An MHD Model for Hot Jupiter Upper Atmospheres: Mass/Angular Momentum Loss & Transits

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Introduction

• **hot Jupiters**
  
  \[ M \sim M_{\text{Jup}}; \quad R \sim R_{\text{Jup}}; \quad D < 0.1 \text{ AU} \]
  
  EUV/X-ray heating \( \rightarrow \) large scale heights
  
  stellar tides important

• **can probe atmospheres with transits:**
  
  \( P = 100 \text{ nbar} - 10 \text{ nbar} \): upper atmosphere (\( 10^4 \) K)
  
  H photoionization in thermosphere (Yelle 2004)
  
  \[ H + \gamma \rightarrow e + p \]
  
  \[ P_{\text{gas}} \sim g m_p/\sigma_{pi} \sim 10^{-9} \text{ bar} \left( \frac{g}{300 \text{ cm s}^{-2}} \right) \]

  \( P = 1 \text{ bar} - 1 \text{ mbar} \): optical/NIR continuum (\( 10^3 \) K)
  
  e.g., H\(_2\), H\(_2\)O, He, CH\(_4\)
  
  GCMs, weather, day/night heat flow
  
  \[ H_2 \rightarrow H + H \]

  Vidal-Madjar et al. (2003)
Escape From Hot Jupiters

- upper atmosphere of HD 209458b
  NaI (Charbonneau et al. 2002)
  HI, OI, CII (e.g., Vidal-Madjar et al. 2003, 2004; Ben-Jaffel 2008)
  HI transit depths $\rightarrow$ absorbing area comparable to Roche Lobe

- atmospheric escape models
  EUV heating to $10^4$ K $\rightarrow$ thermally-driven HD outflow
  e.g., Yelle (2004), Tian et al. (2005), Garcia-Munoz (2007), Murray-Clay et al. (2009)

- what about the planetary magnetic field?
Planetary Magnetic Field

- Is field dynamically important? Structure largely unconstrained by observation magnetically dominated for Jupiter/Saturn strength fields:

\[ P_{\text{gas}} \simeq g m_p / \sigma_{\text{pi}} \simeq 10^{-9} \text{ bar} \left( \frac{g}{300 \text{ cm s}^{-2}} \right) \]
\[ P_{\text{mag}} \simeq B_{\text{Jup}}^2 / 8\pi \simeq 10^{-6} \text{ bar} \]

- Expected field strength field scales with energy flux, not \( \Omega \) Jupiter/low-mass stars in saturated regime large radii \( \Rightarrow \) implies high core flux (Arras & Socrates 2009)

\[ F \propto \rho v^3 \]
\[ \therefore B^2 \sim \rho v^2 \sim \rho^{1/3} F^{2/3} \]
\[ F_{\text{HJ}} \sim 10^3 \sim 10^4 F_J \]

Christensen et al. (2009)
Modeling the Upper Atmosphere

• include planetary magnetic field
  atmospheric structure set by magnetic geometry
  magnetic stress balance with hydrodynamic stresses
  what is the steady-state field structure?
  how does $B \neq 0$ affect mass/angular momentum loss rates?

• include stellar tidal potential
  stellar tide sets Roche distance
  centrifugal terms due to synchronous rotation
  provides additional acceleration for outflow
  how do tides affect mass loss rates?

• how do magnetic fields + tides affect the transit depth in Ly $\alpha$?
Guidance From Rotating, Magnetized Stars

- expected geometry from stellar wind theory
  core dipole field dominates near planet
  currents in magnetosphere comb out field lines in wind zone
  e.g., Mestel (1968), Mestel & Spruit (1987)

Trammell, Arras, & Li (2011)

Pneuman & Kopp et al. (1981)
• “dead zone” structure
  (static) gas confined by magnetic field
cusp radius ($\lambda, \epsilon, \beta$)
dead zone several $R_p$ (can exceed $R_{RL}$)

$$\lambda = \frac{GM_p}{Rc_s^2}$$

$$\epsilon = \left( \frac{\Omega R}{c_s} \right)^2$$

$$\beta = \frac{8\pi P_{gas}}{B^2}$$

$r_d : P_d \simeq \frac{B_w^2}{8\pi}$
Analytic Description of the Magnetosphere

- "wind zone" structure
  magnetic stresses dominate (rigid field line approximation)
  sonic point affected by magnetic geometry:

  \[ r_s \approx \frac{GM_p}{3a^2} = \left( \frac{\lambda}{3} \right) R \]
  \[ \lambda = \frac{GM_p}{Ra^2} \]

  stellar tide + magnetic field sets the sonic point condition:

  \[-a^2 \frac{d \ln B}{ds} = \frac{dU}{ds} \]

  \[ \frac{GM_p}{r^2} + \Omega^2 r = \frac{3a^2}{r} \]

  if minimum potential barrier exceeds thermal pressure, a second "polar dead zone" exists
Analytic Model: Transit Signals in Lyman $\alpha$

\[ N_H(y, z) = \int_{-1.1R_*}^{1.1R_*} dx \ n_H(x, y, z) \]

\[ \tau^{(p)}_\nu(y, z) = \int dx \ n_H(x, y, z) \ \sigma_\nu(x, y, z) \]

\[ T_\nu = \frac{1}{\pi R_*^2} \int dy dz \ e^{-\tau^{(p)}_\nu(y, z)} \]

Trammell, Arras, & Li (2011)
Numerical MHD Model

- library of models in strong field limit with ZEUS-MP code (Hayes et al. 2006)
  evolves fluid quantities on staggered-grid (2D; axisymmetric)

\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v} \quad (1)
\]

\[
\rho \frac{D\mathbf{v}}{Dt} = -\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} - \rho \nabla \mathbf{U} \quad (2)
\]

\[
\rho \frac{D}{Dt} \left( \begin{array}{c}
e \\
\rho \end{array} \right) = -P \nabla \cdot \mathbf{v} \quad (3)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (4)
\]

- hot inner boundary \(\rightarrow\) base of H thermosphere
  (1) density, temperature held fixed at base
  (2) dipole magnetic field anchored on corotating frame
  (3) reflecting B.C.’s for radial velocity (“floating B.C.”)
• example 1D profiles:

(1) density can *increase* outward beyond Roche distance because net gravity points outward

(2) dead zone density can be a factor of 10 higher than in the outflow/wind zone

(3) fluid in equatorial + polar dead zones is *static*
Density Structure in the Magnetosphere

$B_0 = 10 \text{ G}$  $B_0 = 14.1 \text{ G}$  $B_0 = 31.5 \text{ G}$

Trammell, Arras, & Li, in prep.
Velocity Structure in the Magnetosphere

$B_0 = 10 \text{ G}$  $B_0 = 14.1 \text{ G}$  $B_0 = 31.5 \text{ G}$

Trammell, Arras, & Li, in prep.
Velocity Structure in the Magnetosphere

Trammell, Arras, & Li, in prep.

→ in regime of neither isolated planet nor Roche Lobe overflow
Results: Transit Spectra in Lyman $\alpha$

Trammell, Arras, & Li, in prep.

$1 - T_\nu = R_p^2(\nu)/R^2_*$

$\Delta\nu \ [\text{km s}^{-1}]$

$\Delta \lambda \ [\AA]$

Ly $\alpha$

pts./errors from Ben-Jaffel (2008)

→ transit signal from magnetically-confined dead zone
Mass/Angular Momentum Loss Rates

\[
\frac{\delta F}{F} = \frac{\int d\nu I_{\nu}^{(*)}(1 - T_{\nu})e^{-\tau_{\nu}^{(ISM)}}}{\int d\nu I_{\nu}^{(*)}e^{-\tau_{\nu}^{(ISM)}}}
\]

<table>
<thead>
<tr>
<th>Run</th>
<th>(\dot{M})</th>
<th>(\dot{j})</th>
<th>(j/(\dot{M}\Omega_p R_p^2))</th>
<th>(\delta F/F)</th>
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<tbody>
<tr>
<td>Model 1</td>
<td>1.31</td>
<td>5.26</td>
<td>248.4</td>
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<td>1.18</td>
<td>6.87</td>
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<td>1.06</td>
<td>11.43</td>
<td>667.1</td>
<td>0.0599</td>
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<td>0.65</td>
<td>1005.4</td>
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<td>16.92</td>
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<td>0.11</td>
<td>1.14</td>
<td>641.2</td>
<td>0.0217</td>
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<td>Model 7</td>
<td>21.41</td>
<td>19.98</td>
<td>57.9</td>
<td>0.2208</td>
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<td>Model 8</td>
<td>2.13</td>
<td>10.82</td>
<td>166.7</td>
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<tr>
<td>Model 9</td>
<td>1.20</td>
<td>2.51</td>
<td>261.6</td>
<td>0.0522</td>
</tr>
</tbody>
</table>

\(\tau_{J} \sim \frac{J_p}{j} \approx \frac{0.1 M_p R_p^2 \Omega}{j}\)

\(\tau_{M} \sim \frac{M_p}{\dot{M}}\)

\(\lambda = 9.6\)

\(\tau_{J} \sim 150\text{Myr}\)

\(\tau_{M} \sim 330\text{Gyr}\)

Tide strength

Transit depth

\(\epsilon = (\Omega R_p / \alpha)^2\)
Summary

- Tides/magnetic fields constrain upper atmosphere. Upper atmosphere is highly ionized and magnetically-dominated. Sonic point location is affected by magnetic geometry. Tides allow densities to increase outward in the dead zone. Lyα transit depth for HD 209458b implies static gas in the magnetosphere.

- Tides/magnetic fields modify atmospheric escape. Gas outside the Roche Lobe does not directly imply mass loss. MHD model shows that the outflow region is controlled by magnetic stresses. In the strong field limit, mass-loss rate varies by 10%. More efficient angular momentum extraction is observed with higher B-field.

Future: Interactions with stellar wind (e.g., Stone & Proga 2009). Future: Characterization of other hot Jupiters (e.g., HD 189733b).
Escape From Hot Jupiters

- other models
  - transonic wind colliding with stellar wind (e.g., Stone & Proga 2009)
  - HI unbound because outside L1 (e.g., Lai et al. 2010)
  - ballistic escape beyond Roche Lobe

\[ r_L \simeq D \left( \frac{M_p}{3M_*} \right)^{1/3} = \left( \frac{GM_p}{3\Omega^2} \right)^{1/3} \]

- problems with RL overflow
  - assumes sonic point near L1
  - ignores planetary magnetic field

Trammell, Arras, & Li (2011)
Magnetic Field Set by Core Heat Flux

- Larger planets have higher $L$
- $L$ varies by $10^3$-$10^4$ from 1-1.4 $R_p$

\[ F \propto \rho v^3 \]
\[ \therefore B^2 \sim \rho v^2 \sim \rho^{1/3} F^{2/3} \]
\[ F_{\text{HJ}} \sim 10^3 - 10^4 F_J \]
Numerical MHD Models

- library of models in strong field limit:

Table 1. Model Parameters for HD 209458b

<table>
<thead>
<tr>
<th>Value</th>
<th>Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_b$</td>
<td>$1.3 , R_{\text{Jup}}$</td>
<td>planet radius</td>
</tr>
<tr>
<td>$\rho_{\text{ss}}$</td>
<td>$10^{-16} - 10^{-13}$</td>
<td>substellar point mass density</td>
</tr>
<tr>
<td>$a$</td>
<td>8.5-13 km s$^{-1}$</td>
<td>isothermal sound speed</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.001-0.1</td>
<td>plasma $\beta = P_{\text{gas}}/P_{\text{mag}}$ at $\theta = 0$</td>
</tr>
<tr>
<td>$M_p$</td>
<td>0.7 $M_{\text{Jup}}$</td>
<td>planet mass</td>
</tr>
<tr>
<td>$M_{\text{star}}$</td>
<td>1.1 $M_{\odot}$</td>
<td>host star mass</td>
</tr>
<tr>
<td>$D$</td>
<td>0.03-0.08 AU</td>
<td>orbital semi-major axis</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Run</th>
<th>$D$ (AU)</th>
<th>$P_s$ ($\mu$bar)</th>
<th>$a$ (km/s)</th>
<th>$B_0$ (G)</th>
<th>$\beta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0.05</td>
<td>0.04</td>
<td>10.0</td>
<td>10.0</td>
<td>0.01</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.05</td>
<td>0.04</td>
<td>10.0</td>
<td>14.1</td>
<td>0.005</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.05</td>
<td>0.04</td>
<td>10.0</td>
<td>31.5</td>
<td>0.001</td>
</tr>
<tr>
<td>Model 4</td>
<td>0.05</td>
<td>0.04</td>
<td>8.5</td>
<td>10.0</td>
<td>0.01</td>
</tr>
<tr>
<td>Model 5</td>
<td>0.05</td>
<td>0.04</td>
<td>13.0</td>
<td>10.0</td>
<td>0.01</td>
</tr>
<tr>
<td>Model 6</td>
<td>0.05</td>
<td>0.004</td>
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<td>10.0</td>
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<td>Model 7</td>
<td>0.05</td>
<td>0.4</td>
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<td>0.1</td>
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<tr>
<td>Model 8</td>
<td>0.03</td>
<td>0.04</td>
<td>10.0</td>
<td>10.0</td>
<td>0.01</td>
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<tr>
<td>Model 9</td>
<td>0.08</td>
<td>0.04</td>
<td>10.0</td>
<td>10.0</td>
<td>0.01</td>
</tr>
</tbody>
</table>

- uses modified ZEUS-MP code (Hayes et al. 2006)
  evolves fluid quantities on staggered-grid (2D; axisymmetric)
  all runs on 576-core Beowulf cluster at UVa
  Infiniband interconnect + 192 high-memory cores

- structure dependence on model parameters:
  e.g., overall effect of increasing $B_0$
  e.g., overall effect of increasing stellar tide