Mixed Models for Gerontological Research 2001 Gerontological Society of America Preconference Workshop

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1 Today's Tour

1. General Introduction
2. Mixed Model Equations.
3. Example: Nonindependent Samples.
4. Introduction to SAS PROC MIXED.
5. Example: Individual Differences in Intercept.
6. Introduction to Longitudinal Models and Data.
7. Example: Age Related Change in IQ.
8. Predicting Change using Occasion of Measurement.
9. A Momentary Glimpse at Underlying Theory.
10. Individual Differences in Level and Slope.
11. Age–Group Differences in Change.
12. REPEATED versus RANDOM Statement in PROC MIXED.
13. Age Related Differences in Level, Slope and Curvature.
14. Missing Data and Mixed Models.
15. Summary.

[1] All materials, scripts and example datafiles may be found at http://www.nd.edu/~sboker

2 Introduction

2.1 Why Mixed Models?

1. Model relationships between outcome variables and predictors that are grouped somehow within the population.
2. Models for multilevel data.
3. Models for longitudinal data.
4. Models for repeated measures.
5. Models with moderating variables.
6. Models for data that are incomplete or with missing observations.
7. Models for quasiexperimental or epidemiological designs.
8. Models for dynamical systems.

2.2 Some Definitions

1. Random Effects: Coefficients that are associated with individuals or groups and that can be considered to be drawn randomly from a distribution.
   - Uses
     (a) Accounting for interindividual differences.
     (b) Accounting for group differences.
     (c) Accounting for multilevel differences.
   - Examples
     (a) Mean IQ for an age group.
     (b) Change in IQ for an age group.
     (c) Mean IQ for an individual measured at several occasions.
     (d) Change in IQ for an individual measured at several occasions.
     (e) Effect of strength training on performance of daily tasks.
     (f) Effect of coping intervention on severity of depressive symptoms.
2. Fixed effects: Population coefficients or levels of experimental or grouping factors.
   - Uses
     (a) Predicting the strength of association between two other variables.
     (b) Accounting for group differences.
     (c) Predicting change over time.
   - Examples

   (a) Assigned treatment group.
   (b) Level of an observed grouping variable.
   (c) Gender.
   (d) Ethnicity.
   (e) Age.
   (f) Cohort group.
   (g) Zygosity.
   (h) First occasion level of a variable.

3. Mixed Effects: A model with both fixed and random effects.

   - Uses
     (a) Accomodate many types of designs and analyses.
     (b) Relaxes some assumptions of standard methods.
     (c) More realistic models for change, i.e. growth or decline.
     (d) Can simulateously account for individual differences in mean level and individuals differences in change.
   - Examples
     (a) Age dependent rates of change.
     (b) Age dependent strength of association between variables.

4. Other terms for models with mixed effects.

   - Hierarchical Linear Models (HLM).
   - Multilevel Models.
   - Random Effects Models.
   - Random Coefficients Models.

3 Equations for Multilevel Model

1. Need to specify two *levels* of equations: within person (or group) and between person (or group).

2. Consider a simple level one model with one predictor.

$$y_{ij} = b_{0i} + b_{1i}x_{ij} + e_{ij} \ , \tag{1}$$

   where y_{ij} and x_{ij} are the outcome and predictor variables for person i on occasion of measurement j, b_{0i} is the intercept term for person i, b_{1i} is the regression coefficient for person i and e_{ij} is the residual.

3. Note that each individual person has their own intercept and regression coefficient.

4. These intercepts and coefficients are assumed to be distributed normally in the population.

5. The second level models predict the intercepts and regression coefficients.

6. The simplest case just forces these b weights to be equal across individuals,

$$b_{0i} = c_{00} \tag{2}$$
$$b_{1i} = c_{10} \ , \tag{3}$$

   where c_{00} and c_{10} are constants across all individuals.

7. This model is no different than ordinary regression.

8. The same simple one predictor first level model could have other second level models.

9. One interesting second level model accounts for individual differences in these b weights.

$$b_{0i} = c_{00} + u_{0i} \tag{4}$$
$$b_{1i} = c_{10} + u_{1i} \ , \tag{5}$$

   where c_{00} and c_{10} are constants across all individuals and u_{0i} and u_{1i} are constants that are unique to each individual i.

10. An even more interesting second level model accounts for individual differences in these b weights as well as predicting them with a third variable.

$$b_{0i} = c_{00} + c_{01}z_i + u_{0i} \tag{6}$$
$$b_{1i} = c_{10} + c_{11}z_i + u_{1i} \ , \tag{7}$$

    where z_i is a variable measured on individual i that is constant across occasions of measurement.

4 Example: Individual Differences in Intercepts

1. Suppose that an experiment was performed in which 80 participants were each measured on a strength task at 2 occasions.

2. The first occasion of measurement was prior to a weight training intervention and the second occasion was after the training.

3. A naive researcher decides to predict the scores on the strength task using a dummy code for whether the occasion of measurement happened before or after the training.

4. Here is a matrix scatterplot of these data. The dummy coded pre-post training variable is labeled $TRIAL$ and the strength score is labeled Y.

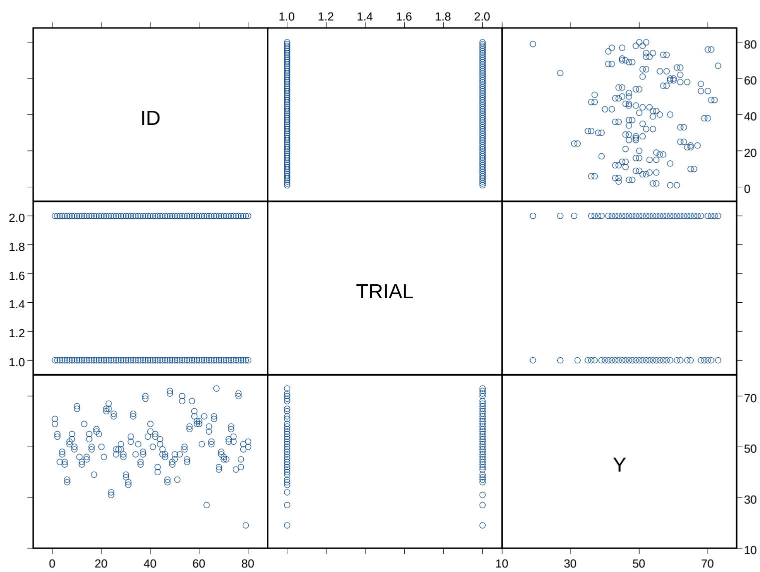

Figure 1: Matrix scatter plot of strength test experiment 1.

5. There seems to be little, if any, relationship between $TRIAL$, whether the strength test was after the training, and Y, the outcome of the strength test.

6. Using SAS, we can take a quick look at the summary statistics for this dataset.

```
PROC MEANS DATA=quirk.exp1data N MEAN STD MIN MAX;
RUN;
```

This results in the following output:

```
Variable   N          Mean       Std Dev       Minimum       Maximum
----------------------------------------------------------------
```

```
ID        160    40.5000000    23.1647097     1.0000000    80.0000000
TRIAL     160     0.5000000     0.5015699     0.0000000     1.0000000
Y         160    50.9062500    10.1577475    19.0000000    73.0000000
AGE       160    75.8250000     5.4663637    65.0000000    84.0000000
```

7. Using the following call to PROC GLM, we can recreate the naive analysis.

```
PROC GLM DATA=quirk.exp1data;
     MODEL y = trial;
RUN;
```

Here is the relevant section of the output from the analysis:

```
    R-Square      Coeff Var      Root MSE       Y Mean
    0.000771       20.00915      10.18591     50.90625

                                    Standard
 Parameter           Estimate          Error    t Value    Pr > |t|
 Intercept        50.06250000     2.54647758      19.66      <.0001
 TRIAL             0.56250000     1.61053383       0.35      0.7274
```

8. It looks like it's time to come up with another theory about the effectiveness of strength training. Or is it? Maybe there was something wrong with the analysis.

9. Let's look at how the first 15 participants fared before and after the training.

10. It looks like there large individual differences in strength prior to the training and that the effect of the training (if any) is small in comparison.

11. Let's try accounting for the individual differences in the starting strength using a mixed model. We'll assume that individuals come into the experiment with a unique strength but that the training affects everyone in the same way.

$$
\begin{aligned}
y_{ij} &= b_{0i} + b_1 x_{ij} + e_{ij} \\
b_{0i} &= c_{00} + u_{0i}
\end{aligned}
$$

where y_{ij} is the measured strength of person i on occasion j and x_{ij} is dummy coded as 1 if occasion j is after the training. There is one random coefficient, the intercept b_{0i}, which has a grand mean value c_{00} and a unique contribution u_{0i} for each person.

12. Now let's use SAS PROC MIXED to fit this model to the data.

```
PROC MIXED DATA=quirk.exp1data;
  MODEL y=trial / S;
  RANDOM int / SUB=id TYPE=un gcorr;
RUN;
```

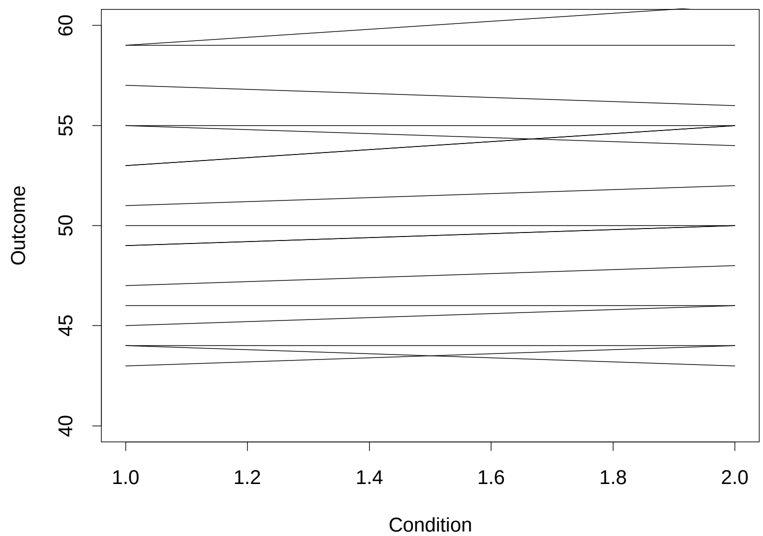

Figure 2: Pre and post test strength for 15 participants in strength test experiment 1.

We will go over the syntax and output in greater detail as the workshop progresses. For now, let's just focus on the relevant section of the SAS output.

```
                  Solution for Fixed Effects

                              Standard
Effect        Estimate         Error       DF     t Value     Pr > |t|
Intercept      50.0625        1.1526       79       43.43       <.0001
TRIAL           0.5625        0.1256       79        4.48       <.0001
```

13. Extra! Extra! Read all about it! Strength training theory saved!
14. The between–persons differences were overwhelming a smaller, but reliable, within–persons effect.
15. As many of you may have alertly noted by now, this same analysis could have been performed using a t–test. It is our hope that this exercise will help you integrate these mixed models equations into your existing statistical knowledge.
16. Scott will return to the relationship between mixed models and other statistical techniques in a bit, but first, let's try another introductory example.

5 Example: Nonindependent Observations

1. A second researcher doesn't believe the results from the strength study above. He thinks that there may have been problems with the reliability of the measure of strength.

2. His theory suggests that there also may be an effect of age such that the strength of older individuals may start at a lower level than that of younger individuals.

3. A second study is designed in which 8 participants are tested for strength 5 times prior to training and 5 times after training.

4. Let's look at the summary statistics for the resulting dataset. A centered age score, *AGEC* is created by subtracting the sample mean age from each person's age.

```
PROC MEANS DATA=quirk.exp1bdata N MEAN STD MIN MAX;
RUN;
```

```
Variable   N          Mean      Std Dev      Minimum      Maximum
-----------------------------------------------------------------
ID        80     4.5000000    2.3057441    1.0000000    8.0000000
TRIAL     80     0.5000000    0.5031546    0.0000000    1.0000000
Y         80    52.0100874    3.5853383   44.5321983   61.2197334
AGE       80    73.5000000    6.0378553   67.0000000   83.0000000
AGEC      80     0.0000000    6.0378553   -9.5000000    6.5000000
```

5. Next let's plot the four variables of interest as a matrix scatter plot. (These plots were created using Splus).

6. This researcher is just as naive, if not more so, as was the previous researcher.

7. First he uses PROC GLM to predict the strength score using the dummy coded TRIAL variable.

```
PROC GLM DATA=quirk.exp1bdata;
     MODEL y = trial;
RUN;
```

Here is the relevant section of the output from the analysis:

```
    R-Square     Coeff Var      Root MSE        Y Mean
    0.014494      5.488194      2.783751      50.72253

                                  Standard
Parameter         Estimate           Error     t Value    Pr > |t|
Intercept      50.38917742      0.44014964      114.48      <.0001
TRIAL           0.66669779      0.62246559        1.07      0.2874
```

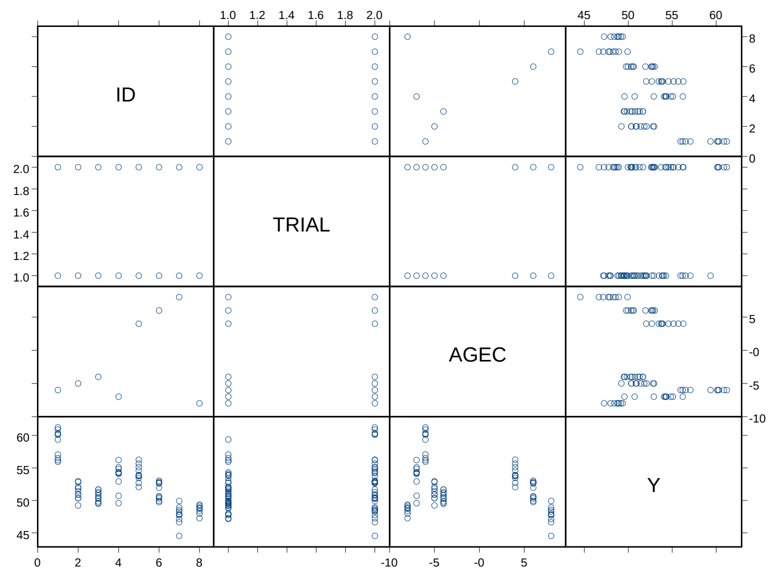

Figure 3: Matrix scatter plot of strength test experiment 2.

8. It doesn't look much better than the naive analysis of the previous experiment. But perhaps there's an effect of age?

```
PROC GLM DATA=quirk.exp1bdata;
     MODEL y = agec;
RUN;
```

Here is the relevant section of the output from the analysis:

```
    R-Square      Coeff Var       Root MSE         Y Mean
    0.205635       4.927310       2.499256       50.72253

                                         Standard
Parameter           Estimate                Error    t Value    Pr > |t|
Intercept        50.72252632           0.27942535     181.52      <.0001
AGEC             -0.20926676           0.04657089      -4.49      <.0001
```

9. Oh yes! Our naive researcher is happy. A helpful reviewer points out that he should have simultaneously tested the effects of age and training as well as their interaction.

```
PROC GLM DATA=quirk.exp1bdat;
     MODEL y = trial agec trial*agec;
RUN;
```

```
R-Square     Coeff Var      Root MSE       Y Mean
0.226597      4.925420      2.498297     50.72253
```

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Intercept	50.38917742	0.39501548	127.56	<.0001
TRIAL	0.66669779	0.55863625	1.19	0.2364
AGEC	-0.17215248	0.06583591	-2.61	0.0108
TRIAL*AGEC	-0.07422856	0.09310604	-0.80	0.4278

10. It seems that there is still an effect of age on strength after accounting for the (nonsignificant) effect of the strength training.

11. To get a better idea of what's going on, let's graph the scores grouped by participant along with a line connecting each participant's means in each condition.

12. The within–subject/within–condition differences are relatively small. But our naive researcher was ignoring the fact that these observations were not independent. Since there are 5 observations for each subject in each condition, each subject was contributing her or his age to the analysis 10 times!

13. Now let's fit a mixed model to these data. The theory suggested that age of the participant might account for some of the variance in strength. We'll assume that individuals come into the experiment with a unique strength that may be related to age, but that the training affects everyone in the same way.

$$
\begin{aligned}
y_{ij} &= b_{0i} + b_1 x_{ij} + e_{ij} \\
b_{0i} &= c_{00} + c_{01} z_i + u_{0i}
\end{aligned}
$$

where y_{ij} is the measured strength of person i on occasion j and x_{ij} is dummy coded as 1 if occasion j is after the training and z_i is the age of person i when entering the study.

14. SAS PROC MIXED can fit this model to the data.

```
PROC MIXED DATA=quirk.exp1bdat;
  MODEL y=trial agec trial*agec;
  RANDOM int / SUB=id TYPE=un gcorr;
RUN;
```

In just a bit, Scott will go over how the model equations are converted into the SAS syntax in more detail. For now, let's focus on the relevant section of the SAS output.

```
                  Solution for Fixed Effects

                          Standard
```

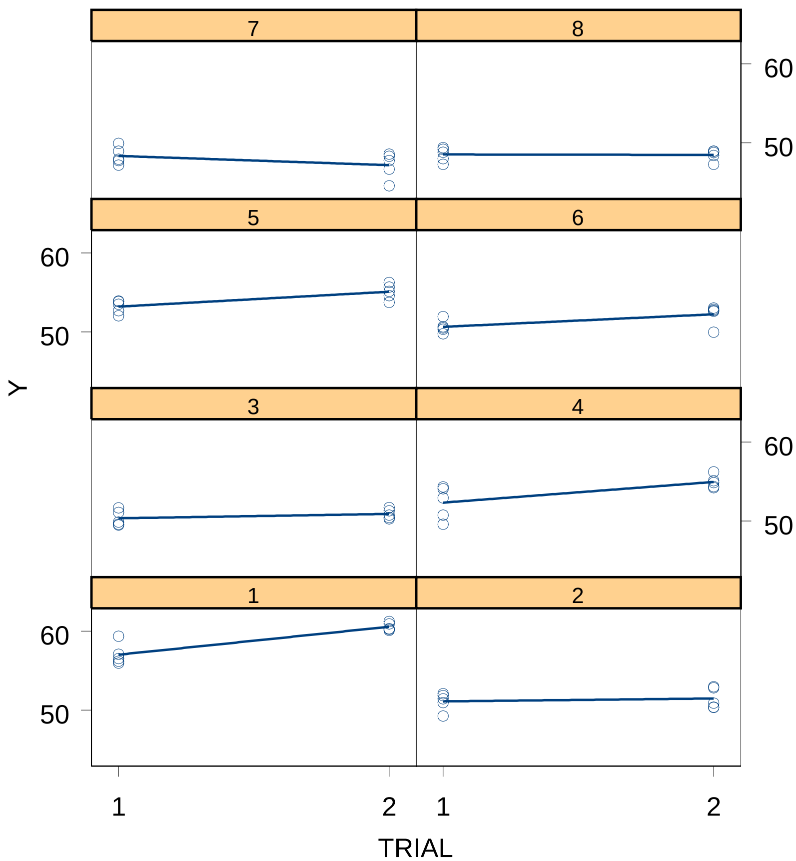

Figure 4: Pre and post training for each of 8 participants in strength test experiment 2.

Effect	Estimate	Error	DF	t Value	Pr > |t|
Intercept	50.3892	0.8726	6	57.75	<.0001
TRIAL	0.6667	0.2916	70	2.29	0.0253
AGEC	-0.1722	0.1454	70	-1.18	0.2405
TRIAL*AGEC	-0.07423	0.04860	70	-1.53	0.1312

15. After correcting for the nonindependence of the within–person observations, accounting for the individual differences in strength at the beginning of the experiment, and allowing an effect of age on those individual differences, it turns out that there *is* still an effect of strength training, but *no* effect of age.

16. This time, ignoring the nonindependence of the ages and strengths of the individuals during repeated measurements gave a spurious answer as well as missing the correct answer!

17. Scott Maxwell will continue the workshop with an introduction to longitudinal modeling using mixed effects.

6 Introduction to Longitudinal Models and Data

- Why longitudinal?
  1. Statistical power
  2. Cohort effects
  3. Missing data mechanisms
- Multilevel models distinguish interindividual change from intraindividual change
- Key conceptual point is that this approach focuses on change on the individual level
- Approach begins by developing a model for each person's trajectory over time

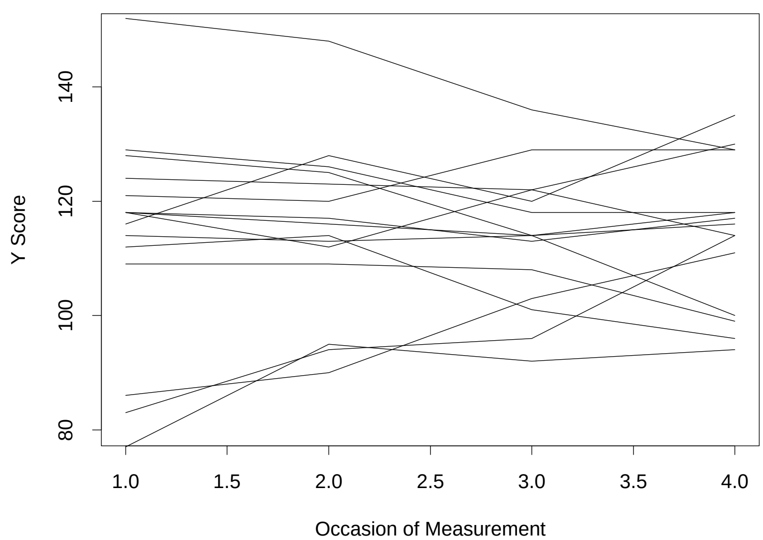

Figure 5: Longitudinal scores for 15 simulated subjects at 4 occasions of measurement.

6.1 HLM "versus" SEM

- HLM models each person's trajectory as a continuous function of time
- For example, in a few minutes we will consider a straight line growth model—presumes each person's trajectory is a straight line, with 2 parameters—a slope and an intercept
- This constitutes our "level-1" model, at the level of the individual—notice this is then a model of intraindividual change

- We can then create a level-2 model, which attempts to explain the level-1 parameters in terms of differences between individuals—here is where interindividual differences come into play
- For example, younger individuals might have different intercepts and slopes from older individuals—or treatment subjects might have different slopes from control subjects
- Traditional SEM models, on the other hand, typically model scores at each time point as a function of scores at other time points—in so doing, the focus is exclusively on interindividual differences
- In other words, SEM attempts to explain how differences between individuals at one time point are related to differences between individuals at other time points
- HLM, on the other hand, begins by focusing on the pattern of change within each person
- 2 further points before proceeding:
  1. In reality, SEM can also be used to build models of intraindividual change through latent growth curve analysis—see, for example, Willett & Sayer (1994)
  2. For further reading on basic differences between HLM and SEM, see Rogosa (1988) and (1995)
     - Rogosa, D. (1988). Myths about longitudinal research. In Schaie, K. W., & Campbell, R. T. (Eds.), Methodological issues in aging research. New York: Springer.
     - updated in Gottman, J. M. (Ed.). The Analysis of Change

6.2 Data collected on 80 individuals of various ages at 3 time points

- Eventually deal with fact that individuals differ in age at Time 1, but initially consider only longitudinal aspect of data
- So for the moment our purpose is to investigate the pattern of scores across 3 time points, irrespective of individual's age—for example, do scores tend to decline from time 1 to time 2 to time 3
- Notice this is also the type of question we would probably want to ask if these 3 time points followed some discernible event, such as an intervention where we assigned individuals to groups, or a naturally occurring event such as the death of a spouse, where we might want to examine individuals at baseline and at 2 (or more) later points in time
- Here are the scores for the 1st 20 individuals

Obs	AGE	y1	y2	y3	old	lin
1	71	103	104	111	0	8
2	67	106	110	113	0	7
3	84	115	122	121	1	6
4	78	123	124	137	1	14
5	65	113	112	118	0	5
6	72	114	105	105	0	-9
7	67	75	83	92	0	17
8	85	59	47	36	1	-23
9	73	109	114	110	1	1
10	71	114	129	125	0	11
11	73	111	114	122	1	11
12	79	116	111	114	1	-2
13	81	131	141	142	1	11
14	69	99	106	108	0	9
15	73	106	113	112	1	6
16	72	108	116	116	0	8
17	69	121	112	119	0	-2
18	74	110	117	119	1	9
19	81	106	100	104	1	-2
20	76	124	123	122	1	-2

- Before jumping into data analysis, need to take a momentary detour to consider data management

  1. The format of the data is in what is called “multivariate form”
  2. Notice that there is one line per individual, and the IQ score at each time point is represented by its own score
  3. In other words, y1 is IQ at time 1, y2 is IQ at time 2, and y3 is IQ at time 3
  4. This arrangement of data is fine for some purposes, but usually not what we need for fitting mixed models
  5. For SAS PROC MIXED (and also for HLM), we need scores in “univariate form” where we have only 1 variable for IQ
  6. Fortunately, don’t need to retype data, but instead can use SAS to reformat the data

```
data quirk.gsauni;
  set quirk.gsamult3;
  wave=1;
  y=y1;
  output;
  wave=2;
  age=age+1;
  y=y2;
```

```
    output;
    wave=3;
    age=age+1;
    y=y3;
    output;

  data quirk.gsauniv;
    set quirk.gsauni;
    wavezero=wave-1;
    wavcmean=wave-2;
    agec75=age-75;
    agec75sq=agec75*agec75;
```

7. This results in the data being reformated as the following.

Obs	id	y	wave	wavezero	wavcmean	AGE	agec75	agec75sq
1	1	103	1	0	-1	71	-4	16
2	1	104	2	1	0	72	-3	9
3	1	111	3	2	1	73	-2	4
4	2	106	1	0	-1	67	-8	64
5	2	110	2	1	0	68	-7	49
6	2	113	3	2	1	69	-6	36
7	3	115	1	0	-1	84	9	81
8	3	122	2	1	0	85	10	100
9	3	121	3	2	1	86	11	121
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
235	79	89	1	0	-1	68	-7	49
236	79	94	2	1	0	69	-6	36
237	79	105	3	2	1	70	-5	25
238	80	163	1	0	-1	67	-8	64
239	80	159	2	1	0	68	-7	49
240	80	154	3	2	1	69	-6	36

Trajectory Plot for 1st 10 Individuals

```
goptions reset=symbol;

symbol1 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol2 color=black interpol=join
        width=1 value=none
        height=0 line=1;
```

```
symbol3 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol4 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol5 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol6 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol7 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol8 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol9 color=black interpol=join
        width=1 value=none
        height=0 line=1;

symbol10 color=black interpol=join
        width=1 value=none
        height=0 line=1;

data tempmix;
  set quirk.gsauniv;
  if id le 10;

proc gplot data=tempmix;
  plot y*wave=id / nolegend haxis=1 to 3 hminor=0;
run;
```

- Fit a variety of models to these data, some traditional and some recent
- Begin with some traditional analyses
- We want to know whether IQ changes as a function of wave, so we could ask whether wave predicts IQ, in which case regression seems to be a natural choice

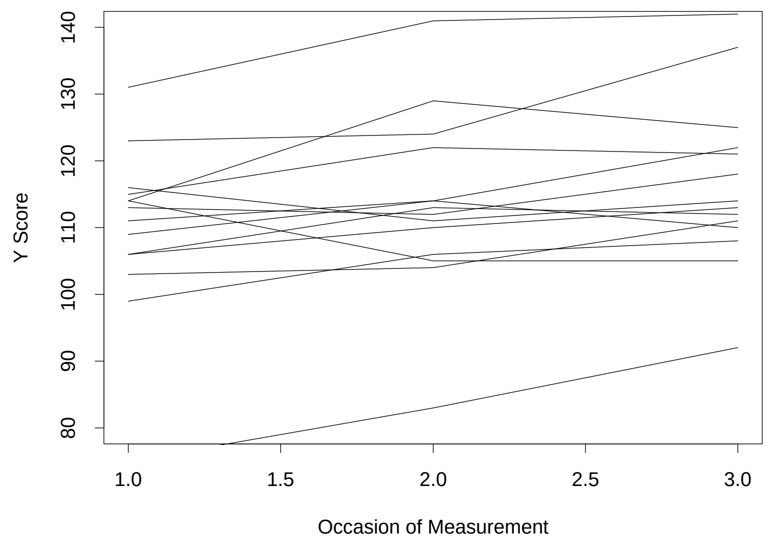

Figure 6: Longitudinal scores for 15 simulated subjects at 3 occasions of measurement.

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \ , \tag{8}$$

where Y_i is the score for person i on IQ and X_i is score for person i at a given wave (i.e, $X = 1$ at wave 1, $X = 2$ at wave 2, and $X = 3$ at wave 3)

- SAS syntax:

```
proc reg data=quirk.gsauniv;
  model y = wave;
```

- (A natural alternative in our actual situation would be to use age instead of wave as a predictor—more on that later)

```
                              The REG Procedure
                                Model: MODEL1
                            Dependent Variable: y

                             Analysis of Variance

                                        Sum of          Mean
Source                   DF            Squares        Square    F Value    Pr > F

Model                     1           96.10000      96.10000       0.23    0.6325
Error                   238              99760     419.15917
Corrected Total         239              99856
```

Root MSE	20.47338	R-Square	0.0010
Dependent Mean	111.00833	Adj R-Sq	-0.0032
Coeff Var	18.44310		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	109.45833	3.49649	31.31	<.0001
wave	1	0.77500	1.61856	0.48	0.6325

- Limitation of this regression analysis
- Repeated measures ANOVA as an alternative
- SAS syntax (notice back to multivariate form of data set):

```
proc glm data=quirk.gsamult3;
  model y1 y2 y3=;
  repeated time 3 (0 1 2) polynomial / summary;
```

The GLM Procedure

Number of observations 80

The GLM Procedure
Repeated Measures Analysis of Variance

Repeated Measures Level Information

Dependent Variable	y1	y2	y3
Level of time	0	1	2

Manova Test Criteria and Exact F Statistics for the Hypothesis of no time Effect
H = Type III SSCP Matrix for time
E = Error SSCP Matrix

S=1 M=0 N=38

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.97482162	1.01	2	78	0.3699
Pillai's Trace	0.02517838	1.01	2	78	0.3699
Hotelling-Lawley Trace	0.02582870	1.01	2	78	0.3699

```
        Roy's Greatest Root         0.02582870       1.01          2         78    0.3699


                                    The GLM Procedure
                          Repeated Measures Analysis of Variance
                  Univariate Tests of Hypotheses for Within Subject Effects

                                                                                  Adj Pr > F
Source                  DF    Type III SS    Mean Square   F Value   Pr > F    G - G    H - F

time                     2     102.633333      51.316667      1.34   0.2641   0.2627   0.2629
Error(time)            158    6039.366667      38.223840


                           Greenhouse-Geisser Epsilon    0.8332
                           Huynh-Feldt Epsilon           0.8490


                                    The GLM Procedure
                          Repeated Measures Analysis of Variance
                          Analysis of Variance of Contrast Variables

time_N represents the nth degree polynomial contrast for time

Contrast Variable: time_1


      Source                      DF     Type III SS     Mean Square    F Value    Pr > F

      Mean                         1       96.100000       96.100000       1.74    0.1913
      Error                       79     4370.900000       55.327848


Contrast Variable: time_2


      Source                      DF     Type III SS     Mean Square    F Value    Pr > F

      Mean                         1        6.533333        6.533333       0.31    0.5797
      Error                       79     1668.466667       21.119831
```

- For later use:

  1. Test of linear trend based on separate error term:
  2. Test of linear trend based on pooled error term (as in univariate approach to repeated measures, using MSAxS as the error term)

6.3 Mixed models

- How can we go about forming a mixed model to fit to our data?

- Need to consider 2 levels:
    1. Within-person trajectory over time
    2. Between-person differences
- For example, typical starting point for within-person trajectory is to assume straight line growth
- We'll see later how to allow for more complicated trajectories, but for now concentrate on straight line model
- Straight line growth translates into the following level-1 model:

$$Y_{it} = \beta_{0i} + \beta_{1i}\text{Time} + \epsilon_{it} \tag{9}$$

- Interpretation of this model (especially i subscript)
- Now pair the level-1 model with an accompanying level-2 model
- Remember that purpose of level-1 model was to account for intraindividual variability—scores vary within a person over time, in our case as a linear function of time (plus error)
- The purpose of the level-2 model is to account for interindividual variability
- Why (if at all) do scores vary between individuals?
- The simplest explanation (usually too simple!)is that individuals do not really differ at all—the only reason they appear to be different is due to the influence of error (as reflected in the level-1 model)
- In this simplistic case, we believe that every individual has the same true trajectory (if we could eliminate error)
- Because we have already assumed that each person's trajectory is linear, another way of saying that each person has the same trajectory is to say that each person has the same intercept and the same slope
- This becomes an example of a level-2 model

$$\beta_{0i} = \gamma_{00} \tag{10}$$

$$\beta_{1i} = \gamma_{10} \tag{11}$$

- What does this mean?
    1. Algebraic interpretation
    2. Geometric interpretation

- Even though this may not be a very plausible model, let's proceed to see how we could fit it using PROC MIXED

- Look again at the model:

$$\begin{aligned} Y_{it} &= \beta_{0i} + \beta_{1i}\text{Time} + \epsilon_{it} \\ \beta_{0i} &= \gamma_{00} \\ \beta_{1i} &= \gamma_{10} \end{aligned}$$

- Notice could rewrite our model as

$$Y_{it} = \gamma_{00} + \gamma_{10}\text{Time} + \epsilon_{it} \tag{12}$$

- This looks an awful lot like an ordinary regression equation—there is only 1 intercept and 1 slope in this model

- Recall that syntax we used with PROC REG was

```
proc reg data=quirk.gsauniv;
  model y = wave;
```

Syntax to fit our mixed model with PROC MIXED is

```
proc mixed data=quirk.gsauniv;
  model y=wave;
```

```
                    The Mixed Procedure

                     Model Information

        Data Set                     QUIRK.GSAUNIV
        Dependent Variable           Y
        Covariance Structure         Diagonal
        Estimation Method            REML
        Residual Variance Method     Profile
        Fixed Effects SE Method      Model-Based
        Degrees of Freedom Method    Residual


                        Dimensions

             Covariance Parameters             1
             Columns in X                      2
             Columns in Z                      0
             Subjects                          1
```

Max Obs Per Subject	240
Observations Used	240
Observations Not Used	0
Total Observations	240

Covariance Parameter Estimates

Cov Parm	Estimate
Residual	419.16

Fit Statistics

-2 Res Log Likelihood	2123.1
AIC (smaller is better)	2125.1
AICC (smaller is better)	2125.1
BIC (smaller is better)	2128.5

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	109.46	3.4965	238	31.31	<.0001
WAVE	0.7750	1.6186	238	0.48	0.6325

The Mixed Procedure

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
WAVE	1	238	0.23	0.6325

- Have we seen an F value of 0.23 and a p-value of .6325 in any of our previous analyses?
- PROC REG output:

Source	DF	Squares	Square	F Value	Pr > F
Model	1	96.10000	96.10000	0.23	0.6325
Error	238	99760	419.15917		
Corrected Total	239	99856			

- Also see how estimated coefficient for wave compares in these 2 analyses:

- From PROC MIXED:

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	109.46	3.4965	238	31.31	<.0001
wave	0.7750	1.6186	238	0.48	0.6325

- From PROC REG:

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	109.45833	3.49649	31.31	<.0001
wave	1	0.77500	1.61856	0.48	0.6325

- Both analyses tell us that estimated coefficient for wave is 0.775
- In other words, our best estimate is that scores increase about 3/4 of a point with each new wave
- The standard error of this estimate is 1.6186 in both analyses
- Fitting a mixed model with no random effects is equivalent to an ordinary least squares regression analysis
- Both implicitly assume that there are no true differences between individuals' trajectories, i.e., we are examining intraindividual change under an assumption of no true interindividual variability
- So far we've seen that PROC MIXED essentially duplicates PROC REG
- The reason is that we have assumed that there that are no true interindividual differences in trajectories
- However, seems more plausible that individuals differ from one another—PROC MIXED easily and naturally allows for this possibility, unlike PROC REG

6.4 A Momentary Glimpse at Underlying Theory

- Matrix form of (univariate) general linear model:

$$\mathbf{Y} = \mathbf{X}\beta + \epsilon \tag{13}$$

where we assume that , $\mathbf{E} \approx \mathbf{N}(0, \sigma^2\mathbf{I})$

- Important point is that errors (and therefore scores on Y) are assumed to be uncorrelated with one another
- This model is amazingly versatile, but only for uncorrelated data
- Why might scores be correlated instead of uncorrelated?
  1. more than one score from each individual, as in a longitudinal study
  2. scores might be clustered together, such as in classrooms, families, or communities
- Presence of correlations among scores creates major difficulties for typical OLS regression analysis
- What's different about mixed models?
- Matrix formulation of general linear mixed model (GLMM):

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{Z}\mathbf{u} + \epsilon \tag{14}$$

- Most obvious difference is presence of $\mathbf{Zu}$, which reflects random effects—we'll see an example of what this means in just a moment
- A less obvious difference is that GLMM also relaxes the assumption on the error term
- In GLMM, $\epsilon \approx \mathbf{N}(0, \mathbf{R})$ where $\mathbf{R}$ can be an arbitrary covariance matrix
- Can be shown that GLMM implies covariance matrix for $\mathbf{Y}$ scores is given by

$$\mathrm{Cov}(\mathbf{Y}) = \mathbf{ZGZ}' + \mathbf{R} \tag{15}$$

- Two practical implications:
  1. GLMM provides 2 mechanisms whereby scores on $\mathbf{Y}$ may be correlated with one another:
     (a) random effects
     (b) correlated errors
  2. requires use of (restricted) maximum likelihood instead of OLS
- We'll start with next simplest model, where individuals are allowed to have different intercepts, but still assume that their slopes do not truly vary
- This leads to a new level-2 model:

$$\beta_{0i} = \gamma_{00} + U_{0i} \tag{16}$$
$$\beta_{1i} = \gamma_{10} \tag{17}$$

- U_{0i} is known as a random effect—now, each person's trajectory is allowed to start at a unique value associated with that individual

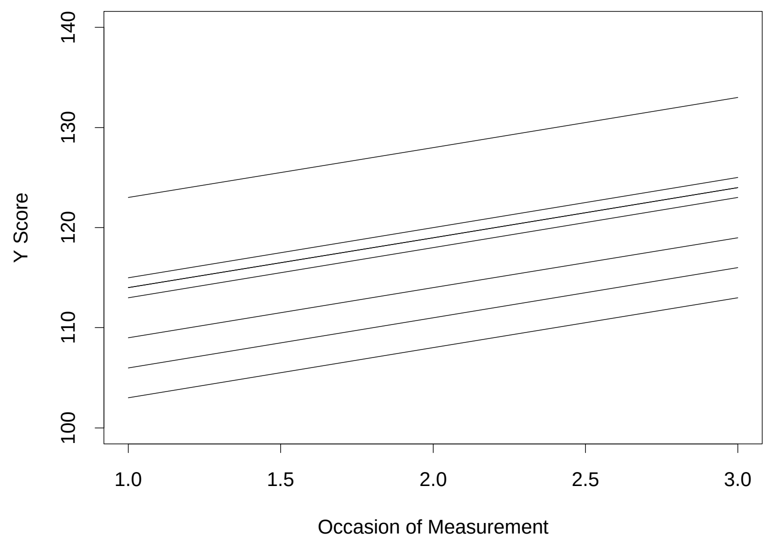

Figure 7: Longitudinal scores with equal slopes for 10 simulated subjects at 3 occasions of measurement.

6.5 How can we fit this model with PROC MIXED?

- PROC MIXED syntax:

```
proc mixed data=quirk.gsauniv;
  model y=wave;
  random int/sub=id;
```

```
                              The Mixed Procedure

                               Model Information

             Data Set                      QUIRK.GSAUNIV
             Dependent Variable            Y
             Covariance Structure          Variance Components
             Subject Effect                ID
             Estimation Method             REML
             Residual Variance Method      Profile
             Fixed Effects SE Method       Model-Based
             Degrees of Freedom Method     Containment


                                  Dimensions

                      Covariance Parameters             2
                      Columns in X                      2
                      Columns in Z Per Subject          1
```

```
Subjects                           80
Max Obs Per Subject                 3
Observations Used                 240
Observations Not Used               0
Total Observations                240


                  Iteration History

Iteration    Evaluations    -2 Res Log Like       Criterion

        0              1      2123.07423000
        1              1      1823.65518475      0.00000000


               Convergence criteria met.


           Covariance Parameter Estimates

             Cov Parm      Subject    Estimate
             Intercept     ID           382.74
             Residual                  38.0245


                    Fit Statistics

           -2 Res Log Likelihood         1823.7
           AIC (smaller is better)       1827.7
           AICC (smaller is better)      1827.7
           BIC (smaller is better)       1832.4



                 The Mixed Procedure

              Solution for Fixed Effects

                        Standard
Effect       Estimate      Error      DF    t Value    Pr > |t|

Intercept      109.46     2.4276      79      45.09      <.0001
WAVE           0.7750     0.4875     159       1.59      0.1139


             Type 3 Tests of Fixed Effects

                     Num     Den
         Effect       DF      DF    F Value    Pr > F

         WAVE          1     159       2.53    0.1139
```

- This F value of 2.53 is identical (within rounding error) to an F we saw earlier

- Specifically,using repeated measures ANOVA to test the linear trend with a pooled error term yielded an F value of 2.51
- The traditional repeated measures ANOVA using MSAxS as an error term is equivalent to assuming a random intercept model (in balanced designs)
- Compound symmetry (or sphericity) is based on a model where individuals differ in intercept but not in slope—sometimes referred to as "parallel growth" Also note the estimated coefficient for wave from this analysis, where we have allowed each individual to have his/her own intercept:

```
                         The Mixed Procedure

                     Solution for Fixed Effects

                                 Standard
Effect          Estimate            Error      DF    t Value    Pr > |t|

Intercept         109.46           2.4276      79      45.09      <.0001
wave              0.7750           0.4875     159       1.59      0.1139
```

- The estimated coefficient is still 0.7750—this equivalence will hold when we have balanced data, but otherwise will not
- However, even with balanced data, how precisely we think we have estimated this coefficient will depend on what random effects we have included in our model
- With a random intercept, standard error for the wave coefficient is 0.4875
- Is this the same value we saw for the standard error when we had no random effects?
  - No, previously it was 1.6186
- Why so much lower now? We have allowed individuals to have different intercepts, but in reality they may also have different slopes
- We can write this level-2 model as

$$\beta_{0i} = \gamma_{00} + U_{0i} \quad (18)$$

$$\beta_{1i} = \gamma_{10} + U_{1i} \quad (19)$$

- This level-2 model has 2 random effects, one for intercept and a second for slope
- Revised PROC MIXED syntax:

```
proc mixed data=quirk.gsauniv;
  model y=wave;
  random int wave/sub=id type=un gcorr;
```

The Mixed Procedure

Model Information

Data Set	QUIRK.GSAUNIV
Dependent Variable	Y
Covariance Structure	Unstructured
Subject Effect	ID
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Dimensions

Covariance Parameters	4
Columns in X	2
Columns in Z Per Subject	2
Subjects	80
Max Obs Per Subject	3
Observations Used	240
Observations Not Used	0
Total Observations	240

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	2123.07423000	
1	1	1803.59398079	0.00000000

Convergence criteria met.

Estimated G Correlation Matrix

Row	Effect	Subject	Col1	Col2
1	Intercept	1	1.0000	-0.2198
2	WAVE	1	-0.2198	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	ID	391.82
UN(2,1)	ID	-18.0403
UN(2,2)	ID	17.1952

```
                    The Mixed Procedure

              Covariance Parameter Estimates

             Cov Parm     Subject    Estimate

             Residual                 20.9375


                      Fit Statistics

           -2 Res Log Likelihood          1803.6
           AIC (smaller is better)        1811.6
           AICC (smaller is better)       1811.8
           BIC (smaller is better)        1821.1


             Null Model Likelihood Ratio Test

               DF    Chi-Square      Pr > ChiSq

                3        319.48          <.0001


                 Solution for Fixed Effects

                              Standard
 Effect       Estimate          Error      DF    t Value    Pr > |t|

 Intercept      109.46         2.3470      79      46.64      <.0001
 WAVE           0.7750         0.5880      79       1.32      0.1913


               Type 3 Tests of Fixed Effects

                           Num     Den
          Effect            DF      DF    F Value    Pr > F

          WAVE               1      79       1.74    0.1913
```

- Have we previously seen an F value of 1.74 and a p of .1913?
- Look back at PROC GLM output:

```
Contrast Variable: time_1


      Source                     DF      Type III SS     Mean Square    F Value    Pr > F

      Mean                        1        96.100000       96.100000       1.74    0.1913
      Error                      79      4370.900000       55.327848
```

- With balanced data, fitting a mixed model with a random intercept and slope is equivalent to using a separate error term in the multivariate approach to repeated measures
- Once again, let's also consider our coefficient for wave

```
                          Solution for Fixed Effects

                                Standard
Effect          Estimate          Error       DF    t Value    Pr > |t|

Intercept         109.46         2.3470       79      46.64      <.0001
wave              0.7750         0.5880       79       1.32      0.1913
```

- It still hasn't budged, but notice that standard error is different yet again
- This time it's higher than it was when we regarded slopes as fixed (0.5880 vs. 0.4875)—why?

6.6 Summary of Results

- Specifically, is the mean linear trend over time different from zero?
- Traditional methods:

1. Regression F=0.23
2. ANOVA
   (a) assuming sphericity F=2.51
   (b) without sphericity F=1.74

- Multilevel analysis:

1. No random effects F=0.23
2. Random intercept F=2.53
3. Random intercept and slope F=1.74

Head1	Head2	Head3
Method	Estimated coeff for wave	Std error
no random	0.7750	1.62
random int	0.7750	0.49
random int and slope	0.7750	0.59

1. Estimates remain constant because we have balanced data—otherwise, they would differ
2. In all cases, effect of wave is nonsignificant, but in general some analyses may yield statistically significant results while others do not
3. Which result to believe? No simple answer, but Steve will describe some methods for answering this question later

- So does this mean that PROC MIXED is really nothing more than PROC REG and GLM?
- No—here are some reasons:
  1. Missing data
  2. Continuous time
  3. More complex models

7 A More Complex Model

- In the spirit of more complex models, let's consider one additional type of model for our data
- Recall that individuals differed in age at the beginning of the study—some were as young as 65, others as old as 85—could this be important?
- For example, maybe the young-old change differently from the old-old
- How could we examine this?
- I'll present one relatively simple approach, and then Steve will present some more sophisticated possibilities
- Let's operationalize "young-old" as 75 or less at the end of the study, and "old-old" as over 75 (notice we are dichotomizing a continuous variable—Steve's methods will not require us to do this)
- How could we form a model to test whether young-old change differently from old-old?
- Does this question reflect intraindividual variability or interindividual variability?

$$Y_{it} = \beta_{0i} + \beta_{1i}\text{Time} + \epsilon_{it}$$

- So far, our most complex level-2 model is:

$$\begin{aligned} \beta_{0i} &= \gamma_{00} + U_{0i} \\ \beta_{1i} &= \gamma_{10} + U_{1i} \end{aligned}$$

- What would it mean to say that young-old and old-old change differently?

$$\begin{aligned} Y_{it} &= \beta_{0i} + \beta_{1i}\text{Time} + \epsilon_{it} && (20) \\ \beta_{0i} &= \gamma_{00} + \gamma_{01}X_i + U_{0i} && (21) \\ \beta_{1i} &= \gamma_{10} + \gamma_{11}X_i + U_{1i} && (22) \end{aligned}$$

where X_i is a dummy variable representing group membership (i.e., we might code membership in the young-old group as 0, and in the old-old group as 1)

- We need to rewrite this model to express it in PROC MIXED syntax:

$$Y_{it} = (\gamma_{00} + \gamma_{01}X_i + U_{0i}) + (\gamma_{10} + \gamma_{11}X_i + U_{1i})\text{Time} + \epsilon_{it} \tag{23}$$

Rearranging and collecting terms:

$$Y_{it} = \gamma_{00} + \gamma_{10}\text{Time} + \gamma_{01}X_i + \gamma_{11}X_i\text{Time} + U_{0i} + U_{1i}\text{Time} + \epsilon_{it} \tag{24}$$

- Need to identify fixed and random parts of the model:
  1. Fixed—look for γ coefficients
  2. Random—look for U terms
- Here we have 3 fixed effects:
  1. Time
  2. Group
  3. Group by Time
- We have 2 random effects:
  1. Intercept
  2. Time
- Probably easiest to see by looking back at level-2 models:

$$\begin{aligned} \beta_{0i} &= \gamma_{00} + \gamma_{01}X_i + U_{0i} \\ \beta_{1i} &= \gamma_{10} + \gamma_{11}X_i + U_{1i} \end{aligned}$$

- Syntax for SAS PROC MIXED

```
proc mixed data=quirk.gsauniv;
  class old;
  model y=wavcmean old old*wavcmean/s;
  random int wavcmean/sub=id type=un gcorr;
```

- Why wavcmean instead of wave?
- Wouldn't change result for slope, but will affect outcome and interpretation for intercept; this is topic of "centering," about which Steve will say more later

The Mixed Procedure

Model Information

Data Set	QUIRK.GSAUNIV
Dependent Variable	Y
Covariance Structure	Unstructured
Subject Effect	ID
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Class Level Information

Class	Levels	Values
OLD	2	0 1

Dimensions

Covariance Parameters	4
Columns in X	6
Columns in Z Per Subject	2
Subjects	80
Max Obs Per Subject	3
Observations Used	240
Observations Not Used	0
Total Observations	240

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	2113.86039767	
1	1	1794.23764762	0.00000000

Convergence criteria met.

Estimated G Correlation Matrix

Row	Effect	Subject	Col1	Col2
1	Intercept	1	1.0000	0.1904
2	WAVCMEAN	1	0.1904	1.0000

The Mixed Procedure

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	ID	391.89
UN(2,1)	ID	15.4299
UN(2,2)	ID	16.7625
Residual		20.9375

Fit Statistics

-2 Res Log Likelihood	1794.2
AIC (smaller is better)	1802.2
AICC (smaller is better)	1802.4
BIC (smaller is better)	1811.8

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	319.62	<.0001

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
WAVCMEAN	1	78	2.37	0.1280
OLD	1	80	0.32	0.5750
WAVCMEAN*OLD	1	80	2.26	0.1371

- Is there a traditional analog to our PROC MIXED analysis?
- Consider lin=y3-y1
- What about a t-test on lin?

```
proc ttest data=quirk.gsamult3;
  class old;
  var lin;
```

The TTEST Procedure

Statistics

Variable	Class	N	Lower CL Mean	Mean	Upper CL Mean	Lower CL Std Dev	Std Dev	Upper CL Std Dev	Std Err

```
lin                 34      -0.36   3.5882     7.5368     9.1278    11.317     14.896    1.9408
          0
lin                 46     -2.849   0.0435     2.9362     8.0795     9.741     12.269    1.4362
          1
lin       Diff (1-2)       -1.154   3.5448      8.244     9.0248    10.437     12.377    2.3604
```

```
                                      T-Tests

             Variable    Method           Variances      DF    t Value    Pr > |t|

             lin         Pooled           Equal          78       1.50      0.1372
             lin         Satterthwaite    Unequal      64.8       1.47      0.1469


                               Equality of Variances

                 Variable    Method      Num DF    Den DF    F Value    Pr > F

                 lin         Folded F        33        45       1.35    0.3465
```

```
PROC MIXED test for interaction of age group and wave:

                            Type 3 Tests of Fixed Effects

                                  Num     Den
                  Effect           DF      DF    F Value    Pr > F

                  wavcmean          1      78       2.37    0.1280
                  old               1      80       0.32    0.5750
                  wavcmean*old      1      80       2.26    0.1371
```

8 A 2×3 Design

- Yet one other possibility is to regard these data in terms of a 2×3 design: a between-subjects factor (age group) with 2 levels, and a within-subjects factor (wave) with 3 levels:

```
proc glm data=quirk.gsamult3;
  class old;
  model y1 y2 y3=old/nouni;
  repeated time 3 (0 1 2) polynomial / summary;
```

- Next couple of pages show portion of output for:
  1. main effect of group
  2. linear effect of wave (averaging over group)
  3. interaction of group and linear effect of wave
- Let's look at them each in turn.

1. Main effect of group:

   - GLM output:

```
                     The GLM Procedure
            Repeated Measures Analysis of Variance
         Tests of Hypotheses for Between Subjects Effects

Source              DF    Type III SS    Mean Square   F Value   Pr > F

 old                 1      379.29621      379.29621      0.32   0.5750
 Error              78    93334.68713     1196.59855
```

   - Notice this is identical to PROC MIXED test of "old":

```
           Type 3 Tests of Fixed Effects

                    Num     Den
Effect               DF      DF    F Value    Pr > F

wavcmean              1      78       2.37    0.1280
old                   1      80       0.32    0.5750
wavcmean*old          1      80       2.26    0.1371
```

2. Linear effect of wave, averaging over age group and also comparing age groups:

   - PROC GLM output:

```
Contrast Variable: time_1

      Source              DF    Type III SS    Mean Square   F Value   Pr > F

      Mean                 1     128.925831     128.925831      2.37   0.1280
      old                  1     122.825831     122.825831      2.26   0.1372
      Error               78    4248.074169      54.462489
```

   - Once again, compare to PROC MIXED:

```
           Type 3 Tests of Fixed Effects

                    Num     Den
Effect               DF      DF    F Value    Pr > F

wavcmean              1      78       2.37    0.1280
old                   1      80       0.32    0.5750
wavcmean*old          1      80       2.26    0.1371
```

3. One final variation

   - We have seen that purpose of including random effects in a model is to account for interindividual differences
   - This can be thought of in terms of correlations—to what extent are scores correlated over time, and what is the pattern of correlations?

- PROC MIXED provides 2 methods for modelling interindividual differences, i.e., correlations:
  (a) RANDOM statement
  (b) REPEATED statement
- The REPEATED statement directly models the correlations (or covariances) of scores over time

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{Z}\mathbf{u} + \epsilon \tag{25}$$

- In GLMM, $\epsilon \approx \mathbf{N}(0, \mathbf{R})$ where $\mathbf{R}$ can be an arbitrary covariance matrix
- REPEATED statement specifies form of $\mathbf{R}$
- SAS syntax for REPEATED statement:

```
proc mixed data=quirk.gsauniv;
  class old;
  model y=wavcmean old old*wavcmean/s;
  repeated/sub=id type=un rcorr;
```

- Compare to what we did for random effects model:

```
proc mixed data=quirk.gsauniv;
  class old;
  model y=wavcmean old wave*wavcmean/s;
  random int wavcmean/sub=id type=un gcorr;
```

```
                         The Mixed Procedure

                          Model Information

            Data Set                        QUIRK.GSAUNIV
            Dependent Variable              Y
            Covariance Structure            Unstructured
            Subject Effect                  ID
            Estimation Method               REML
            Residual Variance Method        None
            Fixed Effects SE Method         Model-Based
            Degrees of Freedom Method       Between-Within


                        Class Level Information

              Class    Levels    Values

              OLD           2    0 1


                              Dimensions

                 Covariance Parameters            6
```

Columns in X	6
Columns in Z	0
Subjects	80
Max Obs Per Subject	3
Observations Used	240
Observations Not Used	0
Total Observations	240

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	2113.86039767	
1	2	1793.95807482	0.00000000

Convergence criteria met.

Estimated R Correlation
Matrix for Subject 1

Row	Col1	Col2	Col3
1	1.0000	0.9288	0.8768
2	0.9288	1.0000	0.9335
3	0.8768	0.9335	1.0000

The Mixed Procedure

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	ID	402.67
UN(2,1)	ID	374.66
UN(2,2)	ID	404.11
UN(3,1)	ID	379.46
UN(3,2)	ID	404.73
UN(3,3)	ID	465.17

Fit Statistics

-2 Res Log Likelihood	1794.0
AIC (smaller is better)	1806.0
AICC (smaller is better)	1806.3
BIC (smaller is better)	1820.3

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq

```
                                5        319.90          <.0001


                                Solution for Fixed Effects

                                               Standard
Effect            OLD     Estimate        Error       DF     t Value     Pr > |t|

Intercept                   109.76       2.9396       78       37.34       <.0001
WAVCMEAN                  0.006711       0.7692       78        0.01       0.9931
OLD               0         2.7652       4.5091       78        0.61       0.5415
OLD               1              0            .        .           .            .
WAVCMEAN*OLD      0         1.7924       1.1800       78        1.52       0.1328
WAVCMEAN*OLD      1              0            .        .           .            .


                              Type 3 Tests of Fixed Effects

                                 Num      Den
                  Effect          DF       DF    F Value    Pr > F

                  WAVCMEAN         1       78       2.34    0.1300
                  OLD              1       78       0.38    0.5415
                  WAVCMEAN*OLD     1       78       2.31    0.1328
```

- Results are very similar to results we obtained with random effects model
- Why bother?
- Can fit a wide variety of types of correlational structures (e.g., unrestricted, compound symmetry, autoregressive)

- We've fit a variety of models to our data, but extremely important to keep in mind one limitation of what we have done
- Namely, we have presumed throughout that growth trajectory is linear, i.e., each individual's trajectory over time follows a straight line
- Maybe we should investigate other forms of growth
- This leads to certain issues of how we designed our study
- Design Considerations

  1. How many subjects?
  2. How many timepoints?
  3. N depends largely on complexity of level-2 model
  4. Can think in terms of usual sample size considerations for ANOVA and/or multiple regression

5. Number of timepoints depends on complexity of level-1 model
6. For polynomial models, absolute minimum number of timepoints is: “highest order trend to be fit plus 2”

Trend	Min timepoints
Linear	3
Quadratic	4
Cubic	5

7. These are mathematical minimum values
8. As important as number of timepoints is the duration of the study—need to allow a large enough observation period for change to occur

9 Quadratic Growth

- Suppose we have a fourth wave of measurement in the IQ data.

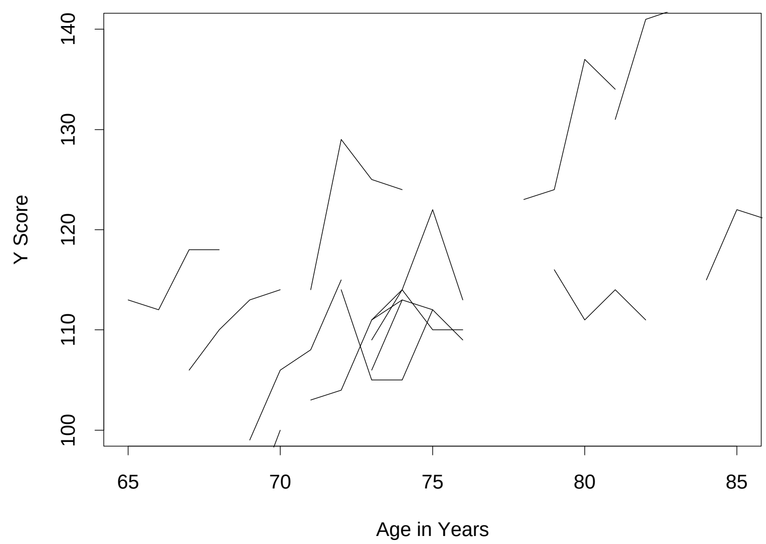

Figure 8: IQ scores for 15 simulated subjects at 4 occasions of measurement plotted by age at time of measurement.

- Now we can test the possibility of curvature in participant's trajectories.
- In order to do this, we must first consider how to represent time and the age of participants in the model. There are two possibilities.
    1. Time as waves of measurement and the age or age category of the participant when the experiment began.
    2. Just use age of the participant at each wave of measurement.
- The first possibility is what Scott had been using to demonstrate the linear growth model.
- The second possibility is what we'll use here.
- If you plan to use age as a predictor, it may turn out to be involved in an interaction term.
- Multiplying variables together can produce spurious effects if the variables aren't centered first.
- Thus we will subtract the mean age (75) from the AGE variable to create a new variable AGEC.

```
DATA quirk.gsadata2;
  SET quirk.gsauniv;
  agec75=age-75;
  agec75sq=agec75*agec75;
RUN;
```

- Notice that we are also creating a squared age variable at the same time. This will be used shortly in the quadratic model.
- First let's start with a model of linear growth in which individuals may have their own slope and intercept, as fit previously, but this time using AGEC75 instead of the TIME variable.

$$
\begin{aligned}
Y_{it} &= \beta_{0i} + \beta_{1i}\text{Agec75} + \epsilon_{it} && (26)\\
\beta_{0i} &= \gamma_{00} + U_{0i} && (27)\\
\beta_{1i} &= \gamma_{10} + U_{1i} && (28)
\end{aligned}
$$

- Substituting the level–2 equations into the level–1 equation gives

$$Y_{it} = \gamma_{00} + \gamma_{10}\text{Agec75} + U_{0i} + U_{1i}\text{Agec75} + \epsilon_{it} \quad (29)$$

- Thus, Agec75 is both a fixed and a random effect in this model.

```
PROC MIXED DATA=quirk.gsadata2;
  MODEL y=agec75 / S;
  RANDOM int agec75/ SUB=id TYPE=un gcorr;
RUN;
```

- This results in the following SAS output.

```
                    The Mixed Procedure

                     Model Information

    Data Set                       QUIRK.GSADATA2
    Dependent Variable             Y
    Covariance Structure           Unstructured
    Subject Effect                 ID
    Estimation Method              REML
    Residual Variance Method       Profile
    Fixed Effects SE Method        Model-Based
    Degrees of Freedom Method      Containment


                        Dimensions

             Covariance Parameters             4
```

Columns in X	2
Columns in Z Per Subject	2
Subjects	80
Max Obs Per Subject	4
Observations Used	320
Observations Not Used	0
Total Observations	320

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	2852.27658900	
1	2	2291.32307560	0.36183382
2	1	2249.94954050	0.93093926
3	1	2222.25404054	0.06650874
4	2	2213.67745040	0.00265853
5	2	2212.52748986	0.00047483
6	1	2212.09820767	0.00001773
7	1	2212.08338783	0.00000003
8	1	2212.08336167	0.00000000

Convergence criteria met.

Estimated G Correlation Matrix

Row	Effect	Subject	Col1	Col2
1	Intercept	1	1.0000	0.8039
2	AGEC75	1	0.8039	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	ID	4.9547
UN(2,1)	ID	5.3042
UN(2,2)	ID	8.7866
Residual		25.3235

Fit Statistics

-2 Res Log Likelihood	2212.1
AIC (smaller is better)	2220.1
AICC (smaller is better)	2220.2
BIC (smaller is better)	2229.6

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	640.19	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	114.75	0.6782	79	169.21	<.0001
AGEC75	-0.05991	0.3471	79	-0.17	0.8634

- But perhaps there is a quadratic effect in these data. Now that we have a fourth wave of measurement we can test this.
- We use the previously squared centered age variable, Agec75sq and write the model equations as

$$Y_{it} = \beta_{0i} + \beta_{1i}\text{Agec75} + \beta_{2i}\text{Agec75sq} + \epsilon_{it} \quad (30)$$
$$\beta_{0i} = \gamma_{00} + U_{0i} \quad (31)$$
$$\beta_{1i} = \gamma_{10} + U_{1i} \quad (32)$$
$$\beta_{2i} = \gamma_{20} + U_{2i} \quad (33)$$

  where we now have three random effects, the intercept, Agec75 and Agec75sq.
- This results in the following SAS input

```
PROC MIXED DATA=quirk.gsadata2;
  MODEL y=agec75 agec75sq / S;
  RANDOM int agec75 agec75sq / SUB=id TYPE=un gcorr;
RUN;
```

- And the following SAS output

The Mixed Procedure

Model Information

Data Set	QUIRK.GSADATA2
Dependent Variable	Y
Covariance Structure	Unstructured
Subject Effect	ID
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Dimensions

Covariance Parameters	7
Columns in X	3
Columns in Z Per Subject	3
Subjects	80
Max Obs Per Subject	4
Observations Used	320
Observations Not Used	0
Total Observations	320

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	2850.52216701	
1	2	2191.88413920	328.82700033
2	1	2188.38078485	22.15815530
3	3	2171.81651469	.
4	1	2169.24016035	4.06340672
5	1	2163.39930193	0.03030413
6	2	2158.82240351	0.00162810
7	2	2157.86415612	0.00018650
8	1	2157.70583993	0.00000357
9	1	2157.70297513	0.00000000

Convergence criteria met.

Estimated G Correlation Matrix

Row	Effect	Subject	Col1	Col2	Col3
1	Intercept	1	1.0000	0.6562	0.09402
2	AGEC75	1	0.6562	1.0000	-0.09911
3	AGEC75SQ	1	0.09402	-0.09911	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	ID	10.1948
UN(2,1)	ID	3.8437
UN(2,2)	ID	3.3649
UN(3,1)	ID	0.1022
UN(3,2)	ID	-0.06189
UN(3,3)	ID	0.1159
Residual		17.7287

Fit Statistics

```
-2 Res Log Likelihood          2157.7
AIC (smaller is better)        2171.7
AICC (smaller is better)       2172.1
BIC (smaller is better)        2188.4


     Null Model Likelihood Ratio Test

       DF    Chi-Square      Pr > ChiSq

        6        692.82          <.0001


          Solution for Fixed Effects

                         Standard
Effect       Estimate       Error      DF    t Value    Pr > |t|

Intercept      114.75      0.6801      79     168.74      <.0001
AGEC75        -0.3238      0.2876      79      -1.13      0.2636
AGEC75SQ      -0.1182     0.05152      79      -2.29      0.0244


        Type 3 Tests of Fixed Effects

                Num     Den
Effect           DF      DF    F Value    Pr > F

AGEC75            1      79       1.27    0.2636
AGEC75SQ          1      79       5.27    0.0244
```

- What do you know. It looks like it was the quadratic parameter that was the significant effect all along.
- Here's what the full data set looks like when plotted.

10 Missing Data and Mixed Models

- One of the benefits of mixed models is that they are adept at handling missing data.
- Let's see what happens when we randomly delete some of the observations in the IQ data.
- Observations were deleted to mimic attrition. On the third occasion one quarter of the participants dropped out. On the fourth occasion, one third of the remaining individuals dropped out.
- Let's see how a mixed model handles these data.

```
PROC MIXED DATA=quirk.gsadata2;
  MODEL y=agec75 agec75sq / S;
  RANDOM int agec75 agec75sq / SUB=id TYPE=un gcorr;
```

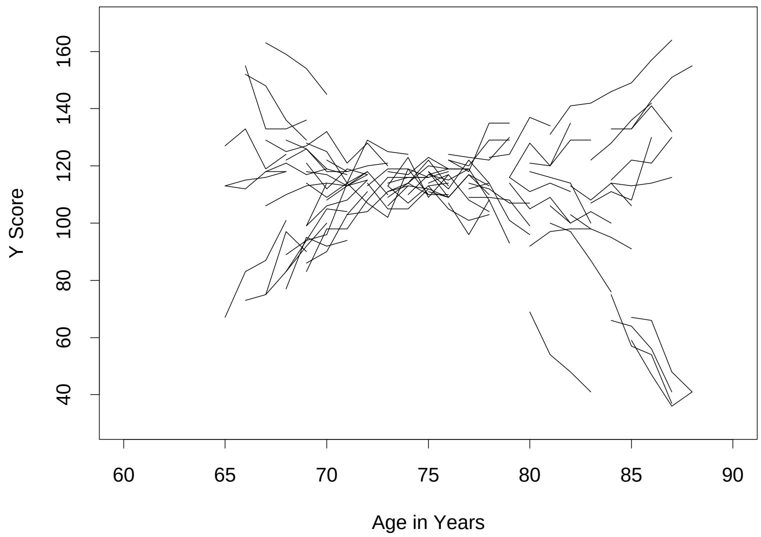

Figure 9: IQ scores for 80 simulated subjects at 4 occasions of measurement plotted by age at time of measurement.

```
RUN;
```

- Results in the following SAS output

```
                         The Mixed Procedure

                          Model Information

Data Set                        QUIRK.GSADATA4
Dependent Variable              Y
Covariance Structure            Unstructured
Subject Effect                  ID
Estimation Method               REML
Residual Variance Method        Profile
Fixed Effects SE Method         Model-Based
Degrees of Freedom Method       Containment


                             Dimensions

         Covariance Parameters             7
         Columns in X                      3
         Columns in Z Per Subject          3
         Subjects                         80
         Max Obs Per Subject               4
         Observations Used               260
```

Observations Not Used	60
Total Observations	320

Iteration History

Iteration	Evaluations	-2 Res Log Like	Criterion
0	1	2292.44852951	
1	4	1785.46873003	.
2	1	1775.20737287	0.00515837
3	3	1773.76097444	.
4	1	1770.62935255	.
5	1	1769.98747908	0.00006309
6	1	1769.94440977	0.00000044
7	1	1769.94412057	0.00000000

Convergence criteria met.

Estimated G Correlation Matrix

Row	Effect	Subject	Col1	Col2	Col3
1	Intercept	1	1.0000		0.2856
2	AGEC75	1		1.0000	
3	AGEC75SQ	1	0.2856		1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate
UN(1,1)	ID	24.9846
UN(2,1)	ID	5.2987
UN(2,2)	ID	0
UN(3,1)	ID	0.5712
UN(3,2)	ID	-0.02898
UN(3,3)	ID	0.1602
Residual		17.4577

Fit Statistics

-2 Res Log Likelihood	1769.9
AIC (smaller is better)	1781.9
AICC (smaller is better)	1782.3
BIC (smaller is better)	1796.2

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq

```
                5        522.50          <.0001


                Solution for Fixed Effects

                             Standard
Effect       Estimate         Error      DF    t Value    Pr > |t|

Intercept      114.20        0.8675      79     131.64      <.0001
AGEC75        -0.3515        0.2701      79      -1.30      0.1969
AGEC75SQ      -0.1202       0.05734      59      -2.10      0.0404

               Type 3 Tests of Fixed Effects

                    Num     Den
        Effect       DF      DF    F Value    Pr > F

        AGEC75        1      79       1.69    0.1969
        AGEC75SQ      1      59       4.39    0.0404
```

- First, notice that although only 40 individuals had complete data ($40 \times 4 = 160$ observations), 240 observations were used by PROC MIXED. This is certainly good news.
- Second, compare the parameter values, standard errors, degrees of freedom and t values for the AGEC75SQ variable.

	Effect	Estimate	S.E.	DF	t Value	Pr > $\|t\|$
Full Data	AGEC75SQ	-0.1182	0.05152	79	-2.29	0.0244
After Dropout	AGEC75SQ	-0.1202	0.05734	59	-2.10	0.0404

- Proc Mixed is useful for missing data problems, but won't cure selection effects on the outcome variable.