular, nonlinear systems that include both divergent and convergent components (i.e. increasing and decreasing variance along different dimensions) will pose difficulty for the linear approximation method due to potential sensitive dependence on initial conditions. Methods for dealing with these types of nonlinear systems as yet require large numbers of occasions of measurement, on the order of thousands of measurements per individual, and thus are currently impractical for many psychological applications.

Conclusion

Are human beings dampened linear oscillators? Probably no more so than they are hydraulic systems, telephone switchboards, computers or any of countless other models that have been used to represent human behavior in the past century or so. The point is that the behavior of the dampened linear oscillator is sufficiently similar to aspects of compelling intraindividual variability in human beings that its usefulness for representing the latter, especially from the standpoint of prediction, needs to be explored and evaluated. For instance, the dampened linear oscillator model has provided evidence of self–regulating oscillations in adolescent substance use (Boker & Graham, 1998) and has been used to elucidate the development of postural control (Boker, 2000).

But the methods presented here are not just applicable to the dampened linear oscillator model. These results underscore the feasibility of fitting and testing the value of different classes of linear and nonlinear dynamical systems models with the kind of data that are typically available in designs that are prevalent in behavioral research. Emotion regulation may have intrinsic self–regulatory mechanisms as well as mechanisms for coupling regulation to environmental changes. There may be self–regulating mechanisms that control use of alcohol and tobacco, which are coupled to other psychological variables, and whose parameters are partially determined by genetics. More complicated systems can be posited, for instance linking changes in an overall mental health inventory to more specific changes in self–efficacy, social support and external life events. Each of these systems may have parameter values at which stability and resiliency is acheived, and other parameter values at which the system becomes dysfunctional in specific symptomatic ways. One point of modeling these systems is to be able to identify and manipulate these parameters in such a way that a dysfunctional system can begin to perform normal self–regulation.

The differential modeling technique discussed in this article may be applied to any structural equation model in which first and/or second derivatives play a role in prediction and/or outcome. Any time one hypothesizes that a psychological system might adapt based on change in itself or its environment, a specific model can be built from that hypothesis. The value of hypothesizing explicit models of process and testing them against meaningful data simply cannot be overestimated for cognitive, developmental, clinical, indeed virtually all areas of behavioral research. Encouraging behavioral scientists, the majority of whom have been trained in the linear, static traditions of correlation and regression (including

ANOVA) analysis, to explore these promising alternatives is a challenge the acceptance of which this demonstration is designed to provoke.

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