Space Science: Atmospheres

General Circulation

Part-6

GENERAL CIRCULATION
HADLEY CELL
ROTATING FRAME
CORIOLIS FORCE
TOTAL TIME DERIVATIVE
GEOSTROPHIC FLOW
GRADIENT WIND
CYCLOSTROPHIC FLOW
INERTIAL FLOW
CYCLONIC
Atmospheric Flow/ Winds
non-uniform heating, requires heat transport (heat engine)

Winds are highly variable here we consider mean flow and global scale winds `general circulation’
DIFFERENTIAL HEATING => FLOW
Both longitudinal + latitudinal dif.
summer differences smaller than winter

Global
Percentage Heat Input Per Day
E   V*   M   J
\(\frac{dT}{T_e}\) 0.68% 0.36% 38% 1.5x10^{-3} %

Therefore: night to day changes in global winds on Mars but not others
Horizontal Pressure Differences + Circulations Cells

In troposphere $T \sim [T_s - \Gamma z]$
and $(p/p_o) = (T/T_o)^x$

Same surface $p_o$ but different lapse rates drives horizontal flow
Often called a thermal wind

General Circulation: Cells
Local Scale: Land / Sea Breeze
Monsoon
Planetary Scale: Hadley Cell
Sea Breeze

This is local effect, but a similar effect drives the monsoons seasonal flow to/away from land

Land Breeze
\[ \ddot{z} = - \left( \frac{g z}{T} \right) [ \Gamma_w - \gamma ] \]

Buoyancy Equation
**Mars Troposphere**

Like the desert

Heating/cooling directly from surface:
no IR absorption unless a dust storm

![Graphs showing temperature profiles](image)

Rapid surface heating

Rapid convection

Like a land/sea breeze but on a planetary scale
VENUS TROPOSPHERE
GIANT HADLEY CELL

Hot air in equatorial regions moves to polar regions (heated at surface or cloud tops?)

Venus also has horizontal day-night circulation!
Therefore, even in absence of rotation the circulation is complex
Day/ Night Cooling

Venus slow rotation
PERIOD ~ 243 days (retrograde)

Tropospheric Winds
~ 4 day rotation

Also winds in thermosphere
Global Circulation on Earth

If slow rotation
(but fast enough so day/night differences small, unlike Venus)

Warm air rises to the tropopause
Equator to pole Hadley Cell
Equatorial low + polar high
What do we see?
Hadley Cells on Earth
NS Flow (starts with negligible ang. motion) has an east to west component ‘easterly’
Opposite in southern hemisphere
SN Flow has a west to east component ‘westerly’ (opposite in SH)
Note: conserving ang. Momentum
westerlies become stronger going north
Effect of Rotation

break-up into cells

surface winds indicated

ITCZ = Intertropical Convergence Zone

H = High Pressure; L = low Pressure
Doldrums (rising air, weak winds ~5° to -5° of equator but can move up and down: ITCZ)

Horse Latitudes: Dry weak wind due to descending air
Jupiter’s Atmosphere shows cell structure

Dominated by circulation cells and zonal winds. These cells are marked by different colored cloud layers.

Black small circle is Io.
Planet Rotation

Write Newton’s Law in the rotating frame in which we measure positions, speeds and accelerations

Result

Acceleration of a parcel of air
= - Pressure Gradient
+ Gravity
+ Coriolis
+ Centrifugal
+ Viscosity

Usually obtain approximate solutions
Hydrostatic Law = Gravity and Pressure

Compare other forces

Ekman Number = Viscous / Inertial
Ekman = 1/Reynolds

Rossby Number = Convection/Coriolis
Rotating Reference Frame: Simple Case

Inertial \((\hat{i}, \hat{j}, \hat{k})\); Rotating\((\hat{i}', \hat{j}', \hat{k}')\)

\[
\hat{i}' = \hat{i} \cos \theta + \hat{j} \sin \theta
\]

\[
\hat{j}' = -\hat{i} \sin \theta + \hat{j} \cos \theta
\]

\[
\vec{\Omega} = \Omega \, \hat{k} \ ; \ \theta = \Omega \, t \ ; \ \Omega = d\theta/dt = \text{angular speed}
\]

\[
\frac{d\hat{i}'}{dt} = [-\hat{i} \sin \theta + \hat{j} \cos \theta] \Omega
\]

\[
\frac{d\hat{j}'}{dt} = [-\hat{i} \cos \theta - \hat{j} \sin \theta] \Omega
\]

Note:

\[
\hat{k} \times \hat{i}' = \hat{j} \cos \theta - \hat{i} \sin \theta
\]

\[
\hat{k} \times \hat{j}' = -\hat{j} \sin \theta - \hat{i} \cos \theta,
\]

Therefore

\[
\frac{d\hat{i}'}{dt} = \vec{\Omega} \times \hat{i}' = \vec{\Omega} \times \hat{i}'
\]

Similarly - -the change in the \(\hat{j}'\) direction

\[
\frac{d\hat{j}'}{dt} = \vec{\Omega} \times \hat{j}'
\]
Momentum (Force/Navier Stokes) Equation

\[ \frac{d\vec{v}}{dt} = -\frac{1}{\rho} \nabla p + \vec{g} + \nu \nabla^2 \vec{v} \]

\( \nu \) = viscosity (\text{length} \times \text{speed} - \text{-like diffusion})

\( \nu \nabla^2 \vec{v} = \text{viscous force per unit mass} \)

Acceleration in Inertial Frame has 3 components in a Rotating Frame

Rotating frame \( \frac{d\vec{v}}{dt} = \left( \frac{d\vec{v}'}{dt} \right)' + 2 \vec{\Omega} \times \vec{v}' - \Omega^2 \vec{R} \)

\[
\left( \frac{d\vec{v}'}{dt} \right)' = -\frac{1}{\rho} \nabla p - 2 \vec{\Omega} \times \vec{v}' + \vec{g}_e + \nu \nabla^2 \vec{v}'
\]

\( \vec{g}_e = \vec{g} + \Omega^2 (\vec{R}) \); \( \vec{R} = \vec{r} - \vec{z} \)

Combine centrifugal with gravity -

means effective gravity is not simply radial,

but then the earth is not quite spherical

Therefore; one new force

Coriolis force

What about \( d/dt \)?
Continuity Equation

variations in air density

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}) \]

Rate of change = \text{- divergence of the flow}

If we follow a parcel of air, then it is useful to rewrite density changes on left

\[ \frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \vec{v} - \vec{v} \cdot \nabla \rho \]

\[ \frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho = -\rho \nabla \cdot \vec{v} \]

change in density due to time dep and flow

\[ \frac{d\rho}{dt} = -\rho \nabla \cdot \vec{v} \quad \text{with} \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \]

For an incompressible fluid

\[ \frac{d\rho}{dt} = 0 ; \quad \nabla \cdot \vec{v} = 0 \]

Total time derivative for a parcel of air

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \]

local rate of change + change due to flow
In Rotating Coordinate follow an air mass

Therefore, use $\nabla \cdot \vec{v} = 0$ and write

**Navier Stokes Eq. in rotating frame (drop primes)**

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho} \nabla p_r - 2\Omega \times \vec{v} + \nu \nabla^2 \vec{v}$$

**Meteorologists:**

$$-\frac{1}{\rho} \nabla p_r = -\frac{1}{\rho} \nabla p + \vec{g}_e$$

reduced pressure, $p_r$, gravity + centrifugal forces
then, as stated earlier. may use isobaric surfaces for $x, y$ that are not necessarily parallel to planet surface
Scaling
make non-dimensional

Use \( \vec{r} \Rightarrow L \vec{r} ; t \Rightarrow \Omega^{-1} t ; \vec{v} \Rightarrow U\vec{v} \)

Therefore, for instance \( p/\rho \Rightarrow \Omega U L (p/\rho) \)

New \( \vec{r}, \vec{v}, t, \) etc. do not have dimensions

Momentum equation again

\[
\left( \frac{d\vec{v}}{dt} \right) - \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -2 \bar{\Omega} \times \vec{v} - \frac{1}{\rho} \nabla p_r + \nu \nabla^2 \vec{v}
\]

Use \( \bar{\Omega} = \Omega k, k \) is rotation axis, and make dimensionless

\[
\rightarrow \frac{\partial \vec{v}}{\partial t} + \varepsilon \vec{v} \cdot \nabla \vec{v} = -2 \hat{k} \times \vec{v} - \frac{1}{\rho} \nabla p_r + E \nabla^2 \vec{v}
\]

only two terms have scaling coefficients!

Rossby Number \( \varepsilon = \frac{\text{convection}}{\text{coriolis}} \rightarrow \frac{U^2}{L} = \frac{U}{\Omega L} \)

Ekman Number \( E = \frac{1}{\text{Reynolds}} = \frac{\text{viscous force}}{\text{rotational inertia force}} \)

\[
\rightarrow \frac{\nu(U/L^2)}{\Omega U} = \frac{\nu}{\Omega L^2}
\]
Size Scales Again

Horizontal vel. \( U \) \( \sim 10 \text{m/s} \)
Vertical vel. \( W \) \( \sim 10 \text{cm/s} \)
Horizontal \( L \) \( \sim 1000 \text{km} = 10^8 \text{cm} \)
Vertical \( H \) \( \sim 10 \text{km} = 10^6 \text{cm} \)

\((\Delta p)_H\) \( \sim 10 \text{mb} = 10^4 \text{ dynes/cm}^2 \)

\((\Delta p)_V\) \( \sim 1 \text{lb} = 10^6 \text{ dynes/cm}^2 \)

\(\Omega\) \( \sim 10^{-4} \text{s} \)

\(g\) \( \sim 10^3 \text{ cm/s} \)

\(\rho\) \( \sim 1 \text{ mg/cm}^3 \)

\(\nu\) \( \sim 0.15 \text{ cm}^2 / \text{s} \)

Scales for vertical terms (accelerations : \( \text{cm/s}^2 \))

\[ \begin{array}{ccccccc}
\frac{\partial W}{\partial t} & \frac{W^2}{H} & 2\Omega U & g & \frac{(\Delta p)_V}{\rho H} & \Omega^2 H \\
10^{-3} & 10^{-4} & 10^{-1} & 10^3 & 10^3 & 10 
\end{array} \]

Scales for horizontal terms (accelerations : \( \text{cm/s}^2 \))

\[ \begin{array}{ccccccc}
\frac{\partial U}{\partial t} & \frac{U^2}{L} & 2\Omega U & 2\Omega W & \frac{(\Delta p)_H}{\rho L} \\
10^{-1} & 10^{-2} & 10^{-1} & 10^{-3} & 10^{-1} 
\end{array} \]
**Geostrophic Flow**

Typical: \( E \ll 1 \) except near surface \( \Rightarrow \) boundary layer

also: \( \varepsilon = \frac{U}{\Omega L} \ll 1 \)

Assume Steady Flow \((\partial \vec{v}/\partial t \sim 0)\)

we obtain the remarkably simple result

\[-\nabla p_r \propto 2 \hat{k} \times \vec{v} ; \hat{k} = \text{planet rotation axis}\]

**Geostrophic Approximation**

- \(-\nabla p_r \Rightarrow \) force
  - Force \( \perp \) to flow (like a magnetic field)
    - OR
    - \( \vec{v} \perp \nabla p_r \)
  - Flow // to isobars
**Vertical Equation**

\[
\frac{\partial v_z}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g_e
\]

typically \( v_z \rightarrow w \)

\[
\frac{\partial w}{\partial t} = 0 ; \quad \text{hydrostatic}
\]

**Horizontal (flow nearly along isobars : x, y)**

\[
\frac{\partial \vec{v}}{\partial t} = -2 \vec{\Omega} \times \vec{v} - \frac{1}{\rho} (\nabla p_r)_{//}
\]

// means in the plane parallel to isobars

often \( \vec{v} = u \hat{i}' + v \hat{j}' \)
Horizontal Flow

Assume surface \(//\) isobars for convenience

\[
\frac{\partial \tilde{v}}{\partial t} = -2 \tilde{\Omega} \times \tilde{v} - \frac{1}{\rho} (\nabla p)_{//}
\]

\(\tilde{v} = u\hat{i}' + v\hat{j}'\)

temporarily use the primes

\[
\begin{align*}
\frac{\partial u}{\partial t} &= f\tilde{v} - \frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{\partial v}{\partial t} &= -fu - \frac{1}{\rho} \frac{\partial p}{\partial y}
\end{align*}
\]

\(f = 2\Omega \sin \phi\)

\(f = 0\) at equator

\(f = 1\) at north pole

\(f = -1\) at south pole

\(f = 2\Omega \sin \phi\) coriolis parameter
Geostrophic Flow
(Steady 1-D flow)

\[ 0 = f v - \frac{1}{\rho} \frac{\partial p}{\partial x} ; \quad v = \frac{1}{\rho f} \frac{\partial p}{\partial x} \]

or

\[ 0 = -f u - \frac{1}{\rho} \frac{\partial p}{\partial y} ; \quad u = -\frac{1}{\rho f} \frac{\partial p}{\partial y} \]

Non-rotating body

Rotating body

u \propto -\frac{\partial p}{\partial y}

v \propto \frac{\partial p}{\partial x}

Oceans: Measure \( p \) vs depth and get \( v \)!
Balanced Wind

Steady 2-D flow about a high

Pressure grad.

OPPOSITE ABOUT A LOW

\[ \text{-Pressure Gradient} = 2 \rho \nu \times \Omega \]

- counter clockwise: cyclonic
- clockwise: anti cyclonic

Remember: flow attempts to relieve pressure in doing so they are coupled
Near surface pressures suggest flow pattern
2 –D Flow: ‘Flat’ Surface

Gradient Wind Equation

\[ \frac{\partial \vec{v}}{\partial t} = -f \hat{k} \times \vec{v} - \frac{1}{\rho} (\nabla p)_{\parallel} \]

// → parallel to surface or along isobars

(drop \( r \) and \( \partial \rightarrow d \) for convenience)

Curved Motion

\[ r = \text{radius of curvature} \]

\[ \hat{k} \times \hat{v} = \hat{n} \]

Write \( \frac{d\vec{v}}{dt} = \hat{v} \frac{dv}{dt} + \frac{d\hat{v}}{dt} \cdot \hat{v} = \hat{v} \cdot \hat{v} + \hat{n} \cdot \frac{v^2}{r} \)

measure along pathlength, \( s \):

1) \( \hat{v} \)

\[ \frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial s} \]; along the flow direction

2) \( \hat{n} \)

\[ \frac{v^2}{r} \pm f \cdot v = -\frac{1}{\rho} \frac{\partial p}{\partial n} \] (± curvative)

Flow nearly // to isobars: ignore \( \hat{v} \) terms
Gradient Wind Equation

\[ \frac{dv}{dt} = 0 \text{ (eq. 1)} \]

discuss direction normal to the flow

\[ \frac{v^2}{r} \pm f v = -\frac{1}{\rho} \frac{\partial p}{\partial n} \]

Case 1: Pure inertial flow

\[ \frac{v^2}{r} \pm f v = 0 \]

If there is a velocity, the motion is curved

Northern

\[ \hat{v} \]

Clockwise anticyclonic

Southern

\[ \hat{n} \]

Counterclock cyclonic

Period = \( \frac{1}{2} \) period of fociualt pendulum
CYCLOSTOPTHOPIC FLOW

Case 2  \( f \approx 0 \); no coriolis

centrifugal = - press. grad.

\[
\frac{v^2}{r} \pm f v = - \frac{1}{\rho} \frac{\partial p}{\partial n}
\]

(near equator or \( r \) small or \( v \) very large)

\[
\frac{v^2}{r} = - \frac{1}{\rho} \frac{\partial p}{\partial n}
\]

Tornadoes-bathroom sinks

Venus : Polar Vorticity
GEOSTROPHIC FLOW

Centripetal term is zero (e.g., large scale flow)

\( \mathbf{L} \) \( \mathbf{H} \) \( \mathbf{L} \)

(anticyclonic)

Northern Hemisphere

Southern Hemisphere (cyclonic)

\( \mathbf{L} \) \( \mathbf{H} \) \( \mathbf{L} \)

Gradient Wind Eq.

\( \frac{v^2}{r} \pm f v = -\frac{1}{\rho} \frac{\partial p}{\partial \eta} \)

Case 3: \( r \) very large (centrifugal force \( \sim 0 \))

\( \pm f v = \frac{1}{\hat{n}} \frac{\rho}{\hat{n}} \)
GRADIENT WIND
(all terms)
Northern Hemisphere \( (f > 0) \)

Regular

Anamolous