Space Science: Atmospheres Part- 6b

Viscosity + Eddy Mixing
Planetary Boundary Layer
Ekman Spiral
Spin Down Time

Reminder

$$\mathbf{k'} \;\; \mathbf{x} \;\; (-\nabla \mathbf{p}_r) \quad = \quad - \; f\rho\,\vec{\mathbf{v}}$$

Steady State

Geostrophic → large scale motion

Near surface: we need to add in drag forces

Result:

Winds near surface are often to the left of the winds aloft in northern henispshere

This defines the ‘boundary layer’

Planetary Boundary Layer

Surface is not smooth on some scale

Scale of ‘roughness’ differs
over mountains, plains, cities
Typical average ~1km (a fraction of H)

treat turbulence induced as a viscosity
Ekman number *not* negligible

$E^{-1} = R_e = L\,V / K$; K = eddy viscosity

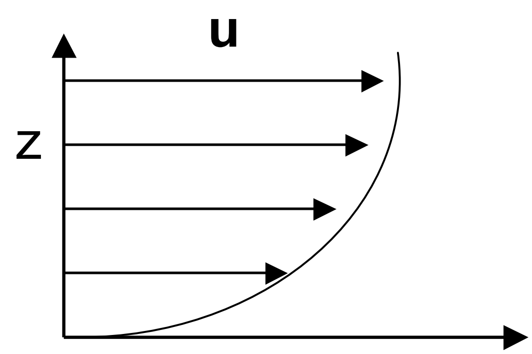
<u>Viscosity</u>
transport of turbulence (momentum) between adjacent layers of fluid

Molecular--primarily above homopause
Eddy --below

No Coriolis Force (Equatorial)

Very Near Surface

Stress $\rho K[\partial u/\partial z]$
$\sim$ const



Momentum Flux due to turbulence/ mixing

$$\tau = \rho K(du/dz)$$

τ is a stress (units of pressure)

Reach a steady state with

Constant Stress between layers

but length/ mixing scale changes near the surface

$$K \sim \overline{L}\,\overline{v}$$

$\overline{L} \approx \kappa z$ near surface; κ von Karman const.

Ekman Spiral

Include Coriolis (assume curvature small) viscosity and pressure gradients

$$\frac{\partial u}{\partial t} = 0 = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + K\frac{\partial^2}{\partial z^2}u$$

$$\frac{\partial v}{\partial t} = 0 = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} + K\frac{\partial^2}{\partial z^2}v$$

2 – D Flow, but coupled vertically!

Boundary conditions

$$u(0) = 0 = v(0)$$

$$u\,(z \rightarrow \infty) = u_g, \quad v(z \rightarrow \infty) = v_g$$

Geostrophic values from

$$f\,v_g = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$f\,u_g = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

Use geostrophic solutions

$$K\frac{\partial^2}{\partial z^2}u = -f(v - v_g)$$

$$K\frac{\partial^2}{\partial z^2}v = f(u - u_g)$$

Trick : Define $\underline{V} \equiv u + i\,v$

Multiply v equation by i and add the two equations

$$K\frac{\partial^2}{\partial z^2}(u + i\,v) = i\,f\,[\,(u - u_g) + i(v - v_g)]$$

$$K\frac{\partial^2}{\partial z^2}(u + i\,v) = i\,f\,[\,u + iv - (u_g + iv_g)]$$

OR

$$\frac{\partial^2}{\partial z^2}\underline{V} = \alpha\,(\underline{V} - \underline{V}_g)$$

$$\alpha \equiv i\,f\,/\,K$$

Solution (cont)

$$\underline{V} - \underline{V}_g = a\, e^{\alpha^{1/2} z} + b e^{-\alpha^{1/2} z}$$

1) $\underline{V}(0) = 0$

2) $\underline{V}(\infty) = \underline{V}_g$

3) $\alpha^{1/2} = \left(\frac{\mathrm{i\,f}}{\mathrm{K}}\right)^{1/2} = \gamma(1+i) \quad : \; [i^{1/2} = \frac{1+i}{\sqrt{2}}]$

$$\gamma = \left(\frac{\mathrm{f}}{2\mathrm{K}}\right)^{1/2}$$

$$\therefore \quad \underline{V} = \underline{V}_g + a\, e^{\gamma(1+i)z} + b\, e^{\gamma(1+i)z}$$

Use (2)

$$\underline{V} = \underline{V}_g + b\, e^{\gamma(1+i)z}$$

Use (1)

$$0 = \underline{V}_g - b$$

$$b = \underline{V}_g$$

$$\therefore \quad \underline{\mathbf{V}} = \underline{\mathbf{V}}_{\mathbf{g}}\, [\, \mathbf{1} - \mathbf{e}^{-\gamma \mathbf{z}}\ \mathbf{e}^{\mathbf{i}\gamma\mathbf{z}}\,]$$

Analytic Solution!

decay and periodic terms

Solution (cont)

Need to go back to u, v ($\underline{V} = u + iv$)

For simplcity : $u_g = u_g$

$v_g = 0$

Solutions!

$$\underline{\mathbf{u = u_g [\, 1 - e^{-\gamma z} \cos \gamma z]}}$$

$$\underline{\mathbf{v = u_g \, e^{-\gamma z} \sin \gamma z}}$$

Ekman was actually an oceanographer

Example $\gamma = [f/2K]^{1/2}$

$\Phi = 30^{\circ}$ Latitude

then

$f = 7 \times 10^{-5}$ /s

If $K = 100 \text{ m}^2$ /s

$\underline{\gamma^{-1} = 1.6 \text{ km}}$

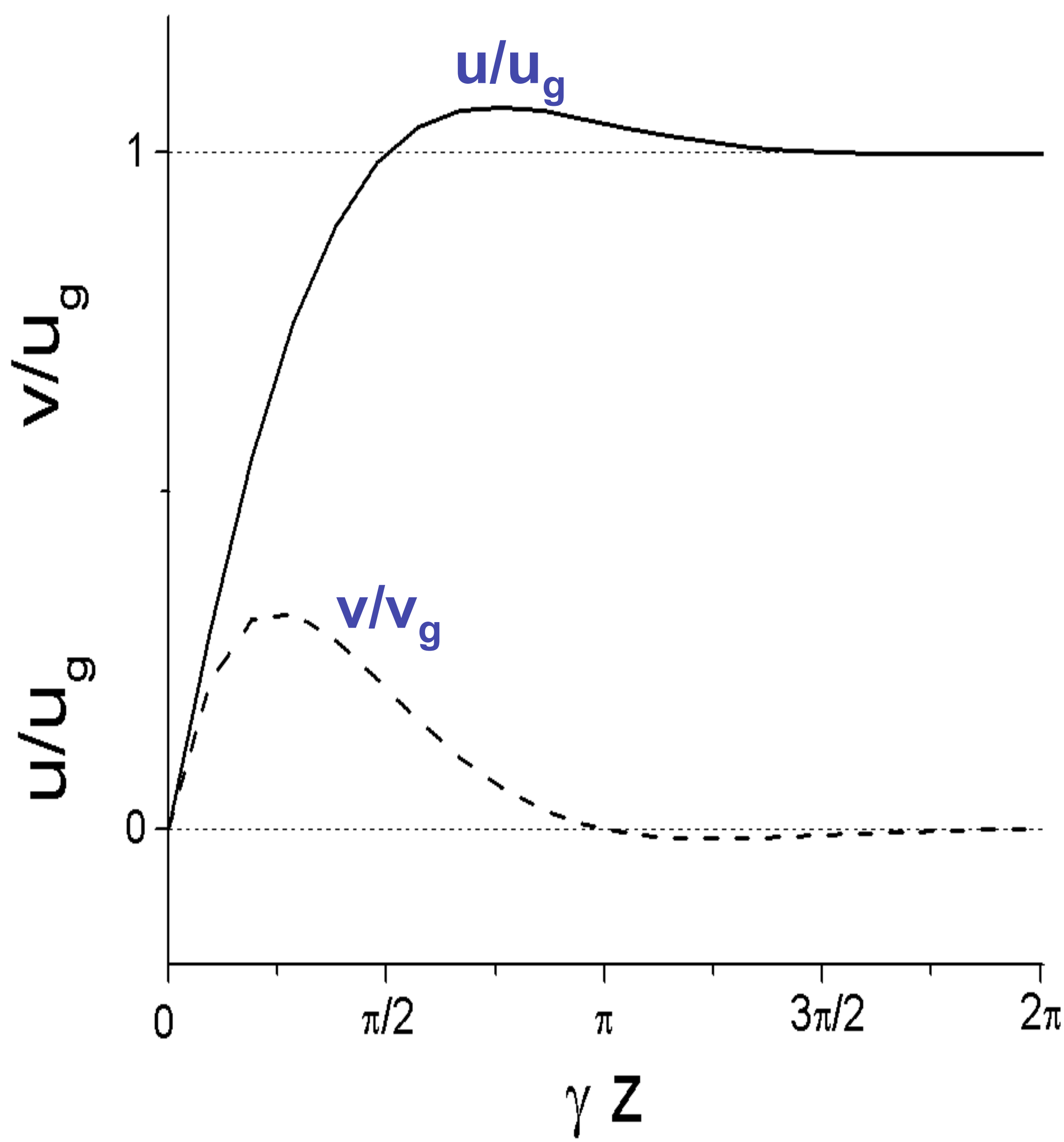
γ^{-1} = **Thickness of Planetary Boundary Layer**

Ekman Spiral

Solutions

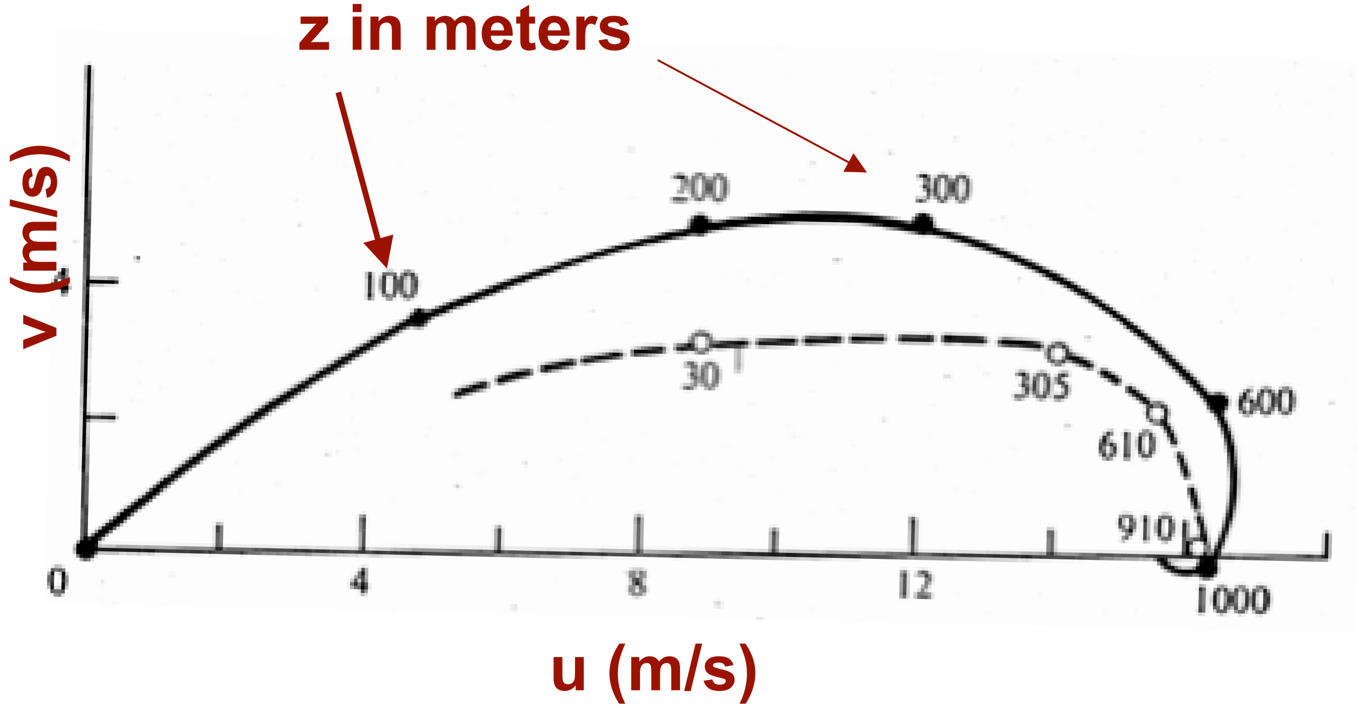
at z >> γ^{-1}

$u=u_g$, $v=0$



Ekman Spiral

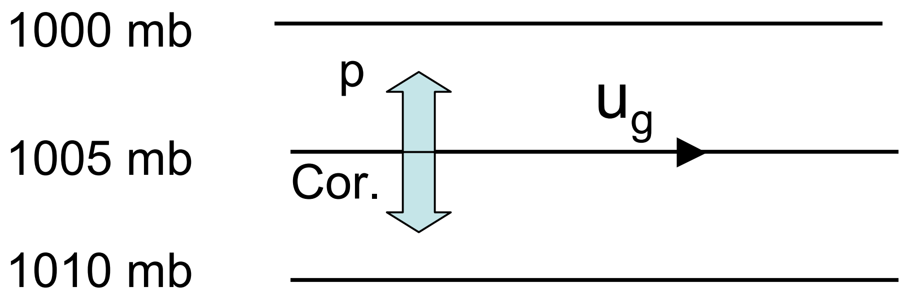
Wind speed and direction vs. altitude



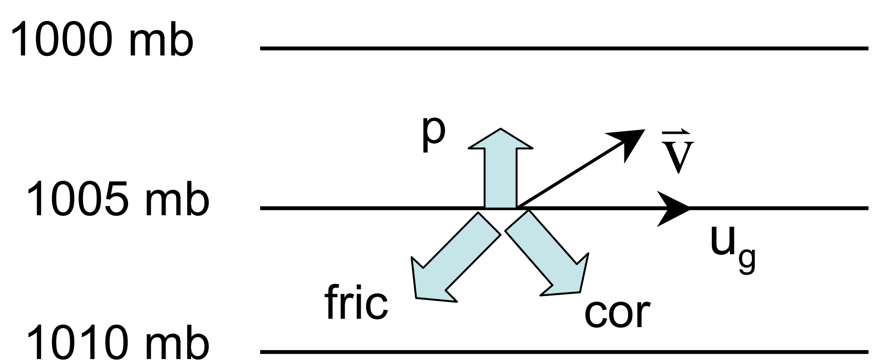
Dashed: Measurements
Solid: Analyitc solution
mid latitude

Force Balance

If at 10 km, Northern hemisphere

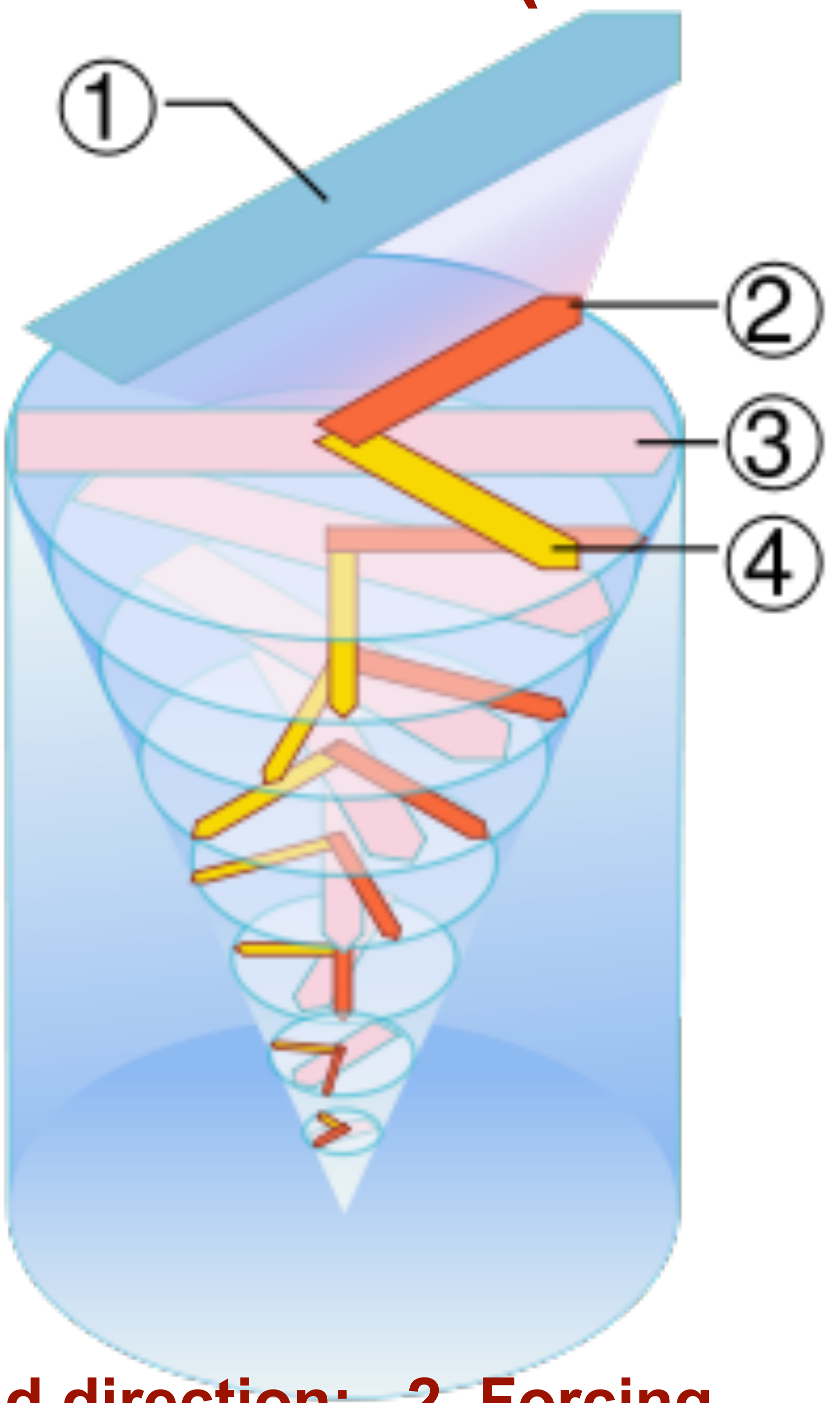


Below ~ 1 km



Surface winds in the Northern Hemisphere tend to blow to the left of winds aloft.

Ocean Layer Driven by Wind Shear (Ekman)



1. Wind direction; 2. Forcing

3. Actual Flow; 4. Geostrophic flow

In ocean waves and Turbulence disrupt

Ekman 'Pumping'

The coupling of the geostrophic layer (often circular motion) to the surface through the planetary boundary layer gives a *spin down time* for the *circular motion* in the troposphere

Not too hard to show

note: boundary layer size ~ γ^{-1}

Spin down time(interaction with the surface

$$\tau_s \sim 2(H\gamma)\, f^{-1}$$

Mid latitudes ~ days

Summary

- Eddy Viscosity
- Planetary Boundary Layer
- Ekman Spiral
- Spin Down Time