Pricing and operation in deregulated electricity market by noncooperative game

L. Geerli a,*, L. Chen b, R. Yokoyama a

a Tokyo Metropolitan University, Minamiosawa 1-1, Hachioji, Tokyo 192-03, Japan
b Osaka Sangyo University, Nakagaito 3-1-1, Daito, Osaka 574-8530, Japan

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Abstract

Pricing structure is becoming considerably important for both electric utility industries and their customers. This paper derives an operation rule for a market model with an electric utility and independent power producers (IPPs) as players of the noncooperative game. The derived operation rules reflecting the competition can be viewed as an extension of the conventional equalizing incremental cost method for the deregulated power systems. As indicated in this paper, the prices of electricity for purchases and sales are equal to the incremental costs of the generators of IPPs but are generally cheaper than the incremental cost of the generators belonging to the utility. To examine the proposed approach, several systems are used as the demonstrated examples in this paper. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Game theory; Equalizing increment cost; Pricing; Power systems; Deregulation; Market

1. Introduction

Electricity markets are experiencing widespread changes that are significantly altering the industry. When new economic mechanisms are being considered for use in the electricity market, there is much talk of ‘gaming’ and whether the mechanism will ‘be gamed’ [1]. Due to the emergence of independent power producers (IPPs), as well as the changing structure of the electricity supply industry, the electric power industry has entered an increasingly competitive environment under which it becomes more realistic to improve economics and reliability of power systems by enlisting market forces [4,8,9]. For example, to maximize his payoffs, a player (e.g. a utility) seeks to displace expensive generation by importing power from neighboring players with lower cost energy. Likewise, a player (an IPP or a utility) with excess generation capacity can choose to export power and receive an immediate return on its investment [10]. Up to now, a considerable number of literatures addressing the competition and deregulation issues have been published [4,8,7,9–12].

This paper aims to propose a game theoretical negotiation method to provide rational decisions of the power exchange for purchases and sales between the utilities and IPPs, as well as prices of the electrical energy under competitive environment, in contrast to the conventional equalizing incremental cost rule. Generally, the game theory is divided into cooperative game and noncooperative game, depending on whether or not the decisions are based on the unanimous agreement among all players [5,10]. For the noncooperative games, the Nash equilibrium solution, saddle solution, minimax principle, Stackelberg strategy and others are mostly used to make the decisions, in contrast to core solution, nucleolus, Shapley value and Nash bargaining solution, which are the solutions of cooperative games [5,6]. Although the solutions of cooperative games belong to the Pareto set from the viewpoint of social welfare, they generally are not easily realized due to requirement of unanimous agreement among all players under competitive environment, comparing with decisions of noncooperative games, which are actually compromise solutions for players.

It is well known that the equalizing increment cost rule for a simplified network model is not only simple but also optimal due to the optimality of social welfare,
as far as each participant in operations of a power system is completely cooperative [14]. However, for the deregulated power systems, this rule may not hold because participants intend to pursue their own benefits, thereby may not always take cooperative attitude. In this paper in order to compare with the equalizing incremental cost rule, we mainly use the Stackelberg strategy and the Nash equilibrium solution to analyze the negotiation process between utilities and IPPs since they are widely recognized as rational decisions for competitive markets in terms of axiom [5,10].

In Section 2, we first derive a common price of electricity for purchases and sales between a utility and IPPs when assuming that all of IPPs construct a coalition who negotiates with the utility for bargaining. By the Stackelberg strategy [6], the Section 2 also provides very clear and simple operation rules, which are similar to the well-known equalizing incremental cost [8] but different due to the fact of competitive environment. In Section 3, we give the prices of electricity, as well as the operation decisions for the situation when each IPP negotiates with the utility independently, which means that IPPs sell their electricity at different prices depending on their negotiations with the utility. In Section 4, a Nash equilibrium [5] is used to analyze the negotiation between IPPs and the utility, which shows that the Nash equilibrium and the Stackelberg strategy have the following problems [3,15]:

Let $Z = (z_1,...,z_m)$ be supply power vector of the utility, where $z_j$, for $j = 1,...,m$ stands for real power output (MWH) of generator-$j$, and $m$ is total number of generators for the utility. Define $g_j(z_j)$ for $j = 1,...,m$ as the incremental cost ($\$/per MWH) of generator-$j$ of the utility. Then $\int_M g_j(z)dz$ is the cost function ($\$\$) for generator-$j$ of the utility. Assume that there are $n$ IPPs and define $X_i = (x_{i1},...,x_{in})$ to be supply power of IPP-$i = 1,...,n$, where $z_j$ for $j = 1,...,n_i$ represents real power output (MWH) of generator-$j$ and $n_i$ is the total number of generators for the IPP-$i$. Let $f_j(x_j)$ for $i = 1,...,n$; $j = 1,...,n_i$ be the incremental cost ($\$/per MWH) of generator-$j$ for the IPP-$i$, and $\partial f_j/\partial x_j > 0$. Then $\int_M f_j(x)dx$ is the cost function ($\$\$) for generator-$j$ of the IPP-$i$. Since this section analyses, the negotiation between a coalition of IPPs and a utility, all of electricity is assumed to be sold at the same price from IPPs to the utility. Then let $y$ be the price ($\$/per MWH) and $h(d)$ be earnings ($) of the utility from selling electricity to customers and is not related with $Z, X_i$. IPPs will sell electric power to utility by maximizing their profits, which can be formulated as the following problems [3,15]:

\[
\text{IPP - } i: \text{Maximize } \quad y \sum_{j=1}^{n_i} x_{ij} - \sum_{j=1}^{n_i} \int_0^{x_{ij}} f_j(x)dx \tag{1}
\]

where $\int_M f_j(x)dx$ is the cost function ($\$\$) for generator-$j$ of the IPP-$i$, and IPP-$i$ is assumed to decide $x_{ij}$, $j = 1,...,n_i$.

On the other hand, the utility intends to maximize its payoffs by utilizing existing generators and lowering the purchasing price of electricity from IPPs, which can be described as the following problem:

\[
\begin{align*}
\text{Utility:} \\
\text{Maximize } & \quad h(d) - y \sum_{j=1}^{m} \sum_{i=1}^{n_i} x_{ij} - \sum_{j=1}^{m} \int_0^{z_j} g_j(z)dz \\ \\
\text{s.t. } & \quad \sum_{j=1}^{m} \sum_{i=1}^{n_i} x_{ij} + \sum_{j=1}^{m} z_j = d \tag{2}
\end{align*}
\]

where Eq. (3) means that the utility has the obligation to serve all customers’ load, and the utility is assumed to decide $y, z_j, j = 1,...,m$. Note that we use a presumed condition in Eqs. (2) and (3), i.e. the purchased electric power $x_{ij}(y)$ is supposed to be a function of $y$. Since the
price of electricity for sales directly influences the payoffs of IPPs, it is reasonable for the utility to assume that the quantity of the purchased electricity energy from IPPs depends on its price. Notice that \( x_i(y) \) is not a predetermined function but an implicit function of \( y \), which is decided by the game at the equilibrium.

The optimality conditions for Eq. (1) is straightforward by differentiating the objective function with respect to \( x_i \), i.e.

\[
y = f_y(x_i) \quad (i = 1, \ldots, n; j = 1, \ldots, n_i)
\]

According to the Implicit Function Theorem and Eq. (4), there exists a differentiable function \( x_j(y) \) at the neighborhood of \( x_0 \) when \( |df/dx_1| \neq 0 \). Differentiating Eq. (4) again, we have the following relations

\[
dx_j = \frac{1}{df_y(x_i)/dx_{ij}} \quad (i = 1, \ldots, n; j = 1, \ldots, n_i)
\]

On the other hand, for Eqs. (2) and (3) of the utility, we establish a Lagrangian function

\[
L(Z,y,\lambda) = h(d) - y \sum_{i=1}^{n} \sum_{j=1}^{n_i} x_{ij}(y) - \sum_{j=1}^{m} \int_{0}^{z_j} g_j(z)dz
\]

\[
+ \lambda \left[ \sum_{i=1}^{n} \sum_{j=1}^{n_i} x_{ij}(y) + \sum_{j=1}^{m} z_j - d \right]
\]

Note that \( h(d) \) is a constant, then the optimality conditions for Eqs. (2) and (3) can be obtained by

\[
\partial L/\partial z_j \quad (j = 1, \ldots, m)
\]

\[
\partial L/\partial \gamma \quad (j = 1, \ldots, m)
\]

\[
\lambda = g_j(y) \quad (j = 1, \ldots, m)
\]

Substituting Eq. (5) into Eq. (8) and combining Eq. (4) with Eq. (9), we have the solution [6]

\[
y = f_y(x_i) \quad (i = 1, \ldots, n_i)
\]

\[
\lambda = g_j(y) \quad (j = 1, \ldots, m)
\]

\[
\lambda - y = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n_i} x_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n_i} 1/d_{ij}}
\]

\[
\sum_{i=1}^{n} \sum_{j=1}^{n_i} x_{ij} + \sum_{j=1}^{m} z_j = d
\]

which are actually conditions of the equilibrium of the game.

Observing Eqs. (10) and (11), the generators for IPPs and the utility obviously have the same incremental cost \( y \) and \( \lambda \), respectively. As shown in Eq. (12), however, their incremental costs are different due to competition between the utility and the coalition of IPPs. Since \( df_y/dx_{ij} \) is assumed to be positive, the incremental cost for utility is generally larger than that for IPPs according to Eq. (12). That implies that the generators of IPPs have to supply power economically at the operation point, comparing to those of the utility because the utility owns the transmission network and is generally burdened with the obligation to serve the customers’ loads. The price \( y \) of electricity for purchases and sales is taken as the same value of the equalizing incremental cost for IPPs as indicated in Eq. (10). It is evident that the discrepancy between the two incremental costs narrows if \( df_y/dx_{ij} \) have small value, which means that the IPPs can benefit more if they have generators with smaller \( df_y/dx_{ij} \) (i.e. flat cost function). Therefore, Eqs. (10)–(13) can be viewed as an extended equalizing incremental cost rule for the deregulated power systems.

Note that we can have similar equations as Eqs. (10)–(13) when taking the upper and lower limits of each variable into consideration, even though more Lagrangian multipliers have to be introduced. In this paper, we assume that the utility decides the purchased prices based on the outputs of IPPs while IPPs determine their outputs depending on the purchased prices. Therefore, this model mainly targets the system where the utility is large enough comparing to each IPP and almost dominates the market.

### 3. Negotiation between a utility and individual IPPs

This section gives a noncooperative solution by supposing that each IPP individually negotiates with the utility [3]. That means each IPP sells its electricity to the utility at the price different from those of other IPPs. Let \( Y = (y_1, \ldots, y_m) \) where \( y_i \) for \( i = 1, \ldots, m \) represents the price at which the utility purchases the electricity from IPP-\( i \). All other definitions are the same as those in Section 2.

Assuming that IPP-\( i \) and the utility decide \( x_i(y) = 1, \ldots, n_i \) and \( Y,Z \), respectively, then we have the following problems for IPPs:

\[
\text{IPP-}i: \quad \text{Maximize} \quad y_i \sum_{j=1}^{n_i} x_{ij} - \int_{0}^{z_j} f_{ij}(x)dx
\]

and for the utility:

\[
\text{Utility:} \quad \text{Maximize} \quad h(d) - \sum_{i=1}^{n} y_i \sum_{j=1}^{n_i} x_{ij}(y_i)
\]

\[
- \int_{0}^{z_j} g_j(z)dz
\]

s.t. \( \sum_{i=1}^{n} \sum_{j=1}^{n_i} x_{ij}(y_i) + \sum_{j=1}^{m} z_j = d \)

Hence in the same way as the derivations of the Section 2, we have the following conditions for a noncooperative solution.
\[ y_i, f_y(x_{ij}) \quad (i = 1, \ldots, n; j = 1, \ldots, n_i) \]

\[ \lambda = g_j(z_j) \quad (j = 1, \ldots, m) \]

\[ \lambda - y_i = \frac{\sum_{j=1}^{n_i} x_{ij}}{\sum_{j=1}^{n_i} 1/d_f(x_{ij})} \quad (i = 1, \ldots, n) \]

\[ \sum_{i=1}^{n} \sum_{j=1}^{n_i} x_{ij} + \sum_{j=1}^{n_i} z_j = d. \]

Observing Eq. (15), it is evident that the generators for the utility have the same incremental cost \( \lambda \), which coincides with the conventional results. However, according to Eq. (14), the generators for IPPs have different incremental costs because IPPs individually negotiate with the utility by using their own strategies, while the equalizing incremental cost is still hold for the generators within an IPP. The price \( y_i \) of electricity for purchases and sales are also different for each IPP from Eq. (14). Eq. (16) indicates that the flatter the fuel cost functions of the generators for IPP-i is (i.e. the smaller \( df_i/y_{ij} \)), the less the discrepancy of the incremental costs between the utility and IPP-i is liable to be. On the other hand, Eqs. (14) and (16) also show that the cheaper the incremental cost of generators for the IPP is, the more an IPP can sell power. Note that we can derive a similar non cooperative solution when assuming every generator in IPPs sells power at its price \( y_i \), analogue to the derivation of Eqs. (14)–(17). Therefore, Eqs. (14)–(17) can also be viewed as an extended equalizing incremental cost rule for the deregulated power systems.

4. Nash equilibrium

In this section, we only analyze the negotiation between a utility and individual IPPs by using Nash equilibrium [5,3]. Therefore, we use the same definition described in Section 3.

IPP-i assumes that the price \( y_i \) is a function of the electric power \( X_i \) selling to the utility. Assuming that IPP-i decides \( x_{ij} \) for \( j = 1, \ldots, n_i \), then the strategy of IPP-i can be formulated as:

\[
\text{IPP - i:} \quad \text{Maximize} \quad y_i (X_i) \sum_{j=1}^{n_i} x_{ij} \]

\[- \sum_{j=1}^{n_i} \int_{0}^{x_{ij}} f_y(x) \, dx \]

On the other hand, the utility assumes that the purchased electric power \( x_{ij} \) from IPP-i is a function of \( y_i \).

Then the strategy of the utility can be described as:

\[
\text{Utility:} \quad \text{Maximize} \quad h(d) - \sum_{j=1}^{n_i} y_i \sum_{j=1}^{n_i} x_{ij}(y_i) - \sum_{j=1}^{m} \int_{0}^{z_j} g_j(z) \, dz \]

s.t. \[ \sum_{j=1}^{n_i} \int_{0}^{x_{ij}} f_y(x) \, dx + \sum_{j=1}^{n_i} x_{ij}(y_i) = d \]

where the utility decides \( Y \) and \( Z \).

Supports \( \hat{c}y_i / \hat{c}x_{ij} \approx 1/dx_{ij}/dy_i \), we have the following Nash equilibrium solution [5].

\[
y_i = \frac{1}{\sum_{j=1}^{n_i} x_{ij}} \int_{0}^{x_{ij}} f_y(x) \, dx + \frac{\sum_{j=1}^{n_i} i x_{ij}}{\sum_{j=1}^{n_i} x_{ij}} y_i(0) \quad (i = 1, \ldots, n) \]

\[ \lambda = g_j(z_j) \quad (j = 1, \ldots, m) \]

\[ \lambda - y_i = \frac{\sum_{j=1}^{n_i} x_{ij}}{\sum_{j=1}^{n_i} x_{ij}/f_{ij} - y_i} \quad (i = 1, \ldots, n) \]

\[ \sum_{i=1}^{n} \sum_{j=1}^{n_i} x_{ij} + \sum_{j=1}^{n_i} z_j = d \]

where \( y_i(0) \) is the electricity price when \( X_i = 0 \).

The equations above are obviously different from the Stackelberg strategy shown in Eqs. (14)–(17). For the Stackelberg strategy, as indicated in Eq. (14), each generator in IPPs operates at equalizing incremental cost, which is also taken as the purchased price, while the purchased prices in the Nash game are determined by not only the incremental costs but also outputs of generators as shown in Eq. (18). Besides, comparing Eq. (16) with Eq. (20), the discrepancies of the incremental costs between the utility and IPPs are also different for the Stackelberg strategy and the Nash game. Although this section only considers the negotiation between the utility and individual IPPs, the Nash equilibrium solution can be derived as the same way for the negotiation between the utility and a coalition of IPPs by assuming \( \hat{c}y_i / \hat{c}x_{ij} \approx 1/dx_{ij}/dy_i \).

5. Consideration of transmission network

This section only deals with the negotiation between the utility and individual IPPs. Assume that only the utility owns transmission network. Let \( H(Z, X_1, \ldots, X_n) \) be a transmission loss function (MWH), then we have the following noncooperative game [3,15]:

\[
\text{IPP - i:} \quad \text{Maximize} \quad y_i \sum_{j=1}^{n_i} x_{ij} - \sum_{j=1}^{n_i} \int_{0}^{x_{ij}} f_y(x) \, dx \]

\[ \text{Utility:} \quad \text{Maximize} \quad h(d) - \sum_{i=1}^{n} y_i \sum_{j=1}^{n_i} x_{ij}(y_i) - \sum_{j=1}^{m} \int_{0}^{z_j} g_j(z) \, dz \]
Table 1
Coefficients of generators for 8-generator power system

<table>
<thead>
<tr>
<th>Generator number</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>0.002</td>
<td>11</td>
<td>105</td>
<td>660</td>
</tr>
<tr>
<td>Unit 2</td>
<td>0.002</td>
<td>11</td>
<td>105</td>
<td>660</td>
</tr>
<tr>
<td>Unit 3</td>
<td>0.0019</td>
<td>10</td>
<td>100</td>
<td>960</td>
</tr>
<tr>
<td>Unit 4</td>
<td>0.0019</td>
<td>10</td>
<td>100</td>
<td>960</td>
</tr>
<tr>
<td>IPP</td>
<td>0.0026</td>
<td>9</td>
<td>100</td>
<td>435</td>
</tr>
<tr>
<td>IPP</td>
<td>0.0027</td>
<td>9</td>
<td>100</td>
<td>420</td>
</tr>
<tr>
<td>IPP</td>
<td>0.0028</td>
<td>10</td>
<td>100</td>
<td>310</td>
</tr>
<tr>
<td>IPP</td>
<td>0.0029</td>
<td>10</td>
<td>100</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 2
Coefficients of generators for 40-generator power system

<table>
<thead>
<tr>
<th>Generator number</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>0.002</td>
<td>11</td>
<td>105</td>
<td>325</td>
</tr>
<tr>
<td>Unit 6 ~ 15</td>
<td>0.0019</td>
<td>10</td>
<td>100</td>
<td>610</td>
</tr>
<tr>
<td>Unit 16 ~ 20</td>
<td>0.0018</td>
<td>10</td>
<td>100</td>
<td>360</td>
</tr>
<tr>
<td>Unit 21 ~ 25</td>
<td>0.0029</td>
<td>9</td>
<td>100</td>
<td>635</td>
</tr>
<tr>
<td>Unit 26 ~ 30</td>
<td>0.003</td>
<td>9</td>
<td>100</td>
<td>885</td>
</tr>
<tr>
<td>IPP</td>
<td>0.0026</td>
<td>9</td>
<td>100</td>
<td>235</td>
</tr>
<tr>
<td>IPP</td>
<td>0.0027</td>
<td>9</td>
<td>100</td>
<td>370</td>
</tr>
<tr>
<td>IPP6 ~ 10</td>
<td>0.0028</td>
<td>9</td>
<td>100</td>
<td>500</td>
</tr>
</tbody>
</table>

6. Socially optimal behavior

Next, we will show that the conventional equalizing incremental cost rule optimizes the social welfare and is also a Pareto solution. Consider a power industry society including a utility and several IPPs. Then the problem to maximize the welfare of the society can be formulated as follows [2]:

Society: Maximize

$$\sum_{x_1, x_2, \ldots, x_n} h(d) - \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij}$$

s.t.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} - H = d$$

where \( H = H(z, X_1(Y), \ldots, X_n(Y)) \). Then in the same way as the derivations of the Section 3, we have a noncooperative solution, which can be considered as an extended equalizing incremental cost rule

$$y_i f_j(x_j) \quad (i = 1, \ldots, n; j = 1, \ldots, n_i)$$

$$\hat{\lambda} = \frac{g_j(z_k)}{1 - \frac{\partial H}{\partial z_k}} \quad (j = 1, \ldots, m)$$

$$\hat{\lambda} \left[ 1 - \frac{\sum_{i=1}^{n_i} \frac{\partial H}{\partial x_{ij}}}{\frac{1}{\partial y_i} f_j(x_j) \frac{\partial x_{ij}}{\partial y_i}} \right] - y_i = \frac{\sum_{i=1}^{n_i} x_{ij}}{\sum_{i=1}^{n_i} 1}$$

Eq. (26) is the equalizing incremental cost with the consideration of the transmission loss for all of the generators in the society, which means that the best solution to the economic dispatch problem is the equalizing incremental cost if there is no competition but cooperation.

7. Numerical simulation

7.1. An 8-generator power system

A power system with 8 thermal generators shown in Table 1 is used to illustrate the application, where the utility owns four thermal generators and four IPPs have one generator, respectively. This paper assumes that all of cost function for generators, i.e., \( \int_{a}^{b} g_j(z) \partial z = a_i z^2 + b_i z + c_i \) or \( \int_{a}^{b} f_j(x) \partial x = a_i x^2 + b_i x + c_i \), has the quadratic forms, as shown in Tables 1 and 2.
Table 3
Simulation results for 8-generator power system

<table>
<thead>
<tr>
<th>Demand (MW)</th>
<th>Coalition</th>
<th>Individual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output (MW)</td>
<td>Cost ($ per h)</td>
</tr>
<tr>
<td>Utility</td>
<td>3206.5</td>
<td>38 918</td>
</tr>
<tr>
<td>IPP-1</td>
<td>434.1</td>
<td>4496.9</td>
</tr>
<tr>
<td>IPP-2</td>
<td>418</td>
<td>4334</td>
</tr>
<tr>
<td>IPP-3</td>
<td>224.5</td>
<td>2486.5</td>
</tr>
<tr>
<td>IPP-4</td>
<td>216.8</td>
<td>2404.2</td>
</tr>
<tr>
<td>Total</td>
<td>4500</td>
<td>52 639.6</td>
</tr>
</tbody>
</table>

*IC*: incremental cost.

Table 4
Simulation results for 40-generator power system

<table>
<thead>
<tr>
<th>Demand (MW)</th>
<th>Coalition</th>
<th>Individual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output (MW)</td>
<td>Cost ($ per h)</td>
</tr>
<tr>
<td>Utility</td>
<td>1601.8</td>
<td>19 120.5</td>
</tr>
<tr>
<td>Unit 1~5</td>
<td>6003.3</td>
<td>67 885.4</td>
</tr>
<tr>
<td>Unit 16~20</td>
<td>1779.8</td>
<td>21 272.5</td>
</tr>
<tr>
<td>Unit 21~25</td>
<td>3155.2</td>
<td>34 073.5</td>
</tr>
<tr>
<td>Unit 26~30</td>
<td>4401.2</td>
<td>42 780.5</td>
</tr>
<tr>
<td>IPP</td>
<td>1~2</td>
<td>460.8</td>
</tr>
<tr>
<td>IPP 3~5</td>
<td>1099.1</td>
<td>10210</td>
</tr>
<tr>
<td>IPP 6~10</td>
<td>2498.5</td>
<td>21 834.5</td>
</tr>
<tr>
<td>Total</td>
<td>21 000</td>
<td>221 850.3</td>
</tr>
</tbody>
</table>

*IC*: incremental cost.

Simulation results are summarized in Table 3. According to Table 3, the electricity prices of IPPs are lower than the operation cost of the utility because the utility has the obligation to serve all customers load. While the total outputs of generators for IPPs and the utility are the same for both coalition and individual. Actually, it can be proven that total outputs of IPPs are the same for the negotiations of Sections 2 and 3 as far as the cost functions $f_i(z)$ and $g_j(z)$ are quadratic. For the sake of the simplification, let $h(d) = \pi d$ where $\pi = 80$ ($\$/MWH$) is the electricity rate to consumers.

Since the $df_i/dx_j$ IPP-1 are smaller than those of IPP-2, IPP-3, IPP-4, the IPP-1 supplies more power to the utility, especially for the case of the ‘individual’ according to Table 3. The results also show that the total benefit for the case of the coalition is larger than the case of the individual due to their jointly bargaining with the utility.

7.2. A 40-generator power system

The proposed method also applied to a 40-generator power system with one utility and 10 IPPs. The utility has 30 generators and 10 IPPs own one generator, respectively. Table 4 shows the simulation results where $h(d) = 80d$ is used. In addition to the price $y$, obviously the earning of the utility also depends on the $h(d)$ to consumers. According to Table 4, the total outputs of generators for IPPs and the utility are the same for both coalition and individual. The incremental costs of the utility (12.8 and 12.32$ per h) are higher than the incremental costs of IPPs (10.2 and 10.66$ per h). As shown in Table 4, total benefit for the case of the individual is actually more than that for the case of the coalition, but the benefit of each is not all increased due to their different incremental costs.

8. Conclusions

Game theory is a useful tool for the analysis of electric power market. The derived operation rules reflecting the competition can be viewed as an extension of the conventional equalizing incremental cost method. Besides, we also provided the prices of electricity for purchases and sales, and as indicated in this paper, these prices are generally cheaper than the incremental
costs of the generators belonging to the utility, when presuming that the purchased electric energy is a function of the prices. To examine the proposed approach, two test systems were used as the demonstrated examples. As further research work, the pricing structures and the wheeling for power systems with a variety of practical constraints will be investigated by the game theory.

References