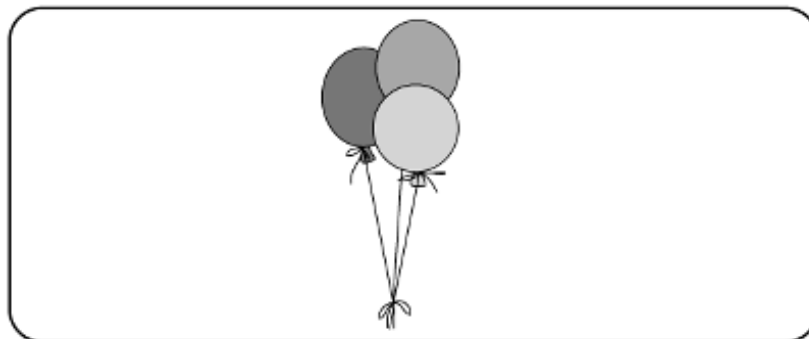


Name _____ Date _____ Partners _____

LAB 12 - THE IDEAL GAS LAW



. . . the hypothesis, that supposes the pressures and expansions to be in reciprocal proportions . . .

–Robert Boyle

OBJECTIVES

- To understand how a gaseous system may be characterized by temperature, pressure, and volume.
- To examine the relationship between any two of these variables when the third is kept constant.
- To understand and be able to use the **Ideal Gas Law**, which describes the relationship between pressure, volume and temperature.
- To use the ideal gas law and helium gas in a constant volume thermometer for a wide range of temperatures to determine the absolute zero of temperature.

OVERVIEW

In introductory physics, we often talk about matter as if it were continuous, but as early as the fifth century B.C., Greek philosophers proposed the idea of “atomism”. They pictured a universe in which everything is made up of tiny “eternal” and “incorruptible” particles, separated by “a void”. Today, we think of these particles as atoms and molecules.

In terms of everyday experience, molecules and atoms are hypothetical entities. In just the past forty years or so, scientists have been able to “see” molecules and atoms using electron microscopes and scanning probe microscopes, but long before atoms and molecules could be “seen”, nineteenth-century scientists used these imaginary, small-scale *microscopic* entities to construct models that account for the large-scale *macroscopic* properties of thermodynamic systems.

Even a small container filled with a gas contains a very large number of molecules (on the order of 600,000,000,000,000,000,000!). Because it is impossible to use Newton’s laws of motion to keep track of what each of these molecules is doing at any moment, we must characterize the behavior of a gas by the *macroscopic* quantities: volume, V ; pressure, p ; and temperature, T . *Kinetic theory* is the area of physics that uses

Newton's laws and averages of molecular behavior to explain the relationship between p , V , and T .

Comment: In SI units, pressure is measured in Pa (N/m^2) and volume is measured in m^3 . Temperature can be measured in $^\circ\text{C}$ (degrees Celsius) or in K (Kelvin).

One of the triumphs of kinetic theory (including an assumption of *non-interacting* gas molecules) is the successful explanation of the well know **Ideal Gas Law**

$$pV = nRT, \quad (1)$$

where p is the gas pressure, V is the volume of the container holding the gas, n is the number of moles¹ of the gas filling the container, R is the **Universal Gas Constant**,

$$R = 8.3145 \text{ J/mol}\cdot\text{K}, \quad (2)$$

and T is the temperature measured on an **absolute scale** (e.g., the **Kelvin scale**).

In Activity 1 you will carry out experiments to relate p and V at constant temperature (*Boyle's Law*). In Activity 2 you will use a constant-volume thermometer and measure pressure for well known temperatures. You will extrapolate these results to zero pressure to determine the absolute zero of temperature.

NOTE: The hot water bath that you will need in Activity 2 needs about half an hour to come to a boil. Before you do anything else, **top it off with water and plug it in**. Make sure to leave enough room in the hot water bath to completely submerge the spherical ball of the gas thermometer without boiling water spilling over the top.

INVESTIGATION 1: BOYLE'S LAW

You will start by examining the relationship between pressure p and volume V by doing measurements on the air in a syringe. Since the syringe is in thermal contact with the surrounding air, if you change the volume of the gas relatively slowly, the gas in the syringe remains in thermal equilibrium with the surroundings. Another way of referring to a process that takes place at constant temperature is to call it *isothermal*.

For isothermal processes, the ideal gas law implies an inverse relationship between volume and pressure:

$$V = (nRT) \frac{1}{p}, \quad (3)$$

This relationship is known as *Boyle's Law*.

To test your prediction you will need

- 20-mL plastic syringe (with the needle removed)
- short piece of Tygon tubing (to attach syringe to pressure sensor)
- pressure sensor

¹ One mole of gas consists of Avogadro's constant ($N_A = 6.022 \times 10^{23}$) molecules.

Activity 1-1: Isothermal Volume Change for a Gas

The approach to obtaining measurements is to trap a volume of air in the syringe and then compress the air slowly to smaller and smaller volumes by pushing in the piston.

The gas should be compressed slowly so it will always have time to come into thermal equilibrium with the room (and thus be at room temperature).

1. Start with the piston at 20 mL. Attach the end of an unsealed syringe to the pressure sensor using the Tygon tubing.
2. Open the experiment file called **L12.1-1 Pressure vs. Volume**.

The software is set up in **prompted event mode** so that you can continuously measure pressure and decide when you want to **keep** a value. Then you can **enter** the corresponding volume. You can see the p , V data in Table 1 and a plot of pressure vs. volume is displayed in Graph 1.



3. **Start** the computer. Click **Keep** to read the pressure. You'll be prompted to enter the volume (it should be 20 ml at this point).
4. Squeeze the piston until you've reduced the volume by 2 ml and, after the pressure reading is stable, take another reading. Repeat this process until you cannot squeeze it any more (you should be below 6 mL or so). It goes rather quickly, but you have to communicate with your partners to indicate that the person holding the piston is ready for the computer person to click on **Keep**.
5. **Print** one graph for your group report.

Question 1-1: Discuss the relationship between p and V .

6. Now open Graph 2, which shows a plot of V vs. $1/p$ which, according to Boyle's law, should be a straight line through the origin. Use the **fit routine** to fit a line and record the parameters.

slope: _____ intercept: _____

7. **Print** one graph for your group report.

Question 1-2: From your data, estimate the volume of gas in the tubing and the sensor.

INVESTIGATION 2: THERMOMETERS AND TEMPERATURE SCALES

It is easy enough to grasp the concept of **temperature** intuitively: something that feels ‘cold’ has a low temperature whereas something that feels ‘hot’ has a high temperature. However, it is not at all trivial to translate such a qualitative understanding into a scientifically sound or even a scientifically practical quantitative definition of temperature.

The most widely used temperature scale today is the **Celsius Scale** (A. Celsius, 1701-1744). One useful representation of the temperature t in degrees Celsius ($^{\circ}\text{C}$) is

$$t = 100 \frac{X_t - X_0}{X_{100} - X_0}, \quad (4)$$

where X is a *property of a material* that varies with temperature. X_0 and X_{100} are, respectively, the values of that property at the temperatures of freezing and boiling water. The temperature of freezing water is defined as 0°C and that of boiling water is defined as 100°C . X_t is the value of the property at the temperature being measured.

The property X can be the length of a metal rod, the electrical resistance of a wire, the pressure of a gas, or one of many other quantities. Each of the resulting ‘thermometers’ would have provided its own scale of temperature which need not have agreed with any other. Of all these thermometers, gas thermometers are unique in providing, under suitable conditions, a scale that is not only (almost) independent of the gas used, but that is also based on well understood physical phenomena and that comes closest to providing an ‘absolute’ temperature scale.

Water is chosen to define the temperature scale because it is widely available and easy to purify, and because its freezing and boiling points straddle the temperature range of greatest importance to human existence. They also mark **phase transitions**. Matter in general, and water in particular, exists in three **phases: solid, liquid and gaseous**.

When heat energy is continuously imparted to a solid, the temperature of that solid will rise to a particular value, the **melting point**, where it remains constant until all the solid has been converted to liquid. Then the temperature will rise again until a second stationary value, the **boiling point**, is reached. Here again the temperature will remain constant as heat energy is added until all the liquid is vaporized. Only then will the temperature of the gas rise with the continued application of heat. The reason for this is that at the phase transitions (melting and boiling points) the added energy serves to change the phase of the material rather than to raise its temperature.

A mixture of two phases of the same material, e.g. ice and water, is thus capable of absorbing heat as well as of giving it off without changing temperature. For this reason, such mixtures are very useful as temperature standards.

There is one caveat: The boiling point of water (indeed of any liquid), and to a much smaller degree its freezing point, are dependent on the ambient pressure which is usually the atmospheric pressure. For the precise calibration of a Celsius thermometer, it is thus necessary to work at one standard atmosphere of pressure (760 Torr)². In our simple experiments, we need not worry too much about the pressure, but you should know that considerable precautions are necessary to obtain precisely reproducible fixed points.

We will use a **constant volume gas thermometer**, which consists of a fixed volume that is filled with a given mass of gas whose pressure p is used as a measure of temperature. In this case (constant volume), the temperature (in °C) is given by

$$t = 100 \frac{p_t - p_0}{p_{100} - p_0}. \quad (5)$$

The resulting Celsius scale is called the **ideal gas Celsius scale**.³ An ideal gas is one in which the interaction between molecules is negligible. This is the case when two conditions are met:

1. The molecules of the gas collide only (relatively) rarely with each other. This happens if the gas is dilute, i.e. if the volume available to the gas molecules is large compared to the volume they would fill if they were tightly packed.
2. During a collision, the molecules just exchange energy and momentum but do not stick to each other. This condition is satisfied by all gases at temperatures well above their condensation point, i.e. the point at which the gas is liquefied.

Helium is an excellent approximation to an ideal gas: it is monatomic, its atoms are small, and its condensation point is the lowest of all known substances. It is for this reason that we shall use helium to fill our **gas thermometer**.

In this activity, we will fill a constant volume with a fixed quantity of gas. The ideal gas law then implies a simple proportionality between temperature and pressure,

$$p = \left(\frac{nR}{V} \right) T, \quad (6)$$

This relationship is known as *Gay-Lussac's Law*.

Kinetic Gas Theory tells us that the molecules of a gas are in constant thermal motion. As the temperature of the gas is lowered, the thermal motion becomes slower and slower. Since a molecule cannot move slower than not at all, there must exist a lowest

²The international unit of pressure is the Pascal (1 Pa = 1 Nm⁻²), but the pressure gauges in this laboratory read in Torr, where 760 Torr = 760 mm Hg = 1 atm = 1.013 × 10⁵ Pa.

³ The zero of the Celsius scale is defined as the temperature at which ice and water coexist at atmospheric pressure. The triple point of water (where all three phases of water – solid, liquid, gas – are in equilibrium) is at 0.01°C.

temperature, an absolute zero of temperature, at which all thermal motion has come to a stop. In an **Absolute Temperature Scale**, this point is chosen as zero.

In an absolute scale, the ratio of two temperatures becomes a meaningful quantity. A temperature, for instance, of 400 K is indeed twice as high as a temperature of 200 K. A similar statement made using Celsius or Fahrenheit temperatures would be quite nonsensical. Liquid nitrogen boils at a very frigid 77 K. It is, nevertheless, almost twenty times ‘hotter’ than liquid helium, which boils at 4.2 K. In the same sense, a temperature of 0.01 K is 100 times colder than one of 1 K and indeed there are physical phenomena that are unnoticeable at, say, 0.1 K but become obvious at 0.001 K. The quest for lower and lower temperatures goes on. According to the **Third Law of Thermodynamics**, it is not possible to ever reach ‘absolute zero’. However, it has been approached to less than 10^{-6} K.

For the **Kelvin Scale** (Lord Kelvin, 1824 - 1907), the increments are taken over from the Celsius scale, i.e. a temperature difference of one Kelvin (notation: 1 K, *not* 1 degree K or °K) is the same as a temperature difference of 1°C.⁴ The Celsius and Kelvin scales (see Figure 1) are related by the following expression:

$$T_{\text{Kelvin}} = t_{\text{Celsius}} + 273.15 \text{ K} \quad (7)$$

The pressure that a gas exerts on the walls of its container is due to the change in momentum of the gas molecules bouncing off of these walls. At the absolute zero of temperature, there is no thermal motion and the pressure of the gas must drop to zero. The ideal gas thermometer will thus register zero at the temperature of 0 K.

The materials you will need for this investigation are

- Safety goggles (one for each student, **WEAR THEM!!!**)
- Gas thermometer
- Hot water pot
- Nalgene container
- Methanol
- Liquid nitrogen
- helium gas
- Kitchen-type thermometer
- Plastic buckets
- Dry ice (frozen carbon dioxide)
- Dewar

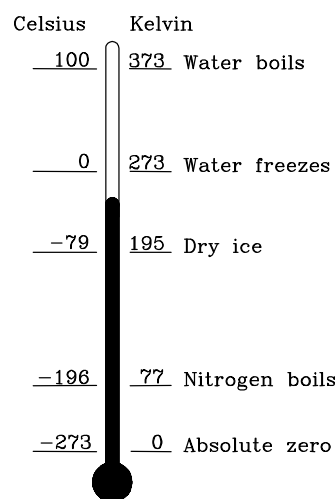


Figure 1

CAUTION: PUT ON SAFETY GOGGLES!!!

Liquid nitrogen and the dry ice - methanol mixture are extremely cold and can damage skin and eyes so *be very careful* while using them. [Boiling water is not particularly good for you either.]

⁴ This account of the definition of the Kelvin scale is somewhat simplified. The actual definition of the scale is based on thermodynamic considerations.

Activity 2-1: Constant volume thermometer

Question 2-1: Discuss why we use helium gas to use for this experiment. Why not just use air, which is composed of mostly nitrogen and oxygen (with a little carbon dioxide and water vapor as well)?

1. Open the experimental file **L12.2-1 Absolute Zero**. Note that you can enter data for temperature and pressure into the table. Measure the room temperature bath and insert that value in the computer and in Table 2-1 (below) now.
2. *Data Studio* is already set up for you to take three sets of data for different initial pressures. Enter your data into the tables in this manual and into *Data Studio*.
3. With the sphere of the gas thermometer *completely* immersed in the ice bath, your instructor will pump it out and then fill it with helium gas to a pressure of about 1,060 Torr at the pump station. This is about 300 Torr above atmospheric pressure (p_{atm}) of 760 Torr. Do not fill the sphere to a pressure greater than 300 Torr above p_{atm} . You can obtain the atmospheric pressure by reading the barometer provided in the lab. **Hold the gas thermometer by the handle while the sphere is in any of the baths!** The sphere tends to float and can readily jump out of the container and break the gauge. Whenever you dip the gas thermometer sphere into any of the temperature baths move it around **gently** to stir the liquid and insure good thermal contact. Tap the gas thermometer gauge with your finger because the needle may stick slightly. Read the pressure carefully. Enter your values into Table 2-1 and into the computer.
4. Next, dip the gas thermometer into the boiling water, make sure the sphere is completely covered, move it about gently until the water boils again vigorously. Read the pressure, tapping the pressure gauge gently to help overcome the friction in the meter movement. Enter your data results into Table 2-1 and into the computer.
5. Measure the temperature of the room temperature bath with the kitchen type thermometer. Run cold water over the bulb until it is near room temperature. Now, dip the gas thermometer into the room temperature bath, move it about and wait for the temperature to equilibrate, and then read the pressure and the bath's temperature (again). [Take care to keep this temperature constant!] Record the bath's temperature in Table 2-1
6. Repeat the procedures for the ice bath and the dry ice and methanol bath.

CAUTION! WIPE UP ANY METHANOL SPILLS IMMEDIATELY!
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7. Dry off the sphere with a paper towel then dip it into the liquid nitrogen, move it about until the nitrogen has stopped boiling vigorously and read the pressure. Rinse the sphere under running water until all the ice has melted.

8. Refer to Appendix A for the temperatures of the baths (you must measure room temperature with a thermometer). The computer will graph the measured pressures against the Celsius scale ranging from -300°C to 100°C . This experiment with initial pressure near 1,060 Torr will be labeled Run 1 in the computer. Do not erase these data.

Table 2-1 Data for initial pressure near 1,060 Torr

Temperature Bath	Temperature ($^{\circ}\text{C}$)	Pressure (Torr)
Ice water (initial fill)		
Boiling water		
Room temperature		
Ice water		
Dry ice/methanol		
LN ₂		

9. You are now ready to work on the next pressure. While one of you is doing that, another student should be preparing a linear fit to the data in Run 1. Expand the axes so you can see the fit to at least -300°C .
10. Your TA may want to do this step for you. Ask him or her to find out. Immerse the bulb in the ice water bath, and after the pressure has become stable open the valve slightly and slowly until the helium begins to escape. Reduce the pressure to about 960 Torr and close the valve. [If you overshoot, go on to the next pressure and return to this pressure later.] Again, tap the gauge with your finger. Write down the exact value of the pressure. This will represent a new value of n for the number of moles of He in the constant volume gas thermometer. Repeat steps 3) through 8) and determine the pressure for all six temperatures again. Enter the data for this initial pressure of 960 Torr into Run 2.

Table 2-2 Data for initial pressure near 960 Torr

Temperature Bath	Temperature ($^{\circ}\text{C}$)	Pressure (Torr)
Ice water (initial fill)		
Boiling water		
Room temperature		
Ice water		
Dry ice/methanol		
LN ₂		

11. Repeat step 10) for an initial pressure of 860 Torr (Run 3) in the ice water bath. Take the data for all six temperatures again. Enter your data in the tables here and in the computer. Perform Linear Fits on all.

Table 2-3 Data for initial pressure near 860 Torr

Temperature Bath	Temperature (°C)	Pressure (Torr)
Ice water (initial fill)		
Boiling water		
Room temperature		
Ice water		
Dry ice/methanol		
LN ₂		

12. Print out one graph with all linear fits and data on it. Include this with your group report.

The three linear fits should intersect (in pairs) somewhere near $p=0$. Usually the intersections are scattered around a bit. They are also usually not quite centered on $p=0$ due to pressure gauge *zero offsets*.

13. Find the three intersections:

1,060 Torr & 960 Torr: p : _____ t : _____

1,060 Torr & 860 Torr: p : _____ t : _____

960 Torr & 860 Torr: p : _____ t : _____

Question 2-2: Use these intersections to estimate the zero offset of your gauge. Show your work and discuss.

$p_{offset} =$ _____ Torr

Question 2-3: Use these intersections to estimate the absolute zero of temperature. Show your work and discuss agreement with the accepted value.

$$t_{abs} = \text{_____} \text{ } ^\circ\text{C}$$

INVESTIGATION 3: MYSTERY GAS

To do this investigation you will need

- Liquid nitrogen in dewar
- Constant volume thermometer with “mystery gas”

Your TA has a special constant volume thermometer just like the one you have been using, but it is filled with a “mystery” gas. Your TA has time may want to go through this investigation with you. If not, please ask the TA for this thermometer and proceed through the following activity.

Activity 3-1: Cooling the Mystery Gas

1. Immerse the sphere into the ice-water bath and record the temperature.

Pressure _____ mm Hg

Question 3-1: What will the pressure gauge read when the sphere is immersed in liquid nitrogen? Show your work.

2. Dry off the sphere and immerse it into the liquid nitrogen dewar and watch the pressure carefully as it decreases. Record the final pressure.

Pressure _____ mm Hg

Question 3-2: Discuss the agreement with your prediction.

Question 3-3: Discuss the physics of your observation. The following questions may be helpful to you in figuring out what you just observed: Did the temperature go to zero? Did the volume become infinite? What is n ?

3. Warm up the metal sphere and clean it off so another group can use it. Place it back in front of the room.

Clean up your lab area!

