A measurement of the adaptation of color vision to the spectral environment

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Abstract

An exploratory factor analysis of the reflectance spectral distributions of a sample of natural and man–made objects yields a factor pattern remarkably similar to psychophysical color matching curves. The goodness of fit indices from a maximum likelihood confirmatory factor model with fixed factor loadings specified by empirical trichromatic color matching data indicate that the human visual system performs near to an optimum value for an ideal trichromatic system composed of three linear components. An unconstrained four factor maximum likelihood model is found to fit significantly better than a three factor unconstrained model, suggesting that a color metric is better represented in four dimensions rather than in a three dimensional space. This fourth factor can be calculated as a nonlinear interaction term between the first three factors; thus a trichromatic input is sufficient to compute a color space of four dimensions. The visual system may exploit this nonlinear dependency in the spectral environment in order to obtain a four dimensional color space without the biological cost of a fourth color receptor.

Introduction

The world of electromagnetic stimuli in which we are immersed seems to have a much more rich and varied structure than our visual system can process. This work is concerned with the information contained in the range of wavelengths of electromagnetic radiation in visible light, wavelengths from approximately 400nm to 680nm (Boynton, 1979). The visible light that is reflected from an object in our everyday world is composed of a mixture of thousands of separate and distinct wavelengths of light. We perceive this complex distribution of wavelengths of visible light reflected from an object as a single percept that we call a *color* (Zeki, 1993). In doing so, it seems that we have lost a great

This work was supported by the Institute for Developmental and Health Research Methodology at the University of Virginia and by the John D. and Catherine T. MacArthur Foundation

deal of the information that was originally present in the reflected light: a distribution has been summarized into a statistic. One consequence of this loss of information is that many different distributions of visible light can produce the same perceived color (Maxwell, 1857).

At any one moment in time, each point in visual space appears to have a color associated with it. Color perception reduces a very high dimensional input with many degrees of freedom, the distribution of energy values of the photons arriving at a point on the retina, into a low dimensional percept with few degrees of freedom, a color. It has been suggested that the visual system attempts to preserve as much of the information in the input as possible during this process of reduction of dimension (Barlow, 1982; Buchsbaum & Gottschalk, 1983). It follows from this premise that the visual system would attempt to preserve, as much as possible, the covariances present in the distributions of photon energies generated by the product of the illuminant spectra with reflectance spectra from objects present in the environment.

Cohen (1964) and Vrhel, Gershon, & Iwan (1994) present principal components analyses and Maloney (1986) presents a multiple regression analysis, each of which demonstrate that the distributions of wavelengths of reflected light in the environment can be efficiently reproduced using linear combinations of a small number of components. It follows from these results that it would be possible for the visual system to preserve most of the information contained in these distributions of wavelengths if color perception were to take advantage of the predictable patterns of covariance between wavelengths of visible light. These principle components analyses demonstrate that the distributions of reflected light in the environment have a largely predictable covariance structure. If the visual system has adapted to that covariance structure, most of the information contained in the distribution of reflected light could be represented in a few linear components: a low dimensional space. The present work describes a method for measuring the correspondence between the low dimensional structure of the distributions of reflected wavelengths of visible light present in the environment and the low dimensional structure of the perception of color.

The visual system adapts to differing brightness and overall spectral content of illumination sources such that a perception of *color constancy* is maintained within a wide range of environmental lighting conditions (Brainard, Wandell, & Chichilnisky, 1993; Jameson & Hurvitch, 1989; Shepard, 1994). Thus we perceive objects to have the same color at sunset as they had at noon, even though the spectrum of light from the sun at sunset contains proportionately more long wavelengths than it does at noon. One theoretical method for accomplishing color constancy would be to remove the overall mean illuminance from the input wavelength distributions while preserving an accurate but low dimensional estimate of their covariance structure. According to this theory, if color constancy is to be achieved, the interaction between the mean and variance of each wavelength must be removed in order to preserve an invariant pattern of covariances between wavelengths reflected from objects in the environment. It is this pattern of covariances that would define the perceived color of an object.

For example, suppose that the spectrum of the light source illuminating a scene was skewed toward the longer wavelengths. By extracting the mean value for each wavelength over the entire visual scene, the effect of the skewed illuminance spectrum would be removed while the covariance pattern between the wavelengths reflected from individual objects would be preserved.

Statistical techniques such as factor analysis are designed to perform exactly this task of finding an efficient reduction in the dimension of data while preserving covariance relations among the input variables (Tucker, 1958). As long as certain distributional and modeling assumptions are met, factor analysis is considered to perform an optimal linear reduction in dimension with respect to a specified cost function (e.g. least squares or maximum likelihood). Confirmatory factor analysis, a form of maximum likelihood latent variable modeling with constraints, allows measurement of goodness of fit of a particular model to a particular data set (McDonald, 1985; Loehlin, 1992). The premise of this paper is that the goodness of fit of the visual system's solution to the problem of reduction of dimension in the perception of distributions of photon energies can be estimated through confirmatory factor analysis, and consequently the degree to which the visual system has adapted to the predictable regularities in the spectral environment can be directly estimated.

Methods

Two sets of data will be used in these analyses. Trichromatic color matching measurements of human vision will be used to fix the factor loadings of a three latent variable confirmatory factor model. This model will be fit to covariance matrices computed from reflectance spectra gathered from several hundred everyday natural and man–made objects. The sources for these two sets of data are briefly described below.

The color matching data

In 1931, the International Commission on Illumination (Commission Internationale de l'Eclairage, or CIE) developed a standard observer model (CIE, 1931) based on work by Wright (1929) and Guild (1931). This model assumes a brightness sensitivity function which is a linear combination of trichromatic stimuli (Wyszecki & Stiles, 1967). Guild's determination of a standard observer measured the spectral sensitivity of three additive primaries in relation to each other by use of a small field (2-degree) bipartite matching to sample experiment where the reference stimulus was a white tungsten incandescent light of color–temperature of about 4800° Kelvin and the primaries (650nm, 530nm, and 460nm) were obtained by appropriate filters. Guild and Wright obtained similar results for their chromaticity coordinates and the means of the data from these two experiments will be used as the empirical color matching data for the analyses reported here.

The spectral distributions

Vrhel et al. (1994) measured the spectral distributions of a diverse sample of natural and man–made objects. They measured reflectance spectra of 64 Munsell chips, 120 Du Pont paint chips and 170 objects from the environment using a high precision spectrometer and illuminance with a color–temperature of 2800° Kelvin to avoid fluorescence effects. A calibrated white reflectance standard (X-RITE reference tile) was used to compensate for the illuminant spectrum when calculating the object reflectance spectrum.

The natural objects included 31 measurements of human hair and skin, and measurements of 78 different samples of rocks, leaves, bark, wood, flowers, fruit, vegetables and grains. The man–made objects included a wide variety of clothing and personal objects. While this sample of environmental reflectance spectra is not necessarily a representative sample of the natural spectral environment, it is relatively broad based and complex. For purposes of these analyses, the natural objects and the man–made objects were each analyzed separately as well as combined together. Since partitioning the objects into natural versus man–made categories did not affect the results, only the analyses from the combined data set will be reported here.

For each object, the reflectance intensity was measured within the 31 different 10nm wavelength bands located between 395nm and 705nm. These measurements comprise 31 different scores representing the reflectance spectrum for each individual object. The factor analyses were performed on a covariance matrix of these scores. Only the 15 even–numbered 10nm bands were used for the maximum likelihood factor analysis since the correlation between successive bands was so high that maximum likelihood estimation procedures did not converge.

The factor models

The path diagram in Figure 1 presents a schematic representation of the common factor model used in these analyses. The variance of each component wavelength band Ω_j can be recovered as a linear combination of the three factors F_1, F_2 and F_3 plus a unique variance U_j. A factor loading L_{ij} is the proportion which the factor F_i contributes to the variance of the wavelength band Ω_j.

Other researchers have analyzed spectral distributions using principal components methods (Cohen, 1964; Vrhel et al., 1994). Principal components extracts the largest source of common variance as the first component, extracts the second component from the residuals left over from the first component, and so on. There are two problems with using this method. The first problem is strictly mathematical; since principal components focuses on fitting the larger sources of common variance it tends to overestimate the goodness of fit and underestimate the number of required components (McArdle, 1990). The second problem is that in order to measure the correspondence between human vision and the spectral environment, it is important to extract components which have the potential of a psychophysical interpretation. Factor analysis with an oblique rotation extracts sources of common variance (factors) without requiring that these factors be orthogonal or that one factor be as large as possible with respect to the others. There is no theoretical reason why it is to be expected that the visual system operates under the same constraints as principal components analysis. In fact, multidimensional scaling experiments (Indow, 1988) can be interpreted to suggest that the first three components should be given approximately equal weight.

Two forms of factor analysis were used in the present work. Exploratory factor analysis was used as a preliminary step in order to examine the structure of the common variance in the spectral distributions. Maximum likelihood latent variable analysis using a factor model (confirmatory factor analysis) was used to test the goodness of fit between the color matching data and the factors extracted from the spectral distributions. Confirmatory factor analysis was also used to test whether the addition of more than three factors would provide a significantly better fit to the spectral distributions.

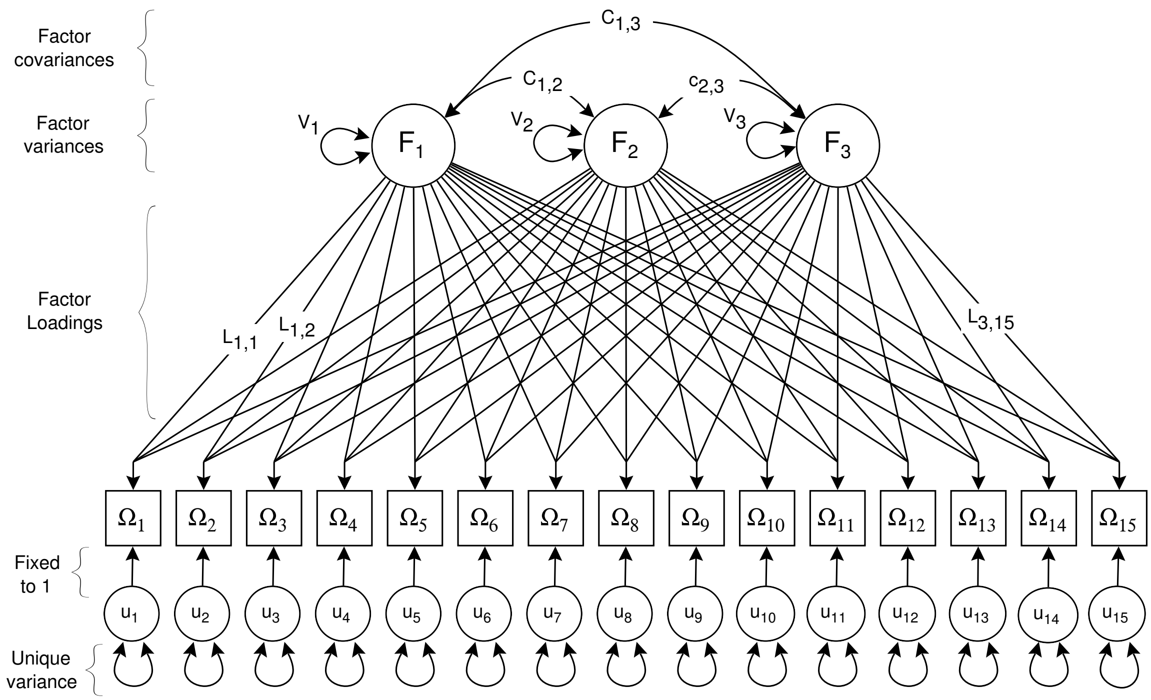

Figure 1. Path diagram of a three common factor model. F_1, F_2 and F_3 represent the three factors (latent variables). $\Omega_1 \ldots \Omega_{15}$ represent the fifteen measured wavelength bands (manifest variables). $U_1 \ldots U_{15}$ represent the fifteen unique components to the variance (uniquenesses). $L_{1,1} \ldots L_{3,15}$ represent the factor loadings, the proportion of the variance of each wavelength band which can be accounted for by each of the common factors. $V_1 \ldots V_3$ and $C_{1,2} \ldots C_{2,3}$ represent the factor variances and covariances respectively.

Results

A preliminary exploratory factor analysis was performed with SAS using principal factors and considering all 31 10nm bands to be manifest variables. This solution was rotated to simple structure using an oblique rotation method (Promax) and the factor loadings were plotted on the same scale as the color matching curves. This factor solution was stable over the natural versus man–made subsets of the data and as illustrated in Figure 2-B, the factor loadings appeared very similar to the empirical color matching curves shown in Figure 2–A.

Maximum likelihood estimation of the common factor model allows the calculation of a goodness of fit index when parameters in a factor model are constrained. In this analysis each factor loading $L_{i,j}$ was constrained to be equal to the corresponding mean color matching coefficient from the Guild and Wright experiments. Then the factor model was fit to the 15 odd–numbered 10nm wavelength bands of reflectance spectra data using SAS's PROC CALIS. In this model the only values which were allowed to be free were the factor variances V_i, factor covariances $C_{i,k}$ and the unique variances U_j of each wavelength band.

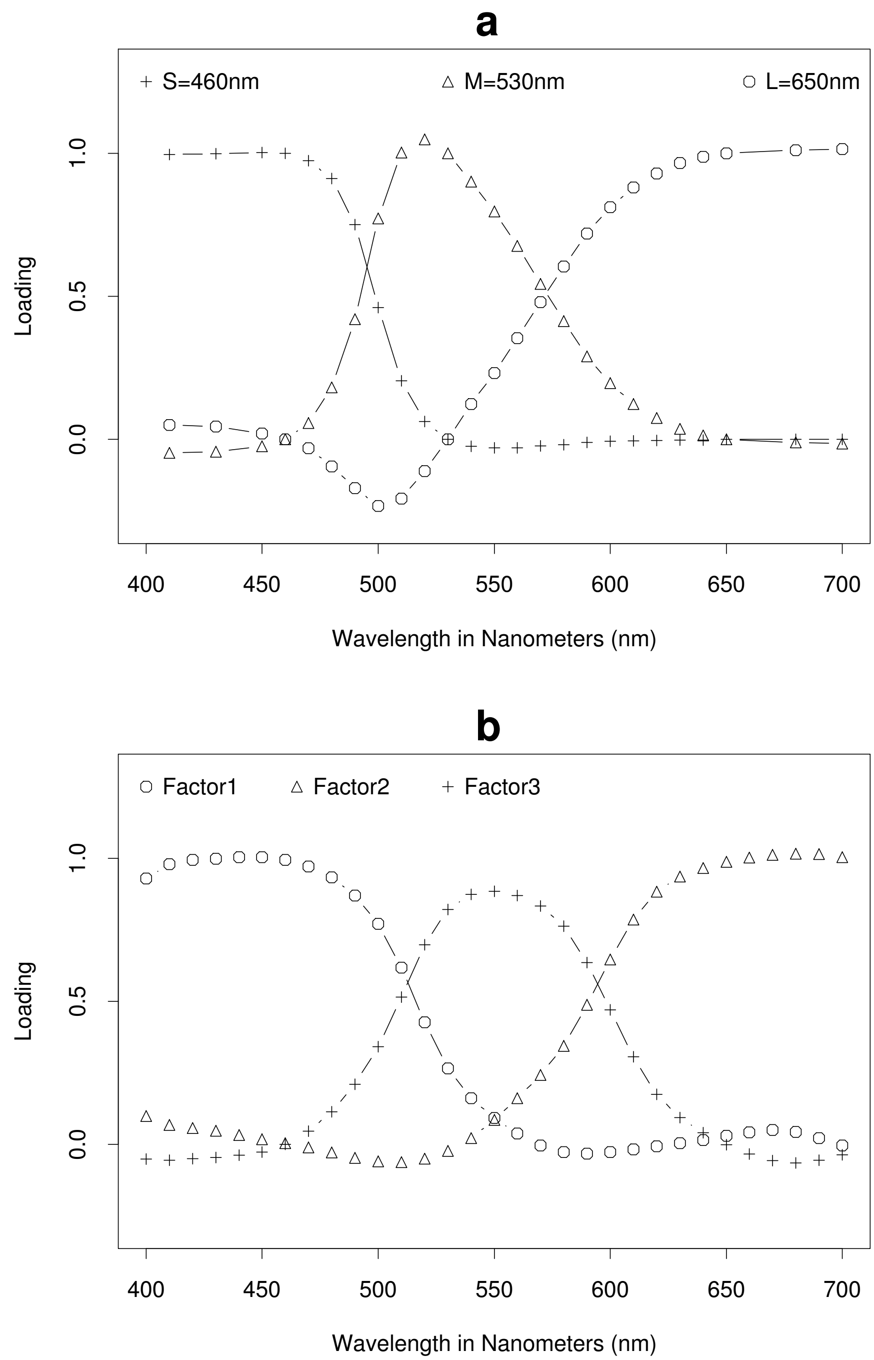

Figure 2. Similarity between color matching curves and factor loadings from exploratory factor analysis of reflectance spectra. (a) Mean color matching curves; data from 3 studies reported in Wright (1928). Each line expresses the amount of light of the appropriate primary wavelength ($L_\lambda = 650$nm, $M_\lambda = 530$nm, $S_\lambda = 460$nm) required to match a sample of wavelength λ on the X axis. (b) Oblique (Promax) rotation of three principal factors solution using measurements of reflectance spectra (in thirty-one 10-nm wavelenth bands from 395 nm to 705 nm) from 354 natural and man-made objects.

Table 1: Goodness of fit indices from confirmatory maximum likelihood factor analysis of a three factor model with loadings constrained to psychophysical color matching coefficients. Note: RMSEA refers to Root Mean Square Error of Approximation, a combined measure of goodness of fit and parsimony.

Model	DF	χ^2	RMSEA (95%)
Null	105	21355	
3 Factor	99	7527	0.46 (0.45–0.47)

Table 1 presents the results of testing the goodness of fit of the factor solution obtained by the adaptation of the eye to the spectral environment under the assumption that the perception of color could be modeled as a linear combination of three wavelengths. At a cost of six degrees of freedom, the χ^2 of the three factor model has been reduced to nearly one third of χ^2 of the Null Model. This indicates that the three factor model is exploiting regularities in the spectral data, but that there is still significant misfit between the model and the data. There are two potential sources of misfit in this model: (1) these constrained loadings may not represent a good fit to the spectral distributions, or (2) no three factor model may be a good fit these spectral distributions. By fitting unconstrained maximum likelihood factor models, these two sources of misfit can be disentangled.

Unconstrained factor models with 3, 4 and 5 factors were fit using maximum likelihood estimation so that χ^2 and RMSEA goodness of fit indices could be computed. Table 2 presents the results of these analyses. At a cost of another 36 degrees of freedom, the unconstrained three factor model achieves a 30% reduction in the χ^2 over the constrained model. According to the the RMSEA 95% confidence intervals, the *constrained* model marginally achieves a significantly better fit than the *unconstrained* model. In fact, the RMSEA of the constrained model in Table 1 is the same as the unconstrained four factor model. This suggests that the psychophysical color matching coefficients are near to an optimum fit for a three factor model of the covariances of spectral distributions from the environment.

Table 2: Goodness of fit indices from unconstrained maximum likelihood factor analyses for the null model and three, four and five factors.

Model	DF	χ^2	RMSEA (95%)
Null	105	21002	
3 Factor	63	5276	0.48 (0.47–0.50)
4 Factor	51	3843	0.46 (0.44–0.47)
5 Factor	40	2619	0.43 (0.41–0.44)

Table 2 shows small but significant improvements in fit for the four and five factor models over the three factor model. Figure 3–A plots the factor patterns for the four factor unconstrained maximum likelihood model. The first three factors in Figure 3–A show a similar factor pattern to the three factors plotted in Figure 2-B. The fourth factor appears to provide a contrast between wavelengths centered around 510nm and wavelengths centered around 590nm.

Could a fourth factor be perceived when the human retina only contains cones with three primary wavelength sensitivities? A nonlinear combination of the three primary factors (F_S, F_M, and F_L being the short, mid, and long wavelength factors) such that $\hat{F_4} = F_M * (F_L - F_S)$ will produce a calculated fourth factor which has a pattern of loadings very similar to the pattern of loadings observed in the four factor unconstrained model (compare Figure 3–B and Figure 3–A). The visual system may exploit this nonlinear dependency in the spectral environment in order to obtain a fourth color dimension without the biological cost of another receptor.

Discussion

These analyses indicate a close correspondence between empirical color matching coefficients and factor models of environmental spectral data. This result is consistent with a hypothesis that the human visual system exploits the reduced rank of the spectral distributions found in the environment in order to reduce the redundancy inherent in the visual input stream. Furthermore, this correspondence indicates that color perception may operate in a manner that is functionally equivalent to factor analysis: by extracting statistical invariants that can be linearly combined to provide estimates of distributions of wavelengths in the spectral environment.

A three factor confirmatory factor analysis where the factor loadings are fixed to be empirical color matching coefficients and fit to environmental spectral data provides an RMSEA fit which is significantly better than that of an unconstrained three factor analysis. This result suggests that human color perception performs near to an optimum value for an ideal trichromatic system composed of three linear components.

A four factor maximum likelihood model fits the reflectance spectral distributions marginally but significantly better than does a three factor maximum likelihood model. In addition, the RMSEA fit of the confirmatory factor analysis using the empirical color matching data is similar to the RMSEA fit of a four factor maximum likelihood model. These two results suggest that the perceptual space of color vision may be better represented by using a four dimensional model.

A fourth factor which is similar to the extracted factors can be calculated as a nonlinear combination of the first three extracted factors, suggesting that a four dimensional representational space could be neurally calculated from trichromatic inputs. A calculated fourth factor has the advantage of allowing each color to be a position in a four dimensional metric space, thus preserving a more accurate estimate of the covariance in the spectral environment while not incurring the biological cost of an additional color receptor.

The results presented here are consistent with the hypothesis that patterns of covariance within the distribution of wavelengths of light reflected from an object comprise the fundamental information defining the color of that object. However, these analyses do not pose a strong test of that hypothesis. A stronger test would require a simultaneous test of color constancy: both the reflectance spectrum measurements and the color matching measurements would need to be gathered under a variety of illumination conditions. A model of both the means and covariances could then be fit to these data thus providing a

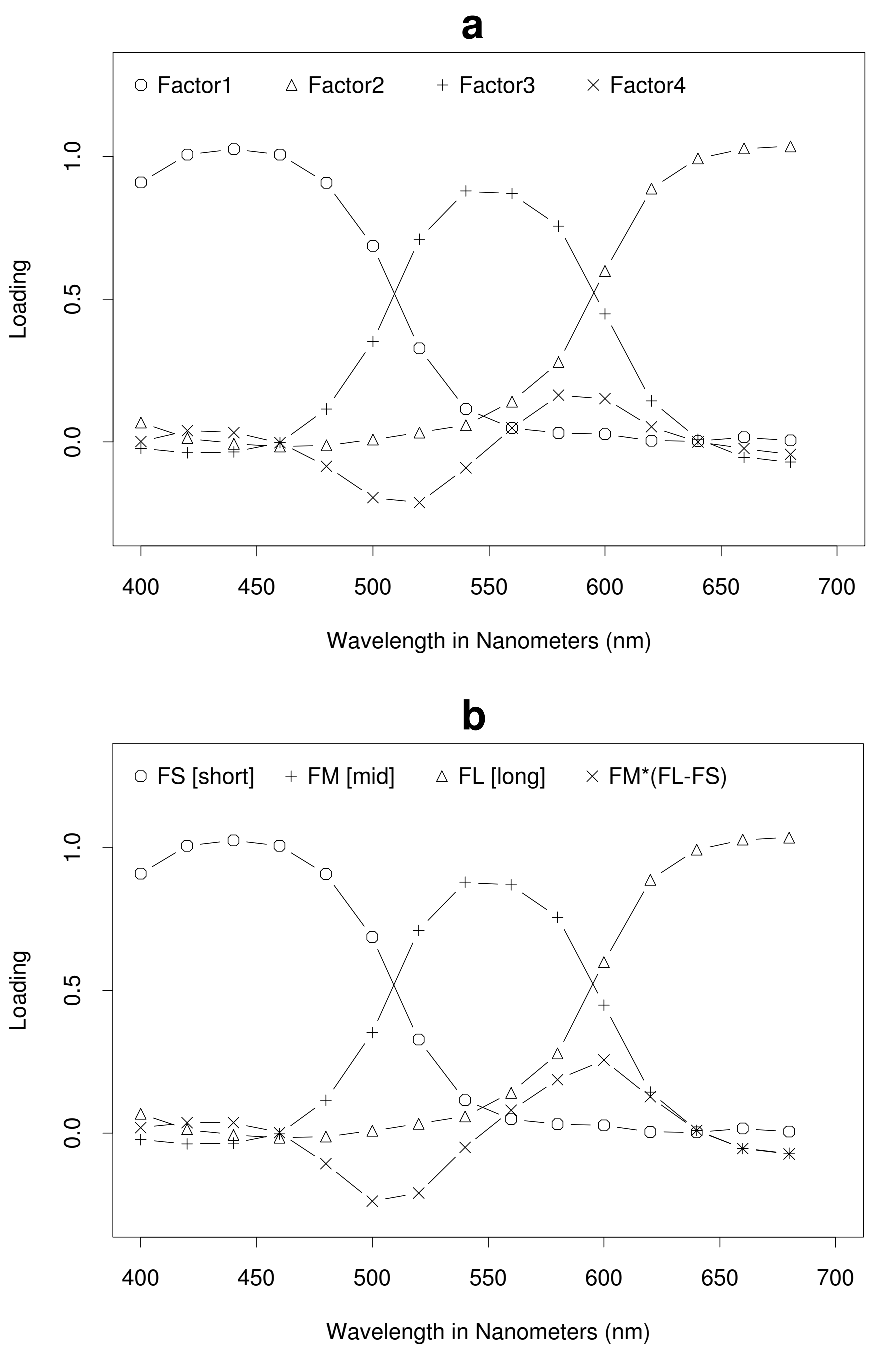

Figure 3. Factor loadings from unconstrained maximum likelihood models fit to fifteen 10-nm bands of reflectance spectra of 354 natural and man-made objects. The factor loadings in (a) are from a four-factor model. Those in (b) are from a three-factor model with a fourth factor calculated as a nonlinear combination of the first three factors.

test of the hypothesis that the overall illumination mean for each wavelength are removed while patterns of wavelength covariance are preserved in the perception of color.

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