

Chapter 15

Multiple Regression

Learning Objectives

1. Understand how multiple regression analysis can be used to develop relationships involving one dependent variable and several independent variables.
2. Be able to interpret the coefficients in a multiple regression analysis.
3. Know the assumptions necessary to conduct statistical tests involving the hypothesized regression model.
4. Understand the role of computer packages in performing multiple regression analysis.
5. Be able to interpret and use computer output to develop the estimated regression equation.
6. Be able to determine how good a fit is provided by the estimated regression equation.
7. Be able to test for the significance of the regression equation.
8. Understand how multicollinearity affects multiple regression analysis.
9. Know how residual analysis can be used to make a judgement as to the appropriateness of the model, identify outliers, and determine which observations are influential.
10. Understand how logistic regression is used for regression analyses involving a binary dependent variable.

Solutions:

1. a. $b_1 = .5906$ is an estimate of the change in y corresponding to a 1 unit change in x_1 when x_2 is held constant.

$b_2 = .4980$ is an estimate of the change in y corresponding to a 1 unit change in x_2 when x_1 is held constant.

2. a. The estimated regression equation is

$$\hat{y} = 45.06 + 1.94x_1$$

An estimate of y when $x_1 = 45$ is

$$\hat{y} = 45.06 + 1.94(45) = 132.36$$

- b. The estimated regression equation is

$$\hat{y} = 85.22 + 4.32x_2$$

An estimate of y when $x_2 = 15$ is

$$\hat{y} = 85.22 + 4.32(15) = 150.02$$

- c. The estimated regression equation is

$$\hat{y} = -18.37 + 2.01x_1 + 4.74x_2$$

An estimate of y when $x_1 = 45$ and $x_2 = 15$ is

$$\hat{y} = -18.37 + 2.01(45) + 4.74(15) = 143.18$$

3. a. $b_1 = 3.8$ is an estimate of the change in y corresponding to a 1 unit change in x_1 when x_2 , x_3 , and x_4 are held constant.

$b_2 = -2.3$ is an estimate of the change in y corresponding to a 1 unit change in x_2 when x_1 , x_3 , and x_4 are held constant.

$b_3 = 7.6$ is an estimate of the change in y corresponding to a 1 unit change in x_3 when x_1 , x_2 , and x_4 are held constant.

$b_4 = 2.7$ is an estimate of the change in y corresponding to a 1 unit change in x_4 when x_1 , x_2 , and x_3 are held constant.

4. a. $\hat{y} = 25 + 10(15) + 8(10) = 255$; sales estimate: \$255,000

- b. Sales can be expected to increase by \$10 for every dollar increase in inventory investment when advertising expenditure is held constant. Sales can be expected to increase by \$8 for every dollar increase in advertising expenditure when inventory investment is held constant.

5. a. The Minitab output is shown below:

The regression equation is
 Revenue = 88.6 + 1.60 TVAdv

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|-------|-------|
| Constant | 88.638 | 1.582 | 56.02 | 0.000 |
| TVAdv | 1.6039 | 0.4778 | 3.36 | 0.015 |

S = 1.215 R-Sq = 65.3% R-Sq(adj) = 59.5%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 1 | 16.640 | 16.640 | 11.27 | 0.015 |
| Residual Error | 6 | 8.860 | 1.477 | | |
| Total | 7 | 25.500 | | | |

- b. The Minitab output is shown below:

The regression equation is
 Revenue = 83.2 + 2.29 TVAdv + 1.30 NewsAdv

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|-------|-------|
| Constant | 83.230 | 1.574 | 52.88 | 0.000 |
| TVAdv | 2.2902 | 0.3041 | 7.53 | 0.001 |
| NewsAdv | 1.3010 | 0.3207 | 4.06 | 0.010 |

S = 0.6426 R-Sq = 91.9% R-Sq(adj) = 88.7%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 2 | 23.435 | 11.718 | 28.38 | 0.002 |
| Residual Error | 5 | 2.065 | 0.413 | | |
| Total | 7 | 25.500 | | | |

| Source | DF | Seq SS |
|---------|----|--------|
| TVAdv | 1 | 16.640 |
| NewsAdv | 1 | 6.795 |

- c. No, it is 1.60 in part (a) and 2.29 above. In part (b) it represents the marginal change in revenue due to an increase in television advertising with newspaper advertising held constant.

- d. Revenue = 83.2 + 2.29(3.5) + 1.30(1.8) = \$93.56 or \$93,560

6. a. The Minitab output is shown below:

The regression equation is
 Speed = 49.8 + 0.0151 Weight

| Predictor | Coef | SE Coef | T | P |
|-----------|----------|----------|------|-------|
| Constant | 49.78 | 19.11 | 2.61 | 0.021 |
| Weight | 0.015104 | 0.006005 | 2.52 | 0.025 |

S = 7.000 R-Sq = 31.1% R-Sq(adj) = 26.2%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|------|-------|
| Regression | 1 | 309.95 | 309.95 | 6.33 | 0.025 |
| Residual Error | 14 | 686.00 | 49.00 | | |
| Total | 15 | 995.95 | | | |

- b. The Minitab output is shown below:

The regression equation is

$$\text{Speed} = 80.5 - 0.00312 \text{ Weight} + 0.105 \text{ Horsepwr}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|-----------|----------|-------|-------|
| Constant | 80.487 | 9.139 | 8.81 | 0.000 |
| Weight | -0.003122 | 0.003481 | -0.90 | 0.386 |
| Horsepwr | 0.10471 | 0.01331 | 7.86 | 0.000 |

$$S = 3.027 \quad R\text{-Sq} = 88.0\% \quad R\text{-Sq}(\text{adj}) = 86.2\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 2 | 876.80 | 438.40 | 47.83 | 0.000 |
| Residual Error | 13 | 119.15 | 9.17 | | |
| Total | 15 | 995.95 | | | |

- c. $\hat{y} = 80.5 - 0.00312(2910) + 0.105(296) = 102.5$

7. a. The Minitab output is shown below:

The regression equation is

$$\text{Price} = 356 - 0.0987 \text{ Capacity} + 123 \text{ Comfort}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|----------|---------|-------|-------|
| Constant | 356.1 | 197.2 | 1.81 | 0.114 |
| Capacity | -0.09874 | 0.04588 | -2.15 | 0.068 |
| Comfort | 122.87 | 21.80 | 5.64 | 0.001 |

$$S = 51.14 \quad R\text{-Sq} = 83.2\% \quad R\text{-Sq}(\text{adj}) = 78.4\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|-------|-------|-------|
| Regression | 2 | 90548 | 45274 | 17.31 | 0.002 |
| Residual Error | 7 | 18304 | 2615 | | |
| Total | 9 | 108852 | | | |

- b. $b_1 = -.0987$ is an estimate of the change in the price with respect to a 1 cubic inch change in capacity with the comfort rating held constant. $b_2 = 123$ is an estimate of the change in the price with respect to a 1 unit change in the comfort rating with the capacity held constant.

- c. $\hat{y} = 356 - .0987(4500) + 123(4) = 404$

8. a. The Minitab output is shown below:

The regression equation is

$$\text{Return} = 247 - 32.8 \text{ Safety} + 34.6 \text{ ExpRatio}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|-------|-------|
| Constant | 247.4 | 110.4 | 2.24 | 0.039 |
| Safety | -32.84 | 13.95 | -2.35 | 0.031 |
| ExpRatio | 34.59 | 14.13 | 2.45 | 0.026 |

$$S = 16.98 \quad R\text{-Sq} = 58.2\% \quad R\text{-Sq}(\text{adj}) = 53.3\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 2 | 6823.2 | 3411.6 | 11.84 | 0.001 |
| Residual Error | 17 | 4899.7 | 288.2 | | |
| Total | 19 | 11723.0 | | | |

b. $\hat{y} = 247 - 32.8(7.5) + 34.6(2) = 70.2$

9. a. The Minitab output is shown below:

The regression equation is

$$\% \text{College} = 26.7 - 1.43 \text{ Size} + 0.0757 \text{ SatScore}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 26.71 | 51.67 | 0.52 | 0.613 |
| Size | -1.4298 | 0.9931 | -1.44 | 0.170 |
| SatScore | 0.07574 | 0.03906 | 1.94 | 0.072 |

$$S = 12.42 \quad R\text{-Sq} = 38.2\% \quad R\text{-Sq}(\text{adj}) = 30.0\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|-------|------|-------|
| Regression | 2 | 1430.4 | 715.2 | 4.64 | 0.027 |
| Residual Error | 15 | 2312.7 | 154.2 | | |
| Total | 17 | 3743.1 | | | |

b. $\hat{y} = 26.7 - 1.43(20) + 0.0757(1000) = 73.8$

Estimate is 73.8%

10. a. The Minitab output is shown below:

The regression equation is

$$\text{Revenue} = 33.3 + 7.98 \text{ Cars}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|-------|-------|
| Constant | 33.34 | 83.08 | 0.40 | 0.695 |
| Cars | 7.9840 | 0.6323 | 12.63 | 0.000 |

$$S = 226.7 \quad R\text{-Sq} = 92.5\% \quad R\text{-Sq}(\text{adj}) = 91.9\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|---------|--------|-------|
| Regression | 1 | 8192067 | 8192067 | 159.44 | 0.000 |
| Residual Error | 13 | 667936 | 51380 | | |
| Total | 14 | 8860003 | | | |

- b. An increase of 1000 cars in service will result in an increase in revenue of \$7.98 million.
- c. The Minitab output is shown below:

The regression equation is

$$\text{Revenue} = 106 + 8.94 \text{ Cars} - 0.191 \text{ Location}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 105.97 | 85.52 | 1.24 | 0.239 |
| Cars | 8.9427 | 0.7746 | 11.55 | 0.000 |
| Location | -0.1914 | 0.1026 | -1.87 | 0.087 |

$$S = 207.7 \quad R\text{-Sq} = 94.2\% \quad R\text{-Sq}(\text{adj}) = 93.2\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|---------|-------|-------|
| Regression | 2 | 8342186 | 4171093 | 96.66 | 0.000 |
| Residual Error | 12 | 517817 | 43151 | | |
| Total | 14 | 8860003 | | | |

11. a. $SSE = SST - SSR = 6,724.125 - 6,216.375 = 507.75$

b. $R^2 = \frac{SSR}{SST} = \frac{6,216.375}{6,724.125} = .924$

c. $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = 1 - (1 - .924) \frac{10-1}{10-2-1} = .902$

- d. The estimated regression equation provided an excellent fit.

12. a. $R^2 = \frac{SSR}{SST} = \frac{14,052.2}{15,182.9} = .926$

b. $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = 1 - (1 - .926) \frac{10-1}{10-2-1} = .905$

- c. Yes; after adjusting for the number of independent variables in the model, we see that 90.5% of the variability in y has been accounted for.

13. a. $R^2 = \frac{SSR}{SST} = \frac{1760}{1805} = .975$
- b. $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = 1 - (1 - .975) \frac{30-1}{30-4-1} = .971$
- c. The estimated regression equation provided an excellent fit.
14. a. $R^2 = \frac{SSR}{SST} = \frac{12,000}{16,000} = .75$
- b. $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = 1 - .25 \frac{9}{7} = .68$
- c. The adjusted coefficient of determination shows that 68% of the variability has been explained by the two independent variables; thus, we conclude that the model does not explain a large amount of variability.
15. a. $R^2 = \frac{SSR}{SST} = \frac{23.435}{25.5} = .919$
- $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = 1 - (1 - .919) \frac{8-1}{8-2-1} = .887$
- b. Multiple regression analysis is preferred since both R^2 and R_a^2 show an increased percentage of the variability of y explained when both independent variables are used.
16. Note: the Minitab output is shown with the solution to Exercise 6.
- a. No; R-Sq = 31.1%
- b. Multiple regression analysis is preferred since both R-Sq and R-Sq(adj) show an increased percentage of the variability of y explained when both independent variables are used.
17. a. $R^2 = \frac{SSR}{SST} = \frac{1430.4}{3743.1} = .382$
- $R_a^2 = 1 - (1 - R^2) \frac{n-1}{n-p-1} = 1 - (1 - .382) \frac{18-1}{18-2-1} = .30$
- b. The fit is not very good
18. Note: The Minitab output is shown with the solution to Exercise 10.
- a. R-Sq = 94.2% R-Sq(adj) = 93.2%
- b. The fit is very good.

19. a. $MSR = SSR/p = 6,216.375/2 = 3,108.188$

$$MSE = \frac{SSE}{n-p-1} = \frac{507.75}{10-2-1} = 72.536$$

b. $F = MSR/MSE = 3,108.188/72.536 = 42.85$

Using F table (2 degrees of freedom numerator and 7 denominator), p -value is less than .01

Because p -value $\leq \alpha = .05$, the overall model is significant.

c. $t = .5906/.0813 = 7.26$

Using t table (7 degrees of freedom), area in tail is less than .005; p -value is less than .01

Because p -value $\leq \alpha$, β_1 is significant.

d. $t = .4980/.0567 = 8.78$

Using t table (7 degrees of freedom), area in tail is less than .005; p -value is less than .01

Because p -value $\leq \alpha$, β_2 is significant.

20. A portion of the Minitab output is shown below.

The regression equation is
 $Y = -18.4 + 2.01 X1 + 4.74 X2$

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|-------|-------|
| Constant | -18.37 | 17.97 | -1.02 | 0.341 |
| X1 | 2.0102 | 0.2471 | 8.13 | 0.000 |
| X2 | 4.7378 | 0.9484 | 5.00 | 0.002 |

S = 12.71 R-Sq = 92.6% R-Sq(adj) = 90.4%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 2 | 14052.2 | 7026.1 | 43.50 | 0.000 |
| Residual Error | 7 | 1130.7 | 161.5 | | |
| Total | 9 | 15182.9 | | | |

- a. Since the p -value corresponding to $F = 43.50$ is $.000 < \alpha = .05$, we reject $H_0: \beta_1 = \beta_2 = 0$; there is a significant relationship.
- b. Since the p -value corresponding to $t = 8.13$ is $.000 < \alpha = .05$, we reject $H_0: \beta_1 = 0$; β_1 is significant.
- c. Since the p -value corresponding to $t = 5.00$ is $.002 < \alpha = .05$, we reject $H_0: \beta_2 = 0$; β_2 is significant.

21. a. In the two independent variable case the coefficient of x_1 represents the expected change in y corresponding to a one unit increase in x_1 when x_2 is held constant. In the single independent variable case the coefficient of x_1 represents the expected change in y corresponding to a one unit increase in x_1 .

- b. Yes. If x_1 and x_2 are correlated one would expect a change in x_1 to be accompanied by a change in x_2 .
22. a. $SSE = SST - SSR = 16000 - 12000 = 4000$
- $$s^2 = \frac{SSE}{n - p - 1} = \frac{4000}{7} = 571.43$$
- $$MSR = \frac{SSR}{p} = \frac{12000}{2} = 6000$$
- b. $F = MSR/MSE = 6000/571.43 = 10.50$
- Using F table (2 degrees of freedom numerator and 7 denominator), p -value is less than .01
- Because p -value $\leq \alpha$, we reject H_0 . There is a significant relationship among the variables.
23. a. $F = 28.38$
- Using F table (2 degrees of freedom numerator and 7 denominator), p -value is less than .01
- Actual p -value = .002
- Because p -value $\leq \alpha$, there is a significant relationship.
- b. $t = 7.53$
- Using t table (7 degrees of freedom), area in tail is less than .005; p -value is less than .01
- Actual p -value = .001
- Because p -value $\leq \alpha$, β_1 is significant and x_1 should not be dropped from the model.
- c. $t = 4.06$
- Actual p -value = .010
- Because p -value $\leq \alpha$, β_2 is significant and x_2 should not be dropped from the model.
24. Note: The Minitab output is shown in part (b) of Exercise 6
- a. $F = 47.83$
- p -value = .000
- Because p -value $\leq \alpha$, we can reject H_0 .

b. For Weight:

$$H_0: \beta_1 = 0 \quad H_a: \beta_1 \neq 0$$

$$p\text{-value} = .386$$

Because $p\text{-value} > \alpha$, we cannot reject H_0

For Horsepower:

$$H_0: \beta_2 = 0 \quad H_a: \beta_2 \neq 0$$

$$p\text{-value} = .000$$

Because $p\text{-value} \leq \alpha$, we can reject H_0

25. a. The Minitab output is shown below:

The regression equation is
Rating = 0.345 + 0.255 TradeEx + 0.132 Use + 0.459 Range

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|------|-------|
| Constant | 0.3451 | 0.5307 | 0.65 | 0.540 |
| TradeEx | 0.25482 | 0.08556 | 2.98 | 0.025 |
| Use | 0.1325 | 0.1404 | 0.94 | 0.382 |
| Range | 0.4585 | 0.1232 | 3.72 | 0.010 |

S = 0.2431 R-Sq = 88.6% R-Sq(adj) = 82.8%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|---------|-------|-------|
| Regression | 3 | 2.74541 | 0.91514 | 15.49 | 0.003 |
| Residual Error | 6 | 0.35459 | 0.05910 | | |
| Total | 9 | 3.10000 | | | |

b. Because the $p\text{-value} = .003 < \alpha = .05$, there is a significant relationship.

c. For TradeEx: Because the $p\text{-value} = .025 < \alpha = .05$, TradeEx is significant.

For Use: Because the $p\text{-value} = .382 > \alpha = .05$, Use is not significant.

For Range: Because the $p\text{-value} = .010 < \alpha = .05$, Range is significant.

The Minitab output after removing Use is shown below:

The regression equation is
Rating = 0.672 + 0.264 TradeEx + 0.485 Range

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|------|-------|
| Constant | 0.6718 | 0.3989 | 1.68 | 0.136 |
| TradeEx | 0.26406 | 0.08432 | 3.13 | 0.017 |
| Range | 0.4853 | 0.1189 | 4.08 | 0.005 |

S = 0.2412 R-Sq = 86.9% R-Sq(adj) = 83.1%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 2 | 2.6928 | 1.3464 | 23.15 | 0.001 |
| Residual Error | 7 | 0.4072 | 0.0582 | | |
| Total | 9 | 3.1000 | | | |

The coefficient of determination for the estimated regression equation developed in part (a) is .886. After the removal of Use, the coefficient of determination is .869. There is very little difference in the fit provided by the two estimated regression equations. But, because Use is not significant, this result is as expected.

26. Note: The Minitab output is shown with the solution to Exercise 10.
- Since the p -value corresponding to $F = 96.66$ is $0.000 < \alpha = .05$, there is a significant relationship among the variables.
 - For Cars: Since the p -value = $0.000 < \alpha = 0.05$, Cars is significant
 - For Location: Since the p -value = $0.087 > \alpha = 0.05$, Location is not significant
27. a. $\hat{y} = 29.1270 + .5906(180) + .4980(310) = 289.8150$
- The point estimate for an individual value is $\hat{y} = 289.8150$, the same as the point estimate of the mean value.
28. a. Using Minitab, the 95% confidence interval is 132.16 to 154.16.
- Using Minitab, the 95% prediction interval is 111.13 to 175.18.
29. a. $\hat{y} = 83.2 + 2.29(3.5) + 1.30(1.8) = 93.555$ or \$93,555

Note: In Exercise 5b, the Minitab output also shows that $b_0 = 83.230$, $b_1 = 2.2902$, and $b_2 = 1.3010$; hence, $\hat{y} = 83.230 + 2.2902x_1 + 1.3010x_2$. Using this estimated regression equation, we obtain

$$\hat{y} = 83.230 + 2.2902(3.5) + 1.3010(1.8) = 93.588 \text{ or } \$93,588$$

The difference ($\$93,588 - \$93,555 = \$33$) is simply due to the fact that additional significant digits are used in the computations. From a practical point of view, however, the difference is not enough to be concerned about. In practice, a computer software package is always used to perform the computations and this will not be an issue.

The Minitab output is shown below:

| Fit | Stdev.Fit | 95% C.I. | 95% P.I. |
|--------|-----------|--------------------|--------------------|
| 93.588 | 0.291 | (92.840, 94.335) | (91.774, 95.401) |

Note that the value of FIT (\hat{y}) is 93.588.

- Confidence interval estimate: 92.840 to 94.335 or \$92,840 to \$94,335

- c. Prediction interval estimate: 91.774 to 95.401 or \$91,774 to \$95,401
30. a. Since weight is not statistically significant (see Exercise 24), we will use an estimated regression equation which uses only Horsepower to predict the speed at 1/4 mile. The Minitab output is shown below:

The regression equation is

$$\text{Speed} = 72.6 + 0.0968 \text{ Horsepwr}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|----------|----------|-------|-------|
| Constant | 72.650 | 2.655 | 27.36 | 0.000 |
| Horsepwr | 0.096756 | 0.009865 | 9.81 | 0.000 |

$$S = 3.006 \quad R\text{-Sq} = 87.3\% \quad R\text{-Sq}(\text{adj}) = 86.4\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 1 | 869.43 | 869.43 | 96.21 | 0.000 |
| Residual Error | 14 | 126.52 | 9.04 | | |
| Total | 15 | 995.95 | | | |

Unusual Observations

| Obs | Horsepwr | Speed | Fit | SE Fit | Residual | St Resid |
|-----|----------|---------|---------|--------|----------|----------|
| 2 | 290 | 108.000 | 100.709 | 0.814 | 7.291 | 2.52R |
| 6 | 450 | 116.200 | 116.190 | 2.036 | 0.010 | 0.00 X |

R denotes an observation with a large standardized residual

X denotes an observation whose X value gives it large influence.

The output shows that the point estimate is a speed of 101.290 miles per hour.

- b. The 95% confidence interval is 99.490 to 103.089 miles per hour.
- c. The 95% prediction interval is 94.596 to 107.984 miles per hour.
31. a. Using Minitab the 95% confidence interval is 58.37% to 75.03%.
- b. Using Minitab the 95% prediction interval is 35.24% to 90.59%.
32. a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ where
 $x_2 = 0$ if level 1 and 1 if level 2
- b. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(0) = \beta_0 + \beta_1 x_1$
- c. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(1) = \beta_0 + \beta_1 x_1 + \beta_2$
- d. $\beta_2 = E(y \mid \text{level 2}) - E(y \mid \text{level 1})$
- β_1 is the change in $E(y)$ for a 1 unit change in x_1 holding x_2 constant.

33. a. two
- b. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ where

| x_2 | x_3 | Level |
|-------|-------|-------|
| 0 | 0 | 1 |
| 1 | 0 | 2 |
| 0 | 1 | 3 |

- c. $E(y | \text{level 1}) = \beta_0 + \beta_1 x_1 + \beta_2(0) + \beta_3(0) = \beta_0 + \beta_1 x_1$
- $E(y | \text{level 2}) = \beta_0 + \beta_1 x_1 + \beta_2(1) + \beta_3(0) = \beta_0 + \beta_1 x_1 + \beta_2$
- $E(y | \text{level 3}) = \beta_0 + \beta_1 x_1 + \beta_2(0) + \beta_3(1) = \beta_0 + \beta_1 x_1 + \beta_3$
- $\beta_2 = E(y | \text{level 2}) - E(y | \text{level 1})$
- $\beta_3 = E(y | \text{level 3}) - E(y | \text{level 1})$
- β_1 is the change in $E(y)$ for a 1 unit change in x_1 holding x_2 and x_3 constant.

34. a. \$15,300
- b. Estimate of sales = $10.1 - 4.2(2) + 6.8(8) + 15.3(0) = 56.1$ or \$56,100
- c. Estimate of sales = $10.1 - 4.2(1) + 6.8(3) + 15.3(1) = 41.6$ or \$41,600
35. a. Let Type = 0 if a mechanical repair
Type = 1 if an electrical repair

The Minitab output is shown below:

The regression equation is
Time = 3.45 + 0.617 Type

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|------|-------|
| Constant | 3.4500 | 0.5467 | 6.31 | 0.000 |
| Type | 0.6167 | 0.7058 | 0.87 | 0.408 |

S = 1.093 R-Sq = 8.7% R-Sq(adj) = 0.0%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|-------|------|-------|
| Regression | 1 | 0.913 | 0.913 | 0.76 | 0.408 |
| Residual Error | 8 | 9.563 | 1.195 | | |
| Total | 9 | 10.476 | | | |

- b. The estimated regression equation did not provide a good fit. In fact, the p -value of .408 shows that the relationship is not significant for any reasonable value of α .
- c. Person = 0 if Bob Jones performed the service and Person = 1 if Dave Newton performed the service. The Minitab output is shown below:

The regression equation is
 Time = 4.62 - 1.60 Person

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 4.6200 | 0.3192 | 14.47 | 0.000 |
| Person | -1.6000 | 0.4514 | -3.54 | 0.008 |

S = 0.7138 R-Sq = 61.1% R-Sq(adj) = 56.2%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 1 | 6.4000 | 6.4000 | 12.56 | 0.008 |
| Residual Error | 8 | 4.0760 | 0.5095 | | |
| Total | 9 | 10.4760 | | | |

- d. We see that 61.1% of the variability in repair time has been explained by the repair person that performed the service; an acceptable, but not good, fit.

36. a. The Minitab output is shown below:

The regression equation is
 Time = 1.86 + 0.291 Months + 1.10 Type - 0.609 Person

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 1.8602 | 0.7286 | 2.55 | 0.043 |
| Months | 0.29144 | 0.08360 | 3.49 | 0.013 |
| Type | 1.1024 | 0.3033 | 3.63 | 0.011 |
| Person | -0.6091 | 0.3879 | -1.57 | 0.167 |

S = 0.4174 R-Sq = 90.0% R-Sq(adj) = 85.0%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 3 | 9.4305 | 3.1435 | 18.04 | 0.002 |
| Residual Error | 6 | 1.0455 | 0.1743 | | |
| Total | 9 | 10.4760 | | | |

- b. Since the p -value corresponding to $F = 18.04$ is $.002 < \alpha = .05$, the overall model is statistically significant.
- c. The p -value corresponding to $t = -1.57$ is $.167 > \alpha = .05$; thus, the addition of Person is not statistically significant. Person is highly correlated with Months (the sample correlation coefficient is $-.691$); thus, once the effect of Months has been accounted for, Person will not add much to the model.

37. a. Let Position = 0 if a guard
 Position = 1 if an offensive tackle.

- b. The Minitab output is shown below:

The regression equation is
 Rating = 11.2 + 0.732 Position + 0.0222 Weight - 2.28 Speed

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 11.223 | 4.523 | 2.48 | 0.022 |
| Position | 0.7324 | 0.2893 | 2.53 | 0.019 |
| Weight | 0.02219 | 0.01039 | 2.14 | 0.045 |
| Speed | -2.2775 | 0.9290 | -2.45 | 0.023 |

S = 0.6936 R-Sq = 47.5% R-Sq(adj) = 40.1%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|------|-------|
| Regression | 3 | 9.1562 | 3.0521 | 6.35 | 0.003 |
| Residual Error | 21 | 10.1014 | 0.4810 | | |
| Total | 24 | 19.2576 | | | |

- c. Since the p -value corresponding to $F = 6.35$ is $.003 < \alpha = .05$, there is a significant relationship between rating and the independent variables.
- d. The value of R-Sq (adj) is 40.1%; the estimated regression equation did not provide a very good fit.
- e. Since the p -value for Position is $t = 2.53 < \alpha = .05$, position is a significant factor in the player's rating.
- f. $\hat{y} = 11.2 + .732(1) + .0222(300) - 2.28(5.1) = 6.96$

38. a. The Minitab output is shown below:

The regression equation is
 Risk = - 91.8 + 1.08 Age + 0.252 Pressure + 8.74 Smoker

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | -91.76 | 15.22 | -6.03 | 0.000 |
| Age | 1.0767 | 0.1660 | 6.49 | 0.000 |
| Pressure | 0.25181 | 0.04523 | 5.57 | 0.000 |
| Smoker | 8.740 | 3.001 | 2.91 | 0.010 |

S = 5.757 R-Sq = 87.3% R-Sq(adj) = 85.0%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 3 | 3660.7 | 1220.2 | 36.82 | 0.000 |
| Residual Error | 16 | 530.2 | 33.1 | | |
| Total | 19 | 4190.9 | | | |

- b. Since the p -value corresponding to $t = 2.91$ is $.010 < \alpha = .05$, smoking is a significant factor.
- c. Using Minitab, the point estimate is 34.27; the 95% prediction interval is 21.35 to 47.18. Thus, the probability of a stroke (.2135 to .4718 at the 95% confidence level) appears to be quite high. The physician would probably recommend that Art quit smoking and begin some type of treatment designed to reduce his blood pressure.

39. a. The Minitab output is shown below:

The regression equation is
 $Y = 0.20 + 2.60 X$

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|------|-------|
| Constant | 0.200 | 2.132 | 0.09 | 0.931 |
| X | 2.6000 | 0.6429 | 4.04 | 0.027 |

S = 2.033 R-Sq = 84.5% R-Sq(adj) = 79.3%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 1 | 67.600 | 67.600 | 16.35 | 0.027 |
| Residual Error | 3 | 12.400 | 4.133 | | |
| Total | 4 | 80.000 | | | |

b. Using Minitab we obtained the following values:

| x_i | y_i | \hat{y}_i | Standardized Residual |
|-------|-------|-------------|-----------------------|
| 1 | 3 | 2.8 | .16 |
| 2 | 7 | 5.4 | .94 |
| 3 | 5 | 8.0 | -1.65 |
| 4 | 11 | 10.6 | .24 |
| 5 | 14 | 13.2 | .62 |

The point (3,5) does not appear to follow the trend of remaining data; however, the value of the standardized residual for this point, -1.65, is not large enough for us to conclude that (3, 5) is an outlier.

c. Using Minitab, we obtained the following values:

| x_i | y_i | Studentized Deleted Residual |
|-------|-------|------------------------------|
| 1 | 3 | .13 |
| 2 | 7 | .91 |
| 3 | 5 | -4.42 |
| 4 | 11 | .19 |
| 5 | 14 | .54 |

$t_{0.025} = 4.303$ ($n - p - 2 = 5 - 1 - 2 = 2$ degrees of freedom)

Since the studentized deleted residual for (3, 5) is $-4.42 < -4.303$, we conclude that the 3rd observation is an outlier.

40. a. The Minitab output is shown below:

The regression equation is
 $Y = -53.3 + 3.11 X$

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | -53.280 | 5.786 | -9.21 | 0.003 |
| X | 3.1100 | 0.2016 | 15.43 | 0.001 |

S = 2.851 R-sq = 98.8% R-sq (adj) = 98.3%

Analysis of Variance

| SOURCE | DF | SS | MS | F | P |
|----------------|----|--------|--------|--------|-------|
| Regression | 1 | 1934.4 | 1934.4 | 238.03 | 0.001 |
| Residual Error | 3 | 24.4 | 8.1 | | |
| Total | 4 | 1598.8 | | | |

- b. Using the Minitab we obtained the following values:

| x_i | y_i | Studentized Deleted Residual |
|-------|-------|------------------------------|
| 22 | 12 | -1.94 |
| 24 | 21 | -.12 |
| 26 | 31 | 1.79 |
| 28 | 35 | .40 |
| 40 | 70 | -1.90 |

$t_{0.025} = 4.303$ ($n - p - 2 = 5 - 1 - 2 = 2$ degrees of freedom)

Since none of the studentized deleted residuals are less than -4.303 or greater than 4.303, none of the observations can be classified as an outlier.

- c. Using Minitab we obtained the following values:

| x_i | y_i | h_i |
|-------|-------|-------|
| 22 | 12 | .38 |
| 24 | 21 | .28 |
| 26 | 31 | .22 |
| 28 | 35 | .20 |
| 40 | 70 | .92 |

The critical value is

$$\frac{3(p+1)}{n} = \frac{3(1+1)}{5} = 1.2$$

Since none of the values exceed 1.2, we conclude that there are no influential observations in the data.

- d. Using Minitab we obtained the following values:

| x_i | y_i | D_i |
|-------|-------|-------|
| 22 | 12 | .60 |
| 24 | 21 | .00 |
| 26 | 31 | .26 |
| 28 | 35 | .03 |
| 40 | 70 | 11.09 |

Since $D_5 = 11.09 > 1$ (rule of thumb critical value), we conclude that the fifth observation is influential.

41. a. The Minitab output appears in the solution to part (b) of Exercise 5; the estimated regression equation is:

$$\text{Revenue} = 83.2 + 2.29 \text{ TVAdv} + 1.30 \text{ NewsAdv}$$

- b. Using Minitab we obtained the following values:

| \hat{y}_i | Standardized Residual |
|-------------|-----------------------|
| 96.63 | -1.62 |
| 90.41 | -1.08 |
| 94.34 | 1.22 |
| 92.21 | -.37 |
| 94.39 | 1.10 |
| 94.24 | -.40 |
| 94.42 | -1.12 |
| 93.35 | 1.08 |

With the relatively few observations, it is difficult to determine if any of the assumptions regarding the error term have been violated. For instance, an argument could be made that there does not appear to be any pattern in the plot; alternatively an argument could be made that there is a curvilinear pattern in the plot.

- c. The values of the standardized residuals are greater than -2 and less than +2; thus, using test, there are no outliers. As a further check for outliers, we used Minitab to compute the following studentized deleted residuals:

| Observation | Studentized Deleted Residual |
|-------------|------------------------------|
| 1 | -2.11 |
| 2 | -1.10 |
| 3 | 1.31 |
| 4 | -.33 |
| 5 | 1.13 |
| 6 | -.36 |
| 7 | -1.16 |
| 8 | 1.10 |

$$t_{.025} = 2.776 \quad (n - p - 2 = 8 - 2 - 2 = 4 \text{ degrees of freedom})$$

Since none of the studentized deleted residuals is less than -2.776 or greater than 2.776, we conclude that there are no outliers in the data.

- d. Using Minitab we obtained the following values:

| Observation | h_i | D_i |
|-------------|-------|-------|
| 1 | .63 | 1.52 |
| 2 | .65 | .70 |
| 3 | .30 | .22 |
| 4 | .23 | .01 |
| 5 | .26 | .14 |
| 6 | .14 | .01 |
| 7 | .66 | .81 |
| 8 | .13 | .06 |

The critical average value is

$$\frac{3(p+1)}{n} = \frac{3(2+1)}{8} = 1.125$$

Since none of the values exceed 1.125, we conclude that there are no influential observations. However, using Cook's distance measure, we see that $D_1 > 1$ (rule of thumb critical value); thus, we conclude the first observation is influential. Final Conclusion: observations 1 is an influential observation.

42. a. The Minitab output is shown below:

The regression equation is

$$\text{Speed} = 71.3 + 0.107 \text{ Price} + 0.0845 \text{ Horsepwr}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|----------|----------|-------|-------|
| Constant | 71.328 | 2.248 | 31.73 | 0.000 |
| Price | 0.10719 | 0.03918 | 2.74 | 0.017 |
| Horsepwr | 0.084496 | 0.009306 | 9.08 | 0.000 |

$$S = 2.485 \quad R\text{-Sq} = 91.9\% \quad R\text{-Sq}(\text{adj}) = 90.7\%$$

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 2 | 915.66 | 457.83 | 74.12 | 0.000 |
| Residual Error | 13 | 80.30 | 6.18 | | |
| Total | 15 | 995.95 | | | |

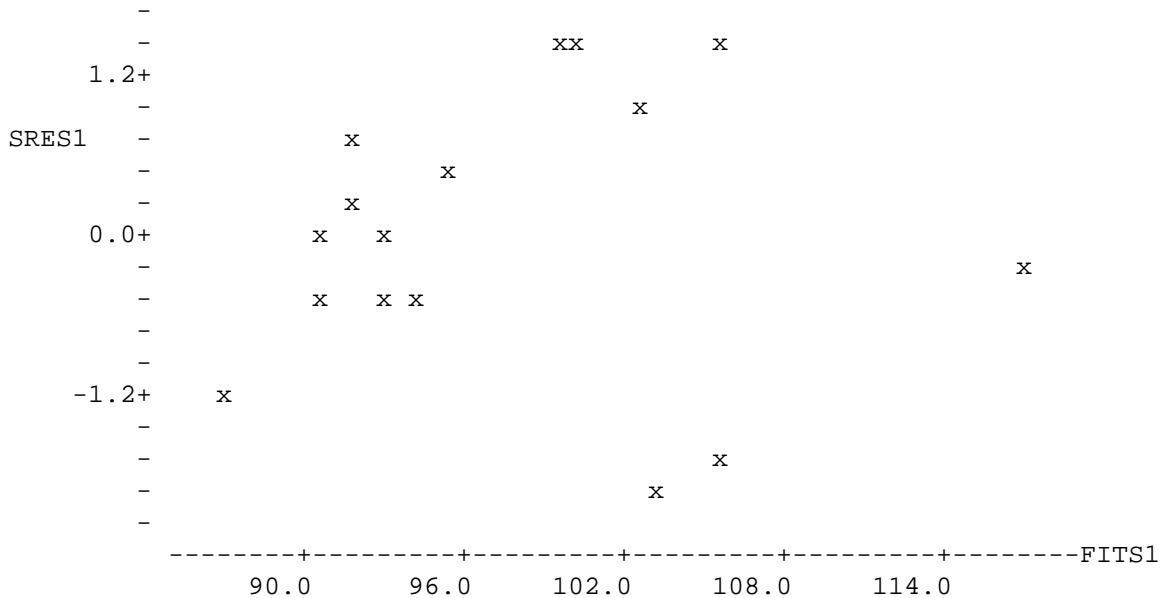
| Source | DF | Seq SS |
|----------|----|--------|
| Price | 1 | 406.39 |
| Horsepwr | 1 | 509.27 |

Unusual Observations

| Obs | Price | Speed | Fit | SE Fit | Residual | St Resid |
|-----|-------|---------|---------|--------|----------|----------|
| 2 | 93.8 | 108.000 | 105.882 | 2.007 | 2.118 | 1.45 X |

X denotes an observation whose X value gives it large influence.

- b. The standardized residual plot is shown below. There appears to be a very unusual trend in the standardized residuals.



- c. The Minitab output shown in part (a) did not identify any observations with a large standardized residual; thus, there does not appear to be any outliers in the data.
- d. The Minitab output shown in part (a) identifies observation 2 as an influential observation.

43. a. The Minitab output is shown below:

The regression equation is

$$\%College = -26.6 + 0.0970 \text{ SatScore}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | -26.61 | 37.22 | -0.72 | 0.485 |
| SatScore | 0.09703 | 0.03734 | 2.60 | 0.019 |

S = 12.83 R-Sq = 29.7% R-Sq(adj) = 25.3%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|------|-------|
| Regression | 1 | 1110.8 | 1110.8 | 6.75 | 0.019 |
| Residual Error | 16 | 2632.3 | 164.5 | | |
| Total | 17 | 3743.1 | | | |

Unusual Observations

| Obs | SatScore | %College | Fit | SE Fit | Residual | St Resid |
|-----|----------|----------|-------|--------|----------|----------|
| 3 | 716 | 40.00 | 42.86 | 10.79 | -2.86 | -0.41 X |

X denotes an observation whose X value gives it large influence.

- b. The Minitab output shown in part a identifies observation 3 as an influential observation.
- c. The Minitab output appears in the solution to Exercise 9; the estimates regression equation is

$$\%College = 26.7 - 1.43 \text{ Size} + 0.0757 \text{ SATScore}$$

- d. The following Minitab output was also provided as part of the regression output for part c.

Unusual Observations

| Obs. | Size | %College | Fit | Stdev.Fit | Residual | St.Resid |
|------|------|----------|-------|-----------|----------|----------|
| 3 | 30.0 | 40.0 | 38.04 | 10.97 | 1.96 | 0.34 X |

X denotes an obs. whose X value gives it large influence.

Observation 3 is still identified as an influential observation.

44. a.
$$E(y) = \frac{e^{\beta_0 + \beta x}}{1 + e^{\beta_0 + \beta x}}$$

- b. It is an estimate of the probability that a customer that does not have a Simmons credit card will make a purchase.
- c. A portion of the Minitab binary logistic regression output follows:

Logistic Regression Table

| Predictor | Coef | SE Coef | Z | P | Odds Ratio | 95% CI | |
|-----------|---------|---------|-------|-------|------------|--------|-------|
| | | | | | | Lower | Upper |
| Constant | -0.9445 | 0.3150 | -3.00 | 0.003 | | | |
| Card | 1.0245 | 0.4235 | 2.42 | 0.016 | 2.79 | 1.21 | 6.39 |

Log-Likelihood = -64.265

Test that all slopes are zero: G = 6.072, DF = 1, P-Value = 0.014

Thus, the estimated logit is $\hat{g}(x) = -0.9445 + 1.0245x$

- d. For customers that do not have a Simmons credit card ($x = 0$)

$$\hat{g}(0) = -0.9445 + 1.0245(0) = -0.9445$$

and

$$\hat{y} = \frac{e^{\hat{g}(0)}}{1 + e^{\hat{g}(0)}} = \frac{e^{-0.9445}}{1 + e^{-0.9445}} = \frac{0.3889}{1 + 0.3889} = 0.28$$

For customers that have a Simmons credit card ($x = 1$)

$$\hat{g}(1) = -0.9445 + 1.0245(1) = 0.0800$$

and

$$\hat{y} = \frac{e^{\hat{g}(1)}}{1 + e^{\hat{g}(1)}} = \frac{e^{0.08}}{1 + e^{0.08}} = \frac{1.0833}{1 + 1.0833} = 0.52$$

- e. Using the Minitab output shown in part (c), the estimated odds ratio is 2.79. We can conclude that the estimated odds of making a purchase for customers who have a Simmons credit card are 2.79 times greater than the estimated odds of making a purchase for customers that do not have a Simmons credit card.

45. a. $\text{odds} = \frac{.3413}{1 - .3413} = .4584$

b. $\text{odds}_1 = \frac{.5790}{1 - .5790} = 1.3753$

$\text{odds}_0 = .4584$ (from part (a))

$$\text{odds ratio} = \frac{\text{odds}_1}{\text{odds}_0} = \frac{1.3753}{.4584} = 3.00$$

- c. The odds ratio for x_2 computed holding annual spending constant at \$2000 is also 3.00. This shows that the odds ratio for x_2 is independent of the value of x_1 .

46. a. $E(y) = \frac{e^{\beta_0 + \beta x}}{1 + e^{\beta_0 + \beta x}}$

- b. A portion of the Minitab binary logistic regression output follows:

Logistic Regression Table

| Predictor | Coef | SE Coef | Z | P | Odds Ratio | 95% CI | |
|-----------|---------|---------|-------|-------|------------|--------|-------|
| | | | | | | Lower | Upper |
| Constant | -2.6335 | 0.7985 | -3.30 | 0.001 | | | |
| Balance | 0.22018 | 0.09002 | 2.45 | 0.014 | 1.25 | 1.04 | 1.49 |

Log-Likelihood = -25.813

Test that all slopes are zero: G = 9.460, DF = 1, P-Value = 0.002

Thus, the estimated logistic regression equation is

$$E(y) = \frac{e^{-2.6355 + 0.22018x}}{1 + e^{-2.6355 + 0.22018x}}$$

- c. Significant result: the p -value corresponding to the G test statistic is 0.0002.
- d. For an average monthly balance of \$1000, $x = 10$

$$E(y) = \frac{e^{-2.6355 + 0.22018x}}{1 + e^{-2.6355 + 0.22018x}} = \frac{e^{-2.6355 + 0.22018(10)}}{1 + e^{-2.6355 + 0.22018(10)}} = \frac{e^{-0.4317}}{1 + e^{-0.4317}} = \frac{0.6494}{1.6494} = 0.39$$

Thus, an estimate of the probability that customers with an average monthly balance of \$1000 will sign up for direct payroll deposit is 0.39.

- e. Repeating the calculations in part (d) using various values for x , a value of $x = 12$ or an average monthly balance of approximately \$1200 is required to achieve this level of probability.
- f. Using the Minitab output shown in part (b), the estimated odds ratio is 1.25. Because values of x are measured in hundreds of dollars, the estimated odds of signing up for payroll direct deposit for customers that have an average monthly balance of \$600 is 1.25 times greater than the estimated odds of signing up for payroll direct deposit for customers that have an average monthly balance of \$500. Moreover, this interpretation is true for any one hundred dollar increment in the average monthly balance.

47. a.
$$E(y) = \frac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2}}{1 + e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2}}$$

- b. For a given GPA, it is an estimate of the probability that a student who did not attend the orientation program will return to Lakeland for the sophomore year.
- c. A portion of the Minitab binary logistic regression output follows:

Logistic Regression Table

| Predictor | Coef | SE Coef | Z | P | Odds Ratio | 95% CI | |
|-----------|--------|---------|-------|-------|------------|--------|-------|
| | | | | | | Lower | Upper |
| Constant | -6.893 | 1.747 | -3.94 | 0.000 | | | |
| GPA | 2.5388 | 0.6729 | 3.77 | 0.000 | 12.66 | 3.39 | 47.35 |
| Program | 1.5608 | 0.5631 | 2.77 | 0.006 | 4.76 | 1.58 | 14.36 |

Log-Likelihood = -40.169

Test that all slopes are zero: G = 47.869, DF = 2, P-Value = 0.000

Thus, the estimated logit is $\hat{g}(x_1, x_2) = -6.893 + 2.5388x_1 + 1.5608x_2$

- d. Significant result: the p -value corresponding to the G test statistic is 0.0000.
- e. Both variables are significant at $\alpha = .01$: the p -value for x_1 is 0.000 and the p -value for x_2 is 0.006
- f. For $x_1 = 2.5$ and $x_2 = 0$

$$\hat{g}(2.5, 0) = -6.893 + 2.5388(2.5) + 1.5608(0) = -0.5460$$

and

$$\hat{y} = \frac{e^{\hat{g}(2.5,0)}}{1 + e^{\hat{g}(2.5,0)}} = \frac{e^{-0.5460}}{1 + e^{-0.5460}} = \frac{0.5793}{1 + 0.5793} = 0.37$$

For $x_1 = 2.5$ and $x_2 = 1$

$$\hat{g}(2.5, 1) = -6.893 + 2.5388(2.5) + 1.5608(1) = 1.0148$$

and

$$\hat{y} = \frac{e^{\hat{g}(2.5,1)}}{1 + e^{\hat{g}(2.5,1)}} = \frac{e^{1.0148}}{1 + e^{1.0148}} = \frac{2.7588}{1 + 2.7588} = 0.73$$

- g. From the Minitab output in part (c) we see that the estimated odds ratio is 4.76 for the orientation program. This means that the odds of students who attended the orientation program continuing are 4.76 times greater than for students who did not attend the program.
- h. We recommend making the orientation program required. From part (e), we see that the odds of continuing are much higher for students who have attended the orientation program.

48. a.
$$E(y) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

- b. A portion of the Minitab binary logistic regression output follows:

Logistic Regression Table

| Predictor | Coef | SE Coef | Z | P | Odds Ratio | 95% CI | |
|-----------|--------|---------|-------|-------|------------|--------|-------|
| | | | | | | Lower | Upper |
| Constant | -2.805 | 1.432 | -1.96 | 0.050 | | | |
| Price | 1.1492 | 0.5143 | 2.23 | 0.025 | 3.16 | 1.15 | 8.65 |

Log-Likelihood = -8.200

Test that all slopes are zero: G = 9.465, DF = 1, P-Value = 0.002

Thus, the estimated logit is $\hat{g}(x) = -2.805 + 1.1492x$

- c. For chocolates that have a price per serving of \$4.00

$$\hat{g}(4) = -2.805 + 1.1492(4) = 1.7918$$

and

$$\hat{y} = \frac{e^{\hat{g}(4)}}{1 + e^{\hat{g}(4)}} = \frac{e^{1.7918}}{1 + e^{1.7918}} = \frac{6.0002}{1 + 6.0002} = 0.86$$

- d. Using the Minitab output shown in part (b), the estimated odds ratio is 3.16. We can conclude that the estimated odds of having a quality rating of very good or excellent for a chocolate that has a price of \$4.00 per serving is 3.16 times greater than the estimated odds for a chocolate with a price of \$3.00 per serving. Moreover, this interpretation is true for any one dollar difference in the price per serving.

49. a. The expected increase in final college grade point average corresponding to a one point increase in high school grade point average is .0235 when SAT mathematics score does not change. Similarly, the expected increase in final college grade point average corresponding to a one point increase in the SAT mathematics score is .00486 when the high school grade point average does not change.

b.
$$\hat{y} = -1.41 + .0235(84) + .00486(540) = 3.19$$

50. a. Job satisfaction can be expected to decrease by 8.69 units with a one unit increase in length of service if the wage rate does not change. A dollar increase in the wage rate is associated with a 13.5 point increase in the job satisfaction score when the length of service does not change.

b.
$$\hat{y} = 14.4 - 8.69(4) + 13.5(6.5) = 67.39$$

51. a. The computer output with the missing values filled in is as follows:

The regression equation is

$$Y = 8.103 + 7.602 X_1 + 3.111 X_2$$

| Predictor | Coef | SE Coef | T |
|-----------|-------|---------|------|
| Constant | 8.103 | 2.667 | 3.04 |
| X1 | 7.602 | 2.105 | 3.61 |
| X2 | 3.111 | 0.613 | 5.08 |

S = 3.35 R-sq = 92.3% R-sq (adj) = 91.0%

Analysis of Variance

| SOURCE | DF | SS | MS | F |
|----------------|----|---------|---------|-------|
| Regression | 2 | 1612 | 806 | 71.82 |
| Residual Error | 12 | 134.67 | 11.2225 | |
| Total | 14 | 1746.67 | | |

- b. Using t table (12 degrees of freedom), area in tail corresponding to $t = 3.61$ is less than .005; p -value is less than .01

Because $p\text{-value} \leq \alpha$, reject $H_0 : \beta_1 = 0$

Using t table (12 degrees of freedom), area in tail corresponding to $t = 5.08$ is less than .005; p -value is less than .01

Because $p\text{-value} \leq \alpha$, reject $H_0 : \beta_2 = 0$

- c. See computer output.

d. $R_a^2 = 1 - (1 - .923) \frac{14}{12} = .91$

52. a. The regression equation is

$$Y = -1.41 + .0235 X_1 + .00486 X_2$$

| Predictor | Coef | SE Coef | T |
|-----------|----------|----------|-------|
| Constant | -1.4053 | 0.4848 | -2.90 |
| X1 | 0.023467 | 0.008666 | 2.71 |
| X2 | .00486 | 0.001077 | 4.51 |

S = 0.1298 R-sq = 93.7% R-sq (adj) = 91.9%

Analysis of Variance

| SOURCE | DF | SS | MS | F |
|----------------|----|---------|-------|-------|
| Regression | 2 | 1.76209 | .881 | 52.44 |
| Residual Error | 7 | .1179 | .0168 | |
| Total | 9 | 1.88000 | | |

- b. Using F table (2 degrees of freedom numerator and 7 degrees of freedom denominator), p -value is less than .01

Because p -value $\leq \alpha$, there is a significant relationship.

c.
$$R^2 = \frac{SSR}{SST} = .937$$

$$R_a^2 = 1 - (1 - .937) \frac{9}{7} = .919$$

good fit

d. $t_{.025} = 2.365$ (7 DF)

for β_1 : p -value is between .02 and .05; reject H_0 : $\beta_1 = 0$

for β_2 : p -value is less than .01; reject H_0 : $\beta_2 = 0$

53. a. The regression equation is

$$Y = 14.4 - 8.69 X_1 + 13.52 X_2$$

| Predictor | Coef | SE Coef | T |
|-----------|--------|---------|-------|
| Constant | 14.448 | 8.191 | 1.76 |
| X1 | -8.69 | 1.555 | -5.59 |
| X2 | 13.517 | 2.085 | 6.48 |

$$S = 3.773 \quad R\text{-sq} = 90.1\% \quad R\text{-sq (adj)} = 86.1\%$$

Analysis of Variance

| SOURCE | DF | SS | MS | F |
|----------------|----|--------|---------|-------|
| Regression | 2 | 648.83 | 324.415 | 22.79 |
| Residual Error | 5 | 71.17 | 14.234 | |
| Total | 7 | 720.00 | | |

b. $F_{.05} = 5.79$ (5 DF)

$F = 22.79 > F_{.05}$; significant relationship.

c.
$$R^2 = \frac{SSR}{SST} = .901$$

$$R_a^2 = 1 - (1 - .901) \frac{7}{5} = .861$$

good fit

d. $t_{.025} = 2.571$ (5 DF)

for β_1 : $t = -5.59 < -2.571$; reject $H_0 : \beta_1 = 0$

for β_2 : $t = 6.48 > 2.571$; reject $H_0 : \beta_2 = 0$

54. a. The Minitab output is shown below:

The regression equation is
Score = 50.6 + 1.56 RecRes

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|-------|-------|
| Constant | 50.610 | 2.064 | 24.52 | 0.000 |
| RecRes | 1.5621 | 0.4226 | 3.70 | 0.002 |

S = 6.339 R-Sq = 43.1% R-Sq(adj) = 40.0%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 1 | 548.92 | 548.92 | 13.66 | 0.002 |
| Residual Error | 18 | 723.29 | 40.18 | | |
| Total | 19 | 1272.21 | | | |

b. The fit provided by the estimated regression equation is not that good; only 43.1% of the variability in Score is explained by the RecRes.

c. The Minitab output is shown below:

The regression equation is
Score = 33.5 + 1.90 RecRes + 2.61 Afford

| Predictor | Coef | SE Coef | T | P |
|-----------|--------|---------|-------|-------|
| Constant | 33.485 | 3.228 | 10.37 | 0.000 |
| RecRes | 1.8998 | 0.2603 | 7.30 | 0.000 |
| Afford | 2.6108 | 0.4545 | 5.74 | 0.000 |

S = 3.803 R-Sq = 80.7% R-Sq(adj) = 78.4%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 2 | 1026.29 | 513.14 | 35.47 | 0.000 |
| Residual Error | 17 | 245.92 | 14.47 | | |
| Total | 19 | 1272.21 | | | |

Because the p -value = .000 $< \alpha = .05$, there is a significant overall relationship. Moreover, the fit provided by the two-independent variable estimated regression is much better.

55. a. The Minitab output is shown below:

The regression equation is
 $\text{Speed} = 97.6 + 0.0693 \text{ Price} - 0.00082 \text{ Weight} + 0.0590 \text{ Horsepwr} - 2.48 \text{ Zero60}$

| Predictor | Coef | SE Coef | T | P |
|-----------|-----------|----------|-------|-------|
| Constant | 97.57 | 11.79 | 8.27 | 0.000 |
| Price | 0.06928 | 0.03805 | 1.82 | 0.096 |
| Weight | -0.000816 | 0.002593 | -0.31 | 0.759 |
| Horsepwr | 0.05901 | 0.01543 | 3.82 | 0.003 |
| Zero60 | -2.4836 | 0.9601 | -2.59 | 0.025 |

S = 2.127 R-Sq = 95.0% R-Sq(adj) = 93.2%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 4 | 946.18 | 236.55 | 52.28 | 0.000 |
| Residual Error | 11 | 49.77 | 4.52 | | |
| Total | 15 | 995.95 | | | |

- b. Since the p -value corresponding to the F test is 0.000, the relationship is significant.
- c. Since the p -values corresponding to the t test for both Horsepwr (p -value = .003) and Zero60 (p -value = .025) are less than .05, both of these independent variables are significant.
- d. The Minitab output is shown below:

The regression equation is
 $\text{Speed} = 103 + 0.0558 \text{ Horsepwr} - 3.19 \text{ Zero60}$

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 103.103 | 9.448 | 10.91 | 0.000 |
| Horsepwr | 0.05582 | 0.01452 | 3.84 | 0.002 |
| Zero60 | -3.1876 | 0.9658 | -3.30 | 0.006 |

S = 2.301 R-Sq = 93.1% R-Sq(adj) = 92.0%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|--------|--------|-------|-------|
| Regression | 2 | 927.12 | 463.56 | 87.54 | 0.000 |
| Residual Error | 13 | 68.84 | 5.30 | | |
| Total | 15 | 995.95 | | | |

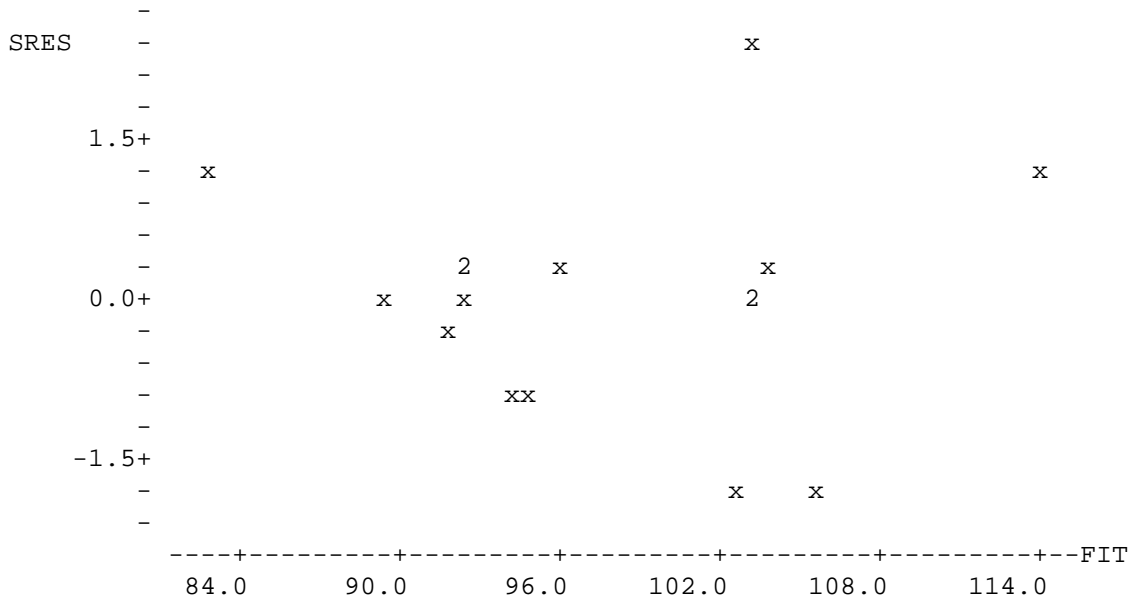
| Source | DF | Seq SS |
|----------|----|--------|
| Horsepwr | 1 | 869.43 |
| Zero60 | 1 | 57.68 |

Unusual Observations

| Obs | Horsepwr | Speed | Fit | SE Fit | Residual | St Resid |
|-----|----------|---------|---------|--------|----------|----------|
| 2 | 290 | 108.000 | 103.352 | 1.015 | 4.648 | 2.25R |
| 12 | 155 | 84.600 | 82.747 | 1.773 | 1.853 | 1.26 X |

R denotes an observation with a large standardized residual
 X denotes an observation whose X value gives it large influence.

e. The standardized residual plot is shown below:



There is an unusual trend in the plot and one observation appears to be an outlier.

- f. The Minitab output indicates that observation 2 is an outlier
- g. The Minitab output indicates that observation 12 is an influential observation.

56. The Minitab output is shown below:

The regression equation is
 CityMPG = 24.1 - 2.10 Displace

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 24.121 | 2.381 | 10.13 | 0.000 |
| Displace | -2.1019 | 0.4850 | -4.33 | 0.000 |

S = 1.446 R-Sq = 36.3% R-Sq(adj) = 34.3%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 1 | 39.276 | 39.276 | 18.78 | 0.000 |
| Residual Error | 33 | 69.009 | 2.091 | | |
| Total | 34 | 108.286 | | | |

Because the p -value = .000 < α = .05, there is a significant relationship.

b. The Minitab output is shown below:

The regression equation is
 CityMPG = 26.4 - 2.44 Displace - 1.20 Drive4

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 26.383 | 2.377 | 11.10 | 0.000 |
| Displace | -2.4387 | 0.4683 | -5.21 | 0.000 |
| Drive4 | -1.2015 | 0.4722 | -2.54 | 0.016 |

S = 1.339 R-Sq = 47.0% R-Sq(adj) = 43.7%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 2 | 50.889 | 25.445 | 14.19 | 0.000 |
| Residual Error | 32 | 57.397 | 1.794 | | |
| Total | 34 | 108.286 | | | |

c. Because the p -value = .016 < α = .05, Drive4 is significant.

d. The Minitab output is shown below:

The regression equation is

$$\text{CityMPG} = 33.3 - 4.15 \text{ Displace} - 1.24 \text{ Drive4} + 2.16 \text{ EightCyl}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|---------|---------|-------|-------|
| Constant | 33.333 | 3.345 | 9.97 | 0.000 |
| Displace | -4.1492 | 0.7584 | -5.47 | 0.000 |
| Drive4 | -1.2368 | 0.4310 | -2.87 | 0.007 |
| EightCyl | 2.1615 | 0.7919 | 2.73 | 0.010 |

S = 1.222 R-Sq = 57.3% R-Sq(adj) = 53.1%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 3 | 62.011 | 20.670 | 13.85 | 0.000 |
| Residual Error | 31 | 46.275 | 1.493 | | |
| Total | 34 | 108.286 | | | |

e. Because the p -value = .000 < α = .05, there is a significant overall relationship.

For Displace: Because the p -value = .000 < α = .05, Displace is significant.

For Drive4: Because the p -value = .007 > α = .05, Drive4 is significant.

For EightCyl: Because the p -value = .010 < α = .05, EightCyl is significant.

57. a. The Minitab output is shown below:

$$\text{Resale\%} = 38.8 + 0.000766 \text{ Price}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|-----------|-----------|------|-------|
| Constant | 38.772 | 4.348 | 8.92 | 0.000 |
| Price | 0.0007656 | 0.0001900 | 4.03 | 0.000 |

S = 5.421 R-Sq = 36.7% R-Sq(adj) = 34.4%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 1 | 477.25 | 477.25 | 16.24 | 0.000 |
| Residual Error | 28 | 822.92 | 29.39 | | |
| Total | 29 | 1300.17 | | | |

Since the p -value corresponding to $F = 16.24$ is .000 < α = .05, there is a significant relationship between Resale% and Price.

b. R-Sq = 36.7%; not a very good fit.

- c. Let Type1 = 0 and Type2 = 0 if a small pickup; Type1 = 1 and Type2 = 0 if a full-size pickup; and Type1 = 0 and Type2 = 1 if a sport utility.

The Minitab output using Type1, Type2, and Price is shown below:

The regression equation is

$$\text{Resale\%} = 42.6 + 9.09 \text{ Type1} + 7.92 \text{ Type2} + 0.000341 \text{ Price}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|-----------|-----------|-------|-------|
| Constant | 42.554 | 3.562 | 11.95 | 0.000 |
| Type1 | 9.090 | 2.248 | 4.04 | 0.000 |
| Type2 | 7.917 | 2.163 | 3.66 | 0.001 |
| Price | 0.0003415 | 0.0001800 | 1.90 | 0.069 |

S = 4.298 R-Sq = 63.1% R-Sq(adj) = 58.8%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|----|---------|--------|-------|-------|
| Regression | 3 | 819.77 | 273.26 | 14.79 | 0.000 |
| Residual Error | 26 | 480.40 | 18.48 | | |
| Total | 29 | 1300.17 | | | |

- d. Since the p -value corresponding to $F = 14.79$ is $.000 < \alpha = .05$, there is a significant relationship between Resale% and the independent variables. Note that individually, Price is not significant at the .05 level of significance. If we rerun the regression using just Type1 and Type2 the value of R-Sq (adj) decreases to 54.4%, a drop of only 4%. Thus, it appears that for these data, the type of vehicle is the strongest predictor of the resale value.