

### A Couple Less-obvious Chapter 7 questions

1) A ski lift is designed with a load limit of 18,000 pounds. It claims to have a capacity of 100 people. Suppose the weights of all the people using the life have a mean of 175 pounds and a standard deviation of 30 pounds. What is the probability that a random group of 100 people will total more than the load limit of 18,000 pounds?

Ans: Let  $X_1$  represent the weight of the first person on the ski lift,  $X_2$  the weight of the second person, etc. The question is

$$P\left(\sum_{i=1}^{100} X_i > 18000\right) = ?$$

$$P\left(\frac{\sum_{i=1}^{100} X_i}{100} > \frac{18000}{100}\right) = ?$$

$$P(\bar{X} > 180) = ?$$

$$E(\bar{X}) = \mu = 175$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{30}{10} = 3$$

$$z = \frac{180 - 175}{3} = 1.67 \quad P(z > 1.67) = .0475$$

2) The managers of Mercury Mufflers find that the time  $t$  (in minutes) required for a worker to replace a muffler varies. Over a period of a year, they collected the following data:

$t$	Relative Frequency
20	10%
30	50%
40	30%
50	10%

a) Calculate the mean and standard deviation of  $t$ .

$$\mu = \sum_x x f(x) = (20).1 + (30).5 + (40).3 + (50).1 = 34$$

$$\sigma = \sqrt{(x - \mu)^2 f(x)}$$

$$= \sqrt{(20 - 34)^2 .1 + (30 - 34)^2 .5 + (40 - 34)^2 .3 + (50 - 34)^2 .1}$$

$$= \sqrt{64} = 8$$

b) They plan to do 50 mufflers with 4 men in a day and hope to finish them all between 9 a.m. and 5 p.m. What proportion of the days will they fail to make it on time?

Ans: Total available man-minutes are given by  $4 \times 8 \times 60 = 1920$ .

If  $X_1$  represents the time it takes to repair the first muffler,  $X_2$  the time it takes to repair the second muffler, etc., the question is:

$$P\left(\sum_{i=1}^{50} X_i > 1920\right) = ?$$

$$P\left(\frac{\sum_{i=1}^{50} X_i}{50} > \frac{1920}{50}\right) = ?$$

$$P(\bar{X} > 38.4) = ?$$

$$E(\bar{X}) = \mu = 34$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{50}} = 1.13$$

$$z = \frac{38.4 - 34}{1.13} = 3.89 \quad P(z > 3.89) \approx 0$$

c) What crucial assumption did you implicitly make? Suggest some circumstances where it would be seriously violated. Then how would the correct answer be different?

Ans: The implicit assumption is that the muffler repair times are independent random variables. This might be a correct assumption if the reason repair times are variable is because of the differences in muffler and car designs. However, repair times may be variable because of some *common cause*, such as an attribute of one of the mechanics. For example, if Frank comes in drunk twice a month, and on those days it takes him 50 minutes to install a muffler because of his impairment, then muffler repair times are not independent random variables. Knowing it took Frank 50 minutes to install the first muffler guarantees he is drunk and that it will take him 50 minutes to install the next muffler, too. If repair times are not independent, they are most likely positively correlated, which means the standard deviation of  $\bar{X}$  will be bigger, and the chance of not finishing all 50 mufflers might be non-negligible.