

Calculating the probability of winning the Virginia **Lotto**

The Virginia **Lotto** © requires you to pick 6 numbers (without replacement, i.e. no duplicates) between 1 and 44. To win a prize you must match either 3, 4, 5 or 6 of the randomly selected numbers. This is a hypergeometric distribution.

The chance of getting exactly 3 numbers correct is:

$$\frac{\binom{6}{3}\binom{38}{3}}{\binom{44}{6}} = \frac{\frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \times \frac{38 \cdot 37 \cdot 36}{3 \cdot 2 \cdot 1}}{\frac{44 \cdot 43 \cdot 42 \cdot 41 \cdot 40 \cdot 39}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{20 \times 8436}{7059052} = .023901$$

Thus the chance of winning a free ticket is 1 in $(.023901)^{-1}$ or 43.839.

The chance of getting exactly 4 numbers correct is:

$$\frac{\binom{6}{4}\binom{38}{2}}{\binom{44}{6}} = \frac{\frac{6 \cdot 5}{2 \cdot 1} \times \frac{38 \cdot 37}{2 \cdot 1}}{\frac{44 \cdot 43 \cdot 42 \cdot 41 \cdot 40 \cdot 39}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{15 \times 703}{7059052} = .001493826$$

Thus the chance of winning the \$50 prize is 1 in $(.001493826)^{-1} = 669.421$

The chance of getting exactly 5 numbers correct is:

$$\frac{\binom{6}{5}\binom{38}{1}}{\binom{44}{6}} = \frac{6 \times 38}{\frac{44 \cdot 43 \cdot 42 \cdot 41 \cdot 40 \cdot 39}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{228}{7059052} = .00003229895$$

so the chance of winning \$1000 is 1 in 30960.754.

The chance of getting all 6 numbers correct is:

$$\frac{\binom{6}{6}\binom{38}{0}}{\binom{44}{6}} = \frac{1}{\frac{44 \cdot 43 \cdot 42 \cdot 41 \cdot 40 \cdot 39}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{1}{7059052} = .00000014166208$$

so the chance of winning the Jackpot is trivially one in 7059052.

To put this in perspective, the odds that an automobile trip will end in death is about one in four million; therefore, if you jump in the car to drive to the Seven-Eleven to buy a lottery ticket, the chance of you dying in a traffic accident on the trip is approximately twice your chance of winning the jackpot.