Differential Structural Equation Modeling of Intraindividual Variability

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Abstract

Recent advances in modeling and analysis have enabled statistical tests of dynamical systems based theories for behavioral phenomena. The use of statistical models to test proposed dynamical or self–regulating explanations of intraindividual variability has the potential to lead to new and more tightly focused understandings of cognitive, developmental and social systems. This chapter provides an introduction to the use of differential structural equation models (dSEM) to fit data from repeated observations of individuals: intraindividual variability. Example data from an experiment in the development of postural control are analyzed in order to give a practical demonstration of these methods. Models are fit to individual trials in a postural control experiment and the resulting estimated parameters are analyzed using a random coefficients approach to elucidate development in the dynamics of intraindividual variability.

Introduction

The world in which we are immersed exhibits both invariant characteristics and continuous change. In order for an organism to successfully flourish in such an environment the predictable regularities of the environment must be recognized while simultaneously the ever-changing characteristics of new conditions must be perceived and adaptations to these perceived changes must be made. If successful adaptation is to be attained, the organism must at some level be able to monitor changes in its internal state and the relationship of that state to the environment. Such monitoring might be a high level cognitive process, or could alternatively be a part of a low level, autonomic, self–regulatory system.

I would like to thank John Nesselroade, Jack McArdle, John Horn, Linda Collins and Aline Sayer for their dynamic and systematic support of the analysis of variability and intraindividual change. I would also like to sincerely thank Bennett Bertenthal for the permission to use the example data presented in this chapter, and Jim Rose for the hard work of collecting postural data from 40 (potentially less than ideally cooperative) infants.

For the purposes of this discussion, self–regulation will be used to describe a process by which an organism modifies its own behavior in a systematic way by using information about change. This information about change might be estimates of differences in internal states of the organism. In another case, information about change might be derived from an estimate of changes that have occurred in the environment. More complex, derived information about change might involve estimates of the relative change in the estimated differences in internal states as compared to the differences that have occurred in the environment during the same time period.

The current chapter examines models which use change as a predictor and/or an outcome variable. This form of model is appealing in that it allows one to test theories of self–regulation or organism–environment coupling (Warren, 1990) as a mechanism for observed patterns of intraindividual variability. In particular, we will focus on a method that models processes in which there is a covariance structure relating the value of a variable, its first derivative and its second derivative.

As a concrete example of first and second derivatives, consider taking an automobile trip from your house to a friend's house 10 miles away. Luckily, there are no stop signs or traffic that will interfere with your trip. Figure 1–a plots the distance of your automobile from your house as a function of time. You start your car about 9 minutes after this thought experiment begins, you accelerate towards your friend's house, reaching a peak velocity of about 55 miles per hour midway in the trip. Then you begin to decelerate, first gradually and then braking decisively, finally arriving at your friend's house about 17 minutes after you started your car. Figure 1–b and c plot the velocity and acceleration of your automobile respectively.

These three quantities: displacement and the first and second derivatives of the displacement can each be thought of as ways of measuring change. Let us consider how they can be applied to psychological variables.

Self–Regulation and Dynamical Disease

It is convenient to think about a first derivative as the "slope" of a variable at a particular instant in time and the second derivative as the "curvature" of a variable at a particular instant in time. For instance, if the variable of interest were depression one could devise an instrument that would, given repeated measurements, provide an estimate of the rate at which a subject was becoming more depressed and whether that rate of change was itself accelerating or decelerating. A normal individual might have a self–regulatory mechanism that would respond to increasing levels of depression by decelerating that increase. Eventually the level of depression would begin to decrease, but the self–regulatory mechanism would need to slow that rate of decrease at just the right time if it were to reach a point of equilibrium at a mood balanced between depression and elation.

Pezard, Nandrino, Renault, and Massioui (1996) advance the hypothesis that clinical depression might be a disease that affects the mechanism of self–regulation, a *dynamical disease* (Glass & Mackey, 1988). A different type of breakdown in such a self–regulatory mechanism might underlie Rapid Cycling Bipolar Disorder (RCBD) (Gottschalk, Bauer, & Whybrow, 1995). In normal subjects, one might characterize the likely day to day changes in mood scores as having a *point attractor*: an equilibrium point somewhere between elation and depression. Individual differences in these equilibrium points would be characterized

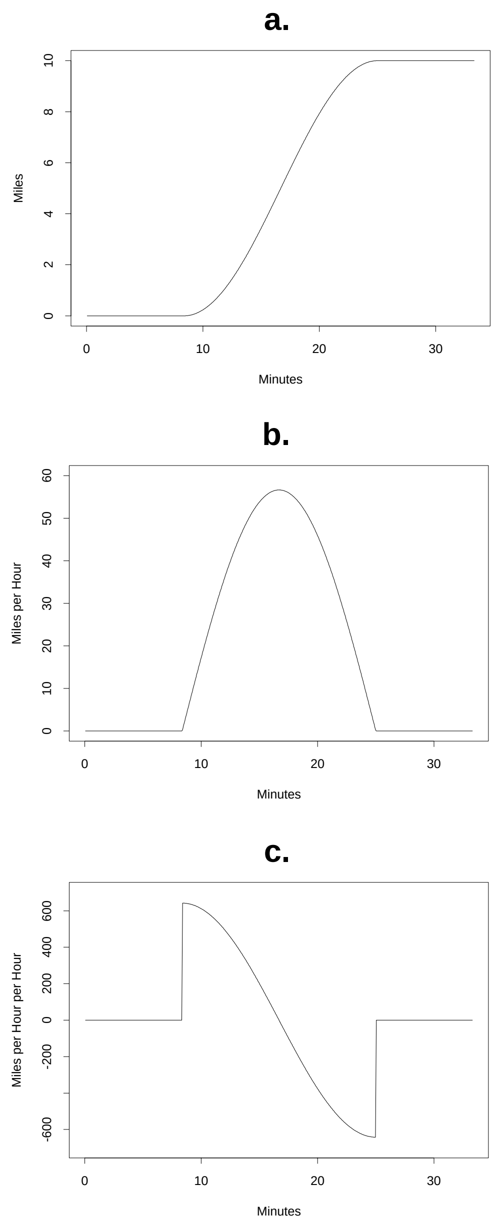

Figure 1. Position and derivatives of automobile traveling between two locations 10 miles apart. a. The abscissa plots time in minutes and the ordinate plots the distance from the original location. b. The ordinate plots the first derivative of the automobile's distance from the original location; the velocity of the automobile at each instant in time. c. The ordinate plots the second derivative of the automobile's distance from the original location; the acceleration (or deceleration) of the automobile at each instant in time.

by a distribution of more or less stable trait scores on this dimension. Thus, given some outside influence that perturbs a normal subject's mood away from equilibrium, we would expect that over time the mood would move back towards equilibrium. Conversely, in RCBD subjects, one might characterize the likely day–to–day changes as some sort of limit cycle attractor. That is, given an outside effect that perturbs an RCBD subject's mood, they might be expected to return to a characteristic pattern of cycling between elation and depression.

A dynamical model produces a prediction about the attractor which best fits the data. This is the basic difference between a dynamical model and a growth curve model which produces a prediction about a best fitting growth curve (what a dynamical modeler would call a single best fitting *trajectory*).

One of the most exciting aspects of the use of dynamical models, and by this we mean models that include change as predictors and outcome variables, is that simply by varying the coefficients of the model we can produce expected attractors that have very different shape (Hubbard & West, 1991; Boker & Graham, 1998). Thus for some dynamical models, the structure of the model does not need to change in order for the expected attractor to change from a point attractor to a limit cycle; only the coefficients need to change (see e.g. Kaplan & Glass, 1995). This means that if, for instance, depression were to be a dynamical disease a single structural equation model might account for normal, clinically depressed and RCBD attractors with between group differences in coefficients for the model.

Developmental Change in Attractors

One does not expect that mechanisms for self–regulation or environmental–organism coupling are limited to any particular time scale except those limits imposed by biology. That is to say, there is no apparent logical reason that the phenomena of self–regulation would be limited to, for instance, the time scale at which the endocrine system operates. Similarly, logic alone offers no reason that the mechanisms of self–regulation could not operate at the time scale of a lifetime. Thus, one might reasonably expect to observe dynamical processes in human behavior operating at time scales measured in milliseconds to time scales measured in decades (Nesselroade & Boker, 1994).

Another possible way that we might observe change is through a developmental change in an existing behavioral attractor (Thelen, 1995; Clark & Phillips, 1993). That is to say, a behavioral attractor might change shape during the process of development. Thus, one pattern in short–term intraindividual variability might morph over time into another pattern in short–term intraindividual variability. Thus we might plausibly study not only individual differences in change and changes in individual differences (Nesselroade, 1991a), but we may be able to model intraindividual change in intraindividual variability using dynamical models.

An example of the way that a behavioral attractor would exhibit developmental change might be observed as an infant becomes capable of self–supported sitting (Bertenthal & Bai, 1989). In this chapter, data from a study of postural control in infants will be used to illustrate techniques that can test hypotheses about the structure of the covariances in short–term intraindividual variability as well as test hypotheses about developmental change in attractors. While this data set contains many observations per individual, these same techniques, given an assumption of homogeneity of attractor shape over individuals, can be applied to multi–wave panel data with few observations per individual (Boker & Graham, 1998).

Example: Development of Postural Control

This analysis will use an example data set that comes from a moving room experiment performed on a group of infants (Bertenthal, Rose, & Bai, 1997). These infants were selected to have ages which straddled the average age of onset of self–supported sitting: 6.5 months (Bayley, 1969). The data from this experiment holds interest for examination using dynamical systems methods since developmental changes in the dynamics of the postural control mechanism can be studied simultaneously with the coupling between perception and action. By studying developmental changes in the coupling between visual perception and postural action we hope to better understand the nature of the postural control system.

Subjects and Procedure

Forty infants participated in the study, 10 in each of 4 age groups: 5, 7, 9 and 13 months. The moving room consisted of a 1.2 m $\times$ 1.2 m $\times$ 2.1 m open–ended enclosure, the walls and ceiling of which were constructed of fiberboard covered in green and white vertically striped cloth and which was mounted on small rubber wheels that rolled on tracks fixed to the floor. A small window in the middle of the front wall of the moving room provided a view to a small electronically activated toy dog that was used to fix the gaze of the infant at the beginning of each trial. A potentiometer was attached to one wall of the moving room such that the position of the room could be measured by the voltage drop through the potentiometer. The position of the room was sampled at 50 Hz and converted to a digital time series using an 8 bit A/D converter thereby creating a time series $R = \{r_1, r_2, r_3, \ldots r_N\}$ representing the room movement.

A forceplate was set in the middle of the floor of the moving room and an infant's bicycle seat was mounted on the center of the forceplate facing parallel to the direction of room movement. The forceplate consisted of a rigid metal plate suspended from 4 pressure transducers that effectively sampled the center of pressure of the infant at 50 Hz synchronously with the measurements of room position.

An infant sat in the infant bicycle seat and at the beginning of each trial, the toy was activated to direct the infant's attention to the front of the room. The walls were then moved in one of six movement conditions for a period of approximately 12 seconds during which time the infant's center of pressure was measured. Four of the movement conditions consisted of a 2×2 design in which the room oscillated at 0.3 Hz or 0.6 Hz and the amplitude of the oscillation was either 9 cm or 18 cm. The remaining two movement conditions were a pseudorandom oscillation and a control condition in which the room did not oscillate. Each infant performed two trials in each movement condition. For the purposes of the present analysis, only the movement conditions in the 2×2 design will be considered.

During each trial, the force plate measured changes in the infant's center of pressure along two axes: the anterior–posterior axis (X) aligned with the movement of the room, and the lateral axis (Y) orthogonal to the movement of the room. As the room began to move, the two axes of center of pressure and the unidirectional position of the room were simultaneously sampled at 50 Hz (20 msec sampling interval) for 10.25 seconds. Thus each trial generated three synchronized time series containing 512 samples: X, Y and R.

Postural Control as a Dynamical System

Figure 2 shows the fore-aft movement of one infant and the corresponding movement of the room plotted against time for one trial in the 0.6 Hz and 9 cm condition. By inspection, it appears that while the infant is accommodating to the room to some degree, the coupling between the room movement and infant center of pressure is not strong. We will model and test this how individual differences in this coupling may be related to the room movement conditions and to age–related change.

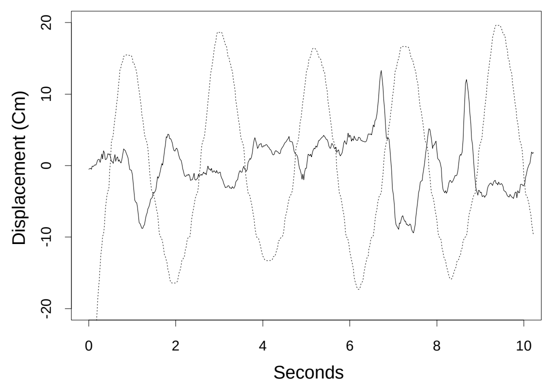

Figure 2. Time series plot of room movement (dotted line) and anterior–posterior center of pressure (COP) (solid line) for one trial for one seven month–old infant.

Note that in Figure 2, the room movement describes an oscillation similar to a sine wave. If the infant's center of pressure (COP) were to exactly mimic a sine wave oscillation, then the resulting time series would be expected to exactly fit a linear oscillator. We will use a dampened linear oscillator as a first approximation to the movement of the infant.

A dampened linear oscillator can be thought of as a pendulum with friction. When a pendulum with friction is given a single push, it will swing back and forth for a while until it comes to rest at its equilibrium point. If we model the infant's COP as a dampened linear oscillator, we are proposing the hypothesis that the postural control system in the infant is attempting to return the COP to a point of equilibrium. Perceptual input that drives the infant's COP away from this point of equilibrium will induce compensatory applications of muscular force that will eventually return the COP to the single point of equilibrium.

Let us explore the consequences of the hypothesis of an undampened linear oscillator for postural control in sitting. A undampened linear oscillator suggests that the acceleration towards the equilibrium point is proportional to the distance from the equilibrium point. This mechanism has face validity in that the farther one is from equilibrium, that is the more off–balance one is, the more force one would tend to exert in order to attempt to return to equilibrium.

Suppose that the infant's COP was at some distance anterior to the equilibrium point. Through some sensory mechanism the infant acquires the information required to produce a compensatory posterior movement towards the equilibrium point. As the infant applies muscular force, the COP will accelerate in a posterior direction towards the equilibrium point.

If all that the postural control system were to use was information about how far the COP was from the point of equilibrium, then the COP would continued to accelerate towards the equilibrium point until it passed the equilibrium point. As long as the COP is anterior to the equilibrium point, the control system would apply muscular force to produce acceleration in the posterior direction. However, as soon as the COP passes the equilibrium point, the control system would begin to exert force in the opposite direction; small amounts at first, but as the COP became farther from the equilibrium point, proportionally more force would be exerted. The COP would decelerate, eventually stop, and begin to accelerate in the anterior direction. This process would continue indefinitely and the maximum velocity of the COP would always be attained exactly at the equilibrium point. The system would always overshoot the equilibrium point and the resulting movement would be exactly that described by a frictionless pendulum as shown in Figure 3–a

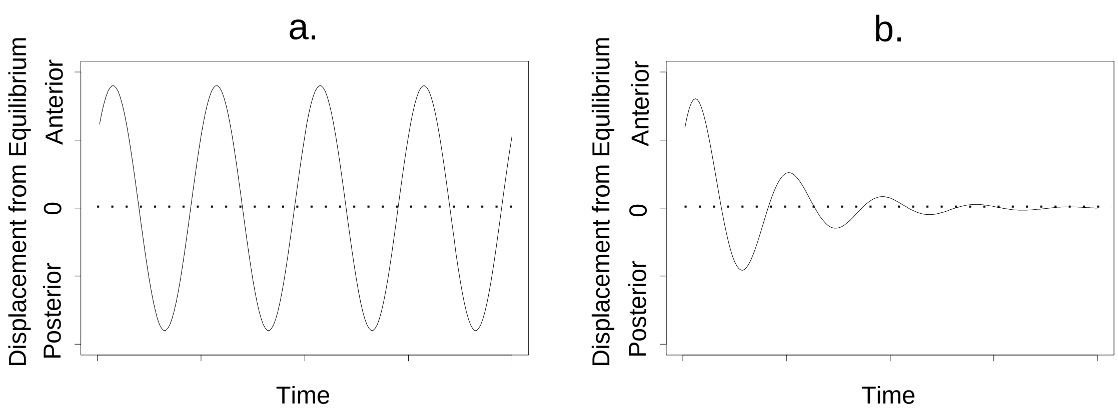

Figure 3. Time series plot of (a) undampened and (b) dampened linear oscillators.

Now let us explore the consequences of adding dampening to the linear oscillator model. As for the undampened oscillator model above, in response to a displacement from equilibrium the postural control system produces a proportionate acceleration towards equilibrium. But in addition, a dampening force will be added: COP velocity produces a proportionate acceleration in the opposite direction. Thus the faster the COP is traveling, the greater the opposing muscular force applied to decelerate the postural movement. In the undampened oscillator we saw that the greatest velocity occurred as the COP was passing over the point of equilibrium. Thus this is the point at which the dampening force will be the greatest. At the extreme values of displacement, the velocity is zero, and so the dampening force will be zero at those points as well. Adding a dampening force to the linear oscillator produces a trajectory as shown in Figure 3–b in which the COP eventually comes to rest at the equilibrium point.

Differential Equation Model of a Linear Oscillator

This dampened oscillator can be expressed as a multiple regression in which the acceleration of the system is predicted by the displacement from equilibrium and the velocity. If we use the standard notation for instantaneous velocity and acceleration this equation can be written as

$$\frac{d^2x(t)}{dt^2} = \zeta\frac{dx(t)}{dt} + \eta x(t) + e(t) \ , \tag{1}$$

where $x(t)$ represents the displacement from equilibrium at time t, $\frac{dx(t)}{dt}$ represents the velocity at time t and $\frac{d^2x(t)}{dt^2}$ represents the acceleration at time t. The parameter η is the square of the frequency of the resulting oscillation, ζ is the dampening (or friction) and e_t is the residual error (see e.g. Thompson & Stewart, 1986).

Many readers may be unfamiliar with the use of "velocity" and "acceleration" in a psychological context. However, velocity is simply linear change; often referred to in growth curve models as slope. In the same context, acceleration would be referred to as curvature. It is important to realize that the techniques applied here to long time series can also be applied to longitudinal data sets with as few as three time points (Boker & Nesselroade, 1999). Ideally, a minimum of six waves of measurement would allow a wider range of models to be tested. However, the techniques applied here are not limited to datasets with hundreds of occasions of measurement.

The three variables in the dampened linear oscillator, displacement, velocity and acceleration, are related as a dynamical system. That is to say, the specific relationships between these three variables specifies a central tendency of a family of trajectories that any one individual might have. This is different than a growth curve type of model in that a growth curve model makes a prediction about a single trajectory of central tendency. The set of equilibria for this family of trajectories is called the system's *attractor*. In this case the attractor is a single point. A best fitting family of trajectories for a dynamical systems model is called an *basin of attraction*, which can be visualized using a *vector field* plot.

One possible shape for the basin of attraction for a dampened linear oscillator is plotted in Figure 4. In this figure, a grid of possible combinations of displacement ($x(t)$) and velocity ($dx(t)/dt$) are plotted. For each of these combinations, an arrow is plotted whose direction and length are determined by the predicted value of the acceleration such that the end of the arrow points to where the expected values of position and velocity will be after a short interval of time.

By selecting a pair of initial values for velocity and displacement, one may follow the arrows and determine the trajectory that would evolve from that original set of starting values. Every trajectory, in this case, will end up at the equilibrium point of zero displacement and zero velocity. Just as there are infinitely many possible starting values for velocity and displacement, there are infinitely many expected trajectories that conform to the shape of a basin of attraction. The shape of the basin of attraction and the attractor itself is determined by the values of the parameters that are estimated from Equation 1. Individual differences in these parameters would indicate that individuals have basins of attraction with different shapes, and may have attractors with different shapes. If these parameters change with development, there would be a developmental change in the shape of the basin of attraction and possibly in the attractor itself (Thelen, 1995; Clark, 1997; Geert, 1997;

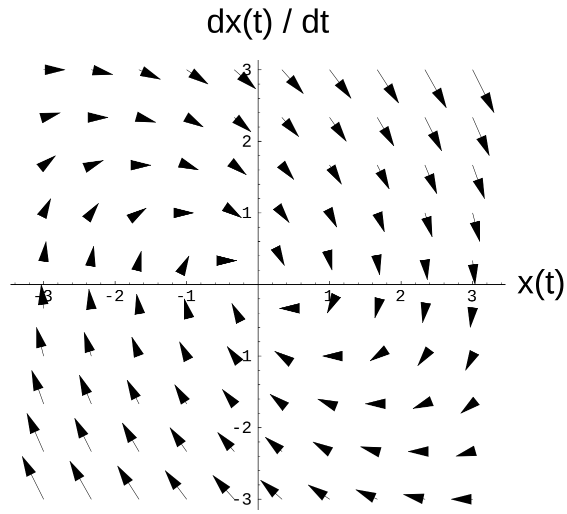

Figure 4. Vector field plot of the point attractor for a dampened linear oscillator with frequency and dampening parameters similar to a pendulum with friction.

Hopkins & Butterworth, 1997).

Local linear approximation to derivatives.

In order to fit a differential equation model to data, at each available occasion of measurement an estimate for the first and second derivatives of the variable must be calculated. Many estimation methods may be used (e.g. Cleveland & Devlin, 1988; Gu, 1990), but local linear approximation is easy and has worked well in simulations (Boker & Nesselroade, 1999).

Suppose the variable X is measured on three occasions resulting in the measurements $x(1)$, $x(2)$ and $x(3)$. A local linear approximation for the derivative of X at the second occasion of measurement is given by the average of the two slopes between $x(1)$ and $x(2)$ and between $x(2)$ and $x(3)$ (see Figure 5),

$$\frac{dx(1+\tau)}{dt} \approx \frac{x(1+2\tau)-x(1)}{2\tau\Delta t} \,. \tag{2}$$

where in this case $\tau = 1$ because $x(1)$, $x(2)$ and $x(3)$ are successive occasions of measurement and Δt is the interval of time between measurements. Every other measurement in a sequence (for instance $x(1)$, $x(3)$ and $x(5)$) could be used if one substituted $\tau = 2$ into Equation 2.

Similarly, the local linear approximation for the second derivative of x at the second occasion of measurement can be calculated from the same triplet of scores $x(1)$, $x(2)$ and $x(3)$ as simply the change in the slopes,

$$\frac{d^2x(1+\tau)}{dt^2} \approx \frac{x(1+2\tau)-2x(1+\tau)+x(1)}{\tau^2\Delta t^2} \,. \tag{3}$$

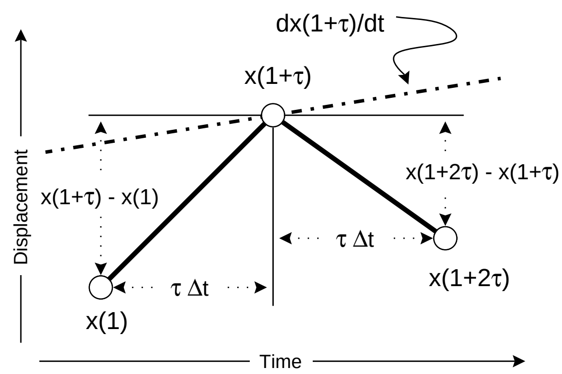

Figure 5. Local linear approximation of the derivative of X at the second occasion of measurement.

Differential Structural Equation Models

The dampened linear oscillator defined in Equation 1 can be considered to be an ordinary multiple regression equation. If we suppose that variables outside the dampened linear oscillator might have an effect on it, a system of linear equations will result. Structural equation modeling provides a powerful tool for estimating parameters of systems of linear equations, and can be effectively used here. We call this technique Differential Structural Equation Models, or dSEM.

In our example data, a model can be fit to each individual trial in which the infant's anterior–posterior COP and room movement were recorded. Each trial consists of 512 data points, so if we assume that the basin of attraction is not changing shape during the trial, we have over 500 observations of the three values: displacement, velocity and acceleration with which to estimate the shape of the basin of attraction.

For each trial, five structural models were fit to the data generated by the room movement and infant anterior–posterior COP. The first model, shown in Figure 6, is one in which the movement of the room provides no prediction of the position, velocity or acceleration of the infant. This model will be used as a null model for comparison to other models that make specific predictions about how the room affects the infants' posture.

The three variables R, dR, and $d2R$ at the top of Figure 6 represent the displacement, velocity and acceleration of the room respectively. The remaining three variables X, dX, and $d2X$ represent the displacement, velocity and acceleration of the infant's anterior–posterior center of pressure (AP–COP). The variables X, dX, and $d2X$ form the hypothesized dampened linear oscillator for the infant such that the coefficient η is the proportional effect of the AP–COP displacement on the acceleration of the AP–COP and the coefficient ζ is the proportional effect of the velocity of the AP–COP on the acceleration of the AP–COP. The loops from a variable to itself represent the variance of that variable, according to the RAM path diagram form introduced by McArdle (McArdle & McDonald, 1984; McArdle & Boker, 1990) and used by the Mx structural equation modeling software (Neale, 1994).

Since the expected movement of the infant is a swaying motion with a frequency equal to the frequency of the moving room, the value of η was fixed so that the frequency of linear

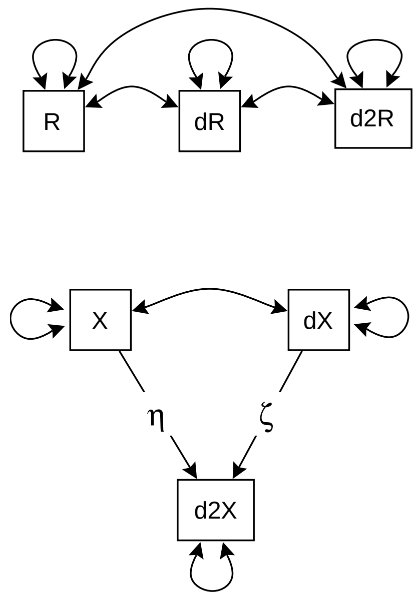

Figure 6. Model A – Differential structural equation model in which the moving room does not predict infant's anterior–posterior center of pressure (AP–COP). The variables in the model: R is room displacement, dR is room velocity, $d2R$ is room acceleration, X is AP–COP displacement, dX is AP–COP velocity, and $d2X$ is AP–COP acceleration.

oscillator comprised by the three variables X, dX, and $d2X$ would be forced to match the frequency of the moving room condition. If the estimated value of ζ is negative, then we expect a dampened linear oscillator. But if the estimated value of ζ is positive, then rather than dampened oscillations over time, we expect to see amplified oscillations over time. If the estimated value of ζ is near zero, then the oscillator is a "frictionless" system; one that does not dampen or amplify over time.

There are two sources of potential for misfit in Model A. First, since the value of η is fixed, then to the degree that the postural control system of the infant is not well described by a linear oscillator at a frequency equal to the frequency of the room, the model will not fit the data. The second source of misfit is the strong assumption of zero covariance between all of the room variables and all of the infant variables. The covariances between the room variables are completely saturated, so the structure of their covariances will be fit exactly and thus cannot contribute to model misfit.

The other four models that were fit to each trial are shown in Figure 7. Each of these models explores a different structure for the effect of the room's displacement, velocity and acceleration on the AP-COP of the infant.

Model B and Model C provide two alternative ways in which the room variables could influence the acceleration of the infants' AP–COP. In Model B, the effect of the room on the AP–COP acceleration is mediated through the displacement and velocity of the AP–COP. On the other hand, in Model C, the effect of the room variables is directly on the AP–COP acceleration. These two models form an interesting dichotomy from a dynamical systems perspective.

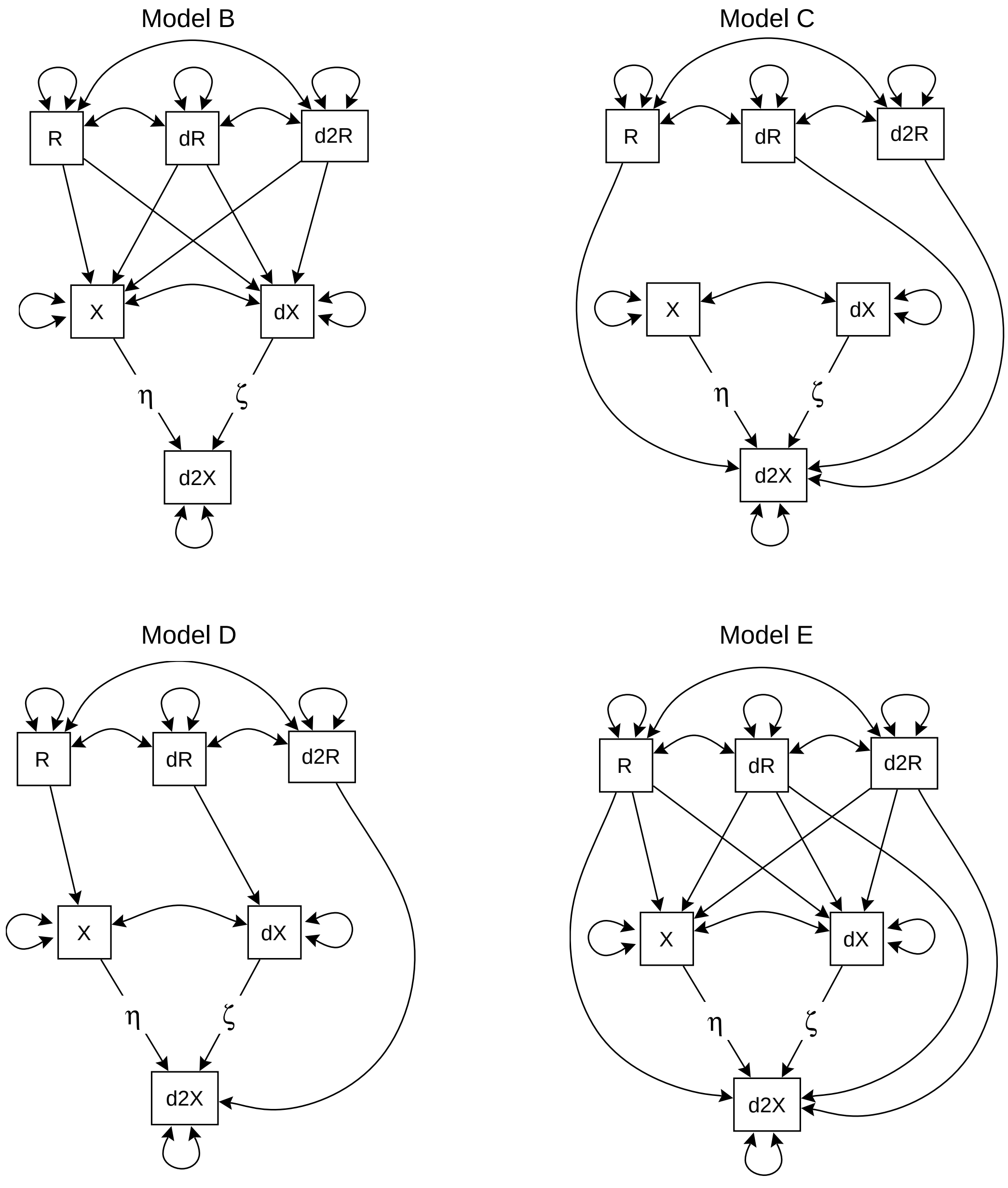

Figure 7. Four differential structural models of the effect of the moving room on the infants' anterior–posterior center of pressure.

In Model B, there is no possibility for the coefficients linking the room variables to the AP–COP variable to affect the estimates of the parameters of the AP–COP oscillator. Thus in Model B, the shape of the basin of attraction for the AP–COP oscillator is independent of the moving room. If Model B were preferred, the theoretical conclusion would be that visual perception is an input to the postural control system, but not an intrinsic part of it.

But in Model C, changes in the coefficients linking the room variables directly to the AP–COP acceleration could cause changes in the AP–COP oscillator coefficients. In this case, the effect of the moving room includes a change in shape of the basin of attraction for the AP–COP oscillator, and if Model C were to be preferred, the theoretical conclusion would be that the perception of the visual stimulus was incorporated as an intrinsic part of the postural control system.

Model D tests the hypothesis that the room displacement only affects the AP–COP displacement, the room velocity only affects the AP–COP velocity and the room acceleration only affects the AP–COP acceleration.

Model E fully saturates the paths between the room and the AP–COP oscillator and thus provides an anchor as the best the model could fit, given that the frequency of the AP–COP oscillator has been fixed. Model E has one degree of freedom, and its misfit will give an indication of how well the fixed frequency oscillator model fits the infants' responses to the moving room.

Results

Each of the five differential structural models were fit to each of the 320 individual trials. In order to remove artifacts possibly associated with the start of the trial and the end of the trial, the first 100 samples and final 200 samples of the trial were ignored, leaving 212 samples of room position and AP–COP from the middle of each trial. The derivatives were calculated using linear interpolation with a time lag of $\tau = 24$. The value of τ was chosen as the value that would induce the least bias in the parameter estimation given the two moving room frequencies. When the correlation between the displacement and its first derivative is minimized, the bias in η is minimized. A τ equal to the number of samples in one quarter the average period (1.92 seconds = 96 samples, $96/4 = 24$) will minimize the correlation between displacement and its first derivative for a sine wave. After choosing $\tau = 24$, first and second derivatives were calculated for each trial using local linear approximation. The resulting number of samples of displacement, velocity and acceleration for room and AP–COP was $N = 165$ for each trial. Next a covariance matrix was calculated for the room and AP–COP displacement, velocity and acceleration (a 6×6 covariance matrix) was calculated for each trial. Finally parameter estimates and fit statistics were obtained were obtained by fitting the previously described models individually to each trial's covariance matrix. Thus for each trial there were obtained individual parameter estimates for each model.

Table 1 presents the overall mean values of the estimated parameters and fit statistics for the five models tested as averaged across all conditions and all infants. On first inspection, none of the fit statistics appear promising, with RMSEAs in the range of 0.42 to 0.97. From this we can conclude that on average, a model of infant postural control based on a linear oscillator with a frequency fixed to the room frequency is not sufficient to account for the variance in the acceleration of the infant. While the mean multiple R^2 values for AP–COP acceleration are in the range 0.40 to 0.46, these are not sufficiently large to suggest that the linear oscillator model is doing an adequate job of accounting for the intrinsic dynamic of the infant.

Table 1: Mean coefficients and fit statistics over all infants, all conditions and all trials.

	Model A	Model B	Model C	Model D	Model E
η	-8.05	-8.05	-8.05	-8.05	-8.05
ζ	-0.01	-0.01	0.00	-0.04	0.00
$R \to X$		-1.49		-1.07	-1.49
$dR/dt \to X$		0.14			0.14
$d^2R/dt^2 \to X$		-0.06			-0.06
$R \to dX/dt$		-2.80			-2.80
$dR/dt \to dX/dt$		-1.03		-0.98	-1.03
$d^2R/dt^2 \to dX/dt$		-0.02			-0.02
$R \to d^2X/dt^2$			9.86		9.86
$dR/dt \to d^2X/dt^2$			0.38		0.38
$d^2R/dt^2 \to d^2X/dt^2$			0.86	0.12	0.86
R^2 for d^2X/dt^2	0.40	0.40	0.46	0.42	0.46
N	165	165	165	165	165
DF	10	4	7	7	1
χ^2	322	182	304	254	165
RMSEA	0.42	0.51	0.49	0.45	0.97

Perhaps it is not so surprising that the movement of an infant would have a large degree of unpredictability, and we might consider that a poorly fitting model is to be expected. If one accepts the linear oscillator model as a first approximation, then the mean values of the parameters η and ζ can be examined. The mean value of $\eta = -8.05$ was predetermined: it is the average of the η's that were for the two moving room frequencies which were fixed when the model was fit. The values of η for the 0.3 Hz (-3.21) and 0.6 Hz (-12.88) conditions were determined empirically by fitting an oscillator model to the moving room data. It is interesting to note that the range of mean values for ζ across the five models is from -0.04 to 0.00. These values are very close to zero, indicating that if the linear oscillator model is used as an approximation to the infants' AP–COP, then the system is essentially "frictionless"; it has neither dampening nor amplification over the course of a single trial.

Whether or not the linear oscillator portion of the model is rejected as a first approximation, there are still several interesting differences between these models. Recall that there are two sources of potential misfit in the model. Model A and Model B are nested and their χ^2 difference is 140 for 6 degrees of freedom. On the other hand, Model A and Model C are nested and their χ^2 difference is only 18 for 3 degrees of freedom. It seems evident that Model B is doing a better job of accounting for the covariance between the room and the infant than is Model A, while Model C does not do much better than Model A, the hypothesis of no influence of the room on the infant AP–COP.

Similarly, the χ^2 difference between the nested models Model B and Model E is 17 for 3 degrees of freedom, while the χ^2 difference between the nested models Model C and Model E is 139 for 6 degrees of freedom. Thus Model B fits almost as well as the model that fully saturates the paths between the room and the infant variables, whereas Model C fits almost as poorly as the model that fixes to zero the effect of the room variables on the infant variables.

Model D which posits that the room displacement affects infant AP–COP displacement, room velocity affects infant AP–COP velocity and room acceleration affects infant AP–COP acceleration has a mean χ^2 fit statistic that is approximately half way between the null room effect model and the fully saturated room effect model.

Examining the mean parameters for Model B suggests that we may be able to create a model that fits almost as well as Model B by fixing to zero the effects of the room velocity and acceleration on infant AP–COP displacement as well as fixing to zero the effect of the room acceleration on infant AP–COP velocity. The parameter values from Model B suggest that the displacement and velocity of the room were the primary visual cues used by these infants to control their sitting posture.

Age–based increase in model misfit

The infants in this study were in one of four age categories at the time of testing: 5, 7, 9 or 13 months. Figure 8 plots the mean χ^2 fit statistic for each model within each age category. There is an age–related increase in the χ^2 for Model A and Model C that appears to reach an asymptote around 9 months of age. There is a much smaller increase in χ^2 for Model B and Model E that follows the same general pattern. Again, the mean fit for Model D is approximately midway between the best fitting and worst fitting models.

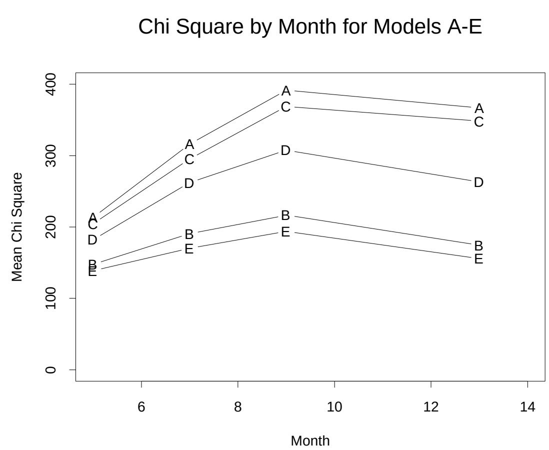

Figure 8. Mean model fit (χ^2) as a function of age of infant for the five models: Model A, Model B, Model C, Model D and Model E.

The difference between the misfit of Model A and Model E increases between the ages of 5 and 9 months. At each age, this difference in misfit is attributable to the degree to which an existing predictive effect of the room on the infant AP–COP is not captured by the null effect model, Model A. Thus, there appears to be a substantial age–related increase in the effect that the room variables have on the infant AP–COP. This may be interpreted as an age–related increase in the infants' response to the moving room, an increase that asymptotes at around 9 months of age. This finding is in accordance with other correlational (Bertenthal et al., 1997) and nonlinear (Boker, 1996) analyses of these data.

Predicting coefficients and fit of trial level differential structural models

The coefficients from Model B which predict the infants' AP-COP from the room variables are likely candidates for a second level of analysis. Since in each trial the room was moved with one of two different frequencies and one of two different amplitudes, it might be that the infant responded to one of these frequencies or amplitudes more than another. It might also be the case that there are age–based differences in these parameters.

As a first approximation to answering these questions, a series of multiple regressions were performed, predicting each of the six parameters linking the room variables to the infant variables in Model B from the Age, Frequency and Amplitude of each trial. The results of these analyses are presented in Table 2.

Table 2: Coefficients from Model B as predicted by unstandardized multiple regressions from Age, Frequency and Amplitude conditions within each trial. Probability values are derived using the simplifying assumption that individual trials are independent. Predictor variables were left in their raw metric: Months of age, Frequency in cycles per second, and Amplitude in centimeters.

	Intercept	Months	Frequency	Amplitude	R^2
$R \to X$	2.076	-0.344	-2.020	0.020	0.116
$p()$	0.017	0.000	0.068	0.593	
$dR/dt \to X$	-0.900	0.040	1.530	0.001	0.160
$p()$	0.000	0.000	0.000	0.900	
$d^2R/dt^2 \to X$	0.041	-0.025	0.101	0.005	0.032
$p()$	0.747	0.003	0.538	0.383	
$R \to dX/dt$	11.037	-0.483	-23.605	0.066	0.199
$p()$	0.000	0.001	0.000	0.495	
$dR/dt \to dX/dt$	1.082	-0.226	-0.640	0.007	0.148
$p()$	0.024	0.000	0.293	0.715	
$d^2R/dt^2 \to dX/dt$	0.387	0.002	-0.994	0.002	0.038
$p()$	0.080	0.873	0.000	0.860	
χ^2	145.943	8.781	-28.318	3.405	0.064
$p()$	0.000	0.000	0.516	0.020	

Scanning down the column of R^2 values, it is interesting to note that the experimental manipulations and age of the infant predict between 11% to 20% of the variance in four of the coefficients for Model B, but only predict less than 4% of the variance for the remaining two coefficients. These two coefficients, $d^2R/dt^2 \to X$ and $d^2R/dt^2 \to dX/dt$ also do not have an intercept that is significantly different than zero. This again suggests that acceleration of the room may not play a part in the infants' postural sway responses.

However, 16% of the variance in the Model B coefficient $dR/dt \to X$ can be accounted for by the age of the infant and experimental conditions. Although the mean value for this coefficient over all conditions and all ages is near zero, the coefficient's value does appear to covary with the age of the infant and with the frequency condition ($p < 0.01$). This suggests that both room displacement and room velocity affect the infants' AP–COP displacement and velocity.

It is interesting to note that the Model B coefficient $dR/dt \rightarrow X$ changes sign depending on the frequency condition. For the mean age of 8.5 months and the slow frequency of 0.3 Hz, this coefficient has a predicted value of -0.10, whereas for the fast frequency of 0.6 Hz, this coefficient has a predicted value of 0.36. Thus the velocity of the room is predictive of AP-COP displacement in the opposite direction of the room velocity in the slow frequency condition, but in the same direction in the fast frequency condition.

The age of the infant is a significant predictor ($p < 0.01$) of the four Model B coefficients with $R^2 > 0.11$, suggesting that there is an age–related change in the effect of the room displacement and velocity on the infants' sway responses. The room frequency is a significant predictor for only two of the selected four Model B coefficients: the effect of the room velocity on the AP–COP displacement, and the effect of the room displacement on the AP–COP velocity. This interesting symmetry may be an important clue to the functional form of a better model for perception–action coupling in visually guided postural control.

Finally, as we had previously noted in the plot of Model B χ^2 as a function of age, there is a significant ($p < 0.01$) age–related change in the the fit of Model B. The linear oscillator model fits the younger infants' data better than it does the older infants data.

Discussion

This chapter illustrated techniques that might be applied to a wide range of longitudinal data. In order for a dynamical systems theoretic approach to be appropriate, one must have a theory that suggests that changes in the individual, in the environment, or in the differences between the individual and the environment are predictive of future behavior of the individual. In order for the differential structural equation modeling technique to be applied, one needs at least several hundred triplets of measurements (a triplet consisting of displacement, first and second derivatives of a variable). These triplets could be sampled from a single individual or across individuals.

In order for second level analyses to be performed, one must have several groups of individuals or several experimental conditions such that in each group there are at least several hundred triplets of observations. This data limitation may seem daunting, but we are attempting to capture variability at short time scales while at the same time estimating longer term change or between groups differences.

We recommend that if one is designing a study in hopes of being able to test theories that predict the attractor that best fits a covariance structure in intraindividual variability or changes in the shape of a basin of attraction, one should gather a *burst measurement* sample on each individual (Nesselroade, 1991b). A burst measurement is a group of repeated observations gathered with a short time interval between observations. The number of observations in the burst will be determined by whether an assumption of homogeneity of attractor shape can be supported, or whether individual or group differences in attractor or basin of attraction shape are to be examined.

If homogeneity of attractor shape can be assumed, then we recommend no less than six observations per burst measurement on an individual. If group differences in attractor or basin of attraction shape are to be examined, we recommend no less than six observations in a burst measurement for an individual and a few hundred bursts per group. If individual differences in attractor or basin of attraction shape are to be examined, we recommend at least 200 measurements per burst measurement on each individual, while the number of individuals may be determined using a power calculation based on expected between–individual differences in the coefficients of the structural model.

We expect that there are a large number of problems in developmental, clinical and cognitive psychology as well as problems in biology, sociology and economics for which differential structural models may prove to be useful. There presently seems to be a change in direction of psychological theory in order to take into account processes by which an individual's behavior may evolve. This, coupled with the growing number of tools available to examine the issues of change make for an especially exciting time to be involved in psychological research.

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