Chapter 12
Static Equilibrium; Elasticity and Fracture
• The Conditions for Equilibrium
• Solving Statics Problems
• Stability and Balance
• Elasticity; Stress and Strain
• Fracture
12-1 The Conditions for Equilibrium

An object with forces acting on it, but with zero net force, is said to be in equilibrium.

The first condition for equilibrium:

\[ \Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0. \]
Example 12-1: Chandelier cord tension.

Calculate the tensions \( \vec{F}_A \) and \( \vec{F}_B \) in the two cords that are connected to the vertical cord supporting the 200-kg chandelier shown. Ignore the mass of the cords.
12-1 The Conditions for Equilibrium

The second condition of equilibrium is that there be no torque around any axis; the choice of axis is arbitrary.

\[ \sum \tau = 0 \]
Balance of net Forces and net Torques

\[
\sum F_x = 0 \quad \sum \tau_x = 0 \\
\sum F_y = 0 \quad \sum \tau_y = 0 \\
\sum F_z = 0 \quad \sum \tau_z = 0
\]

Gravity acts on a single point on a body called the center of gravity. If \( g \) is the same for every point on the body, then \( \text{cog} = \text{com} \).
12-1 The Conditions for Equilibrium

Conceptual Example 12-2: A lever.

This bar is being used as a lever to pry up a large rock. The small rock acts as a fulcrum (pivot point). The force required at the long end of the bar can be quite a bit smaller than the rock’s weight $mg$, since it is the torques that balance in the rotation about the fulcrum. If, however, the leverage isn’t sufficient, and the large rock isn’t budged, what are two ways to increase the leverage?
12-2 Solving Statics Problems

1. Choose one object at a time, and make a free-body diagram by showing all the forces on it and where they act.

2. Choose a coordinate system and resolve forces into components.

3. Write equilibrium equations for the forces.

4. Choose any axis perpendicular to the plane of the forces and write the torque equilibrium equation. A clever choice here can simplify the problem enormously.

5. Solve.
Example 12-3: Balancing a seesaw.

A board of mass $M = 2.0 \text{ kg}$ serves as a seesaw for two children. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of gravity is 2.5 m from the pivot). At what distance $x$ from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.

\[ m_A = 30 \text{ kg} \quad m_B = 25 \text{ kg} \]
Example 12-5: Hinged beam and cable.

A uniform beam, 2.20 m long with mass $m = 25.0$ kg, is mounted by a small hinge on a wall. The beam is held in a horizontal position by a cable that makes an angle $\theta = 30.0^\circ$. The beam supports a sign of mass $M = 28.0$ kg suspended from its end. Determine the components of the force $\vec{F}_H$ that the (smooth) hinge exerts on the beam, and the tension $F_T$ in the supporting cable.
Example 12-6: Ladder.

A 5.0-m-long ladder leans against a smooth wall at a point 4.0 m above a cement floor. The ladder is uniform and has mass \( m = 12.0 \text{ kg} \). Assuming the wall is frictionless (but the floor is not), determine the forces exerted on the ladder by the floor and by the wall.
An object in stable equilibrium may become unstable if it is tipped so that its center of gravity is outside the pivot point. Of course, it will be stable again once it lands!
Hooke’s law: the change in length is proportional to the applied force.

\[ F = k \Delta l \]
This proportionality holds until the force reaches the proportional limit. Beyond that, the object will still return to its original shape up to the elastic limit. Beyond the elastic limit, the material is permanently deformed, and it breaks at the breaking point.
The change in length of a stretched object depends not only on the applied force, but also on its length, cross-sectional area and the material from which it is made.

The material factor, $E$, is called the elastic modulus or Young’s modulus, and it has been measured for many materials.

$$\Delta \ell = \frac{1}{E} \frac{F}{A} \ell_0.$$
### TABLE 12–1 Elastic Moduli

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus, $E$ (N/m$^2$)</th>
<th>Shear Modulus, $G$ (N/m$^2$)</th>
<th>Bulk Modulus, $B$ (N/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solids</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iron, cast</td>
<td>$100 \times 10^9$</td>
<td>$40 \times 10^9$</td>
<td>$90 \times 10^9$</td>
</tr>
<tr>
<td>Steel</td>
<td>$200 \times 10^9$</td>
<td>$80 \times 10^9$</td>
<td>$140 \times 10^9$</td>
</tr>
<tr>
<td>Brass</td>
<td>$100 \times 10^9$</td>
<td>$35 \times 10^9$</td>
<td>$80 \times 10^9$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$70 \times 10^9$</td>
<td>$25 \times 10^9$</td>
<td>$70 \times 10^9$</td>
</tr>
<tr>
<td>Concrete</td>
<td>$20 \times 10^9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brick</td>
<td>$14 \times 10^9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marble</td>
<td>$50 \times 10^9$</td>
<td></td>
<td>$70 \times 10^9$</td>
</tr>
<tr>
<td>Granite</td>
<td>$45 \times 10^9$</td>
<td></td>
<td>$45 \times 10^9$</td>
</tr>
<tr>
<td>Wood (pine) (parallel to grain)</td>
<td>$10 \times 10^9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(perpendicular to grain)</td>
<td>$1 \times 10^9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nylon</td>
<td>$5 \times 10^9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bone (limb)</td>
<td>$15 \times 10^9$</td>
<td>$80 \times 10^9$</td>
<td></td>
</tr>
<tr>
<td>Liquids</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td></td>
<td>$2.0 \times 10^9$</td>
<td></td>
</tr>
<tr>
<td>Alcohol (ethyl)</td>
<td></td>
<td>$1.0 \times 10^9$</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td></td>
<td>$2.5 \times 10^9$</td>
<td></td>
</tr>
<tr>
<td>Gases$^\dagger$</td>
<td></td>
<td></td>
<td>$1.01 \times 10^5$</td>
</tr>
<tr>
<td>Air, H$_2$, He, CO$_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^\dagger$At normal atmospheric pressure; no variation in temperature during process.
Example 12-7: Tension in piano wire.

A 1.60-m-long steel piano wire has a diameter of 0.20 cm. How great is the tension in the wire if it stretches 0.25 cm when tightened?
Stress is defined as the force per unit area.

Strain is defined as the ratio of the change in length to the original length.

Therefore, the elastic modulus is equal to the stress divided by the strain:

\[ E = \frac{F/A}{\Delta l/l_0} = \frac{\text{stress}}{\text{strain}}. \]
In tensile stress, forces tend to stretch the object.
Compressional stress is exactly the opposite of tensional stress. These columns are under compression.
12-4 Elasticity; Stress and Strain

The three types of stress for rigid objects:

- **Tension**: \( \vec{F} \) pulling on a cylinder causing it to stretch.
- **Compression**: \( \vec{F} \) pushing on a cylinder causing it to compress.
- **Shear**: \( \vec{F} \) applying a force to a block causing it to shear.
The shear strain, where $G$ is the shear modulus:

$$\Delta \ell = \frac{1}{G} \frac{F}{A} \ell_0.$$
If an object is subjected to inward forces on all sides, its volume changes depending on its bulk modulus. This is the only deformation that applies to fluids.

\[ \frac{\Delta V}{V_0} = - \frac{1}{B} \Delta P \]

or

\[ B = - \frac{\Delta P}{\Delta V/V_0} \]
If the stress on an object is too great, the object will fracture. The ultimate strengths of materials under tensile stress, compressional stress, and shear stress have been measured.

When designing a structure, it is a good idea to keep anticipated stresses less than 1/3 to 1/10 of the ultimate strength.
12-5 Fracture

### TABLE 12–2 Ultimate Strengths of Materials (force/area)

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile Strength (N/m²)</th>
<th>Compressive Strength (N/m²)</th>
<th>Shear Strength (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron, cast</td>
<td>$170 \times 10^6$</td>
<td>$550 \times 10^6$</td>
<td>$170 \times 10^6$</td>
</tr>
<tr>
<td>Steel</td>
<td>$500 \times 10^6$</td>
<td>$500 \times 10^6$</td>
<td>$250 \times 10^6$</td>
</tr>
<tr>
<td>Brass</td>
<td>$250 \times 10^6$</td>
<td>$250 \times 10^6$</td>
<td>$200 \times 10^6$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$200 \times 10^6$</td>
<td>$200 \times 10^6$</td>
<td>$200 \times 10^6$</td>
</tr>
<tr>
<td>Concrete</td>
<td>$2 \times 10^6$</td>
<td>$20 \times 10^6$</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>Granite</td>
<td></td>
<td>$170 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Wood (pine) (parallel to grain)</td>
<td>$40 \times 10^6$</td>
<td>$35 \times 10^6$</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td>Wood (pine) (perpendicular to grain)</td>
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<td>$5 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Nylon</td>
<td>$500 \times 10^6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bone (limb)</td>
<td>$130 \times 10^6$</td>
<td>$170 \times 10^6$</td>
<td></td>
</tr>
</tbody>
</table>
Example 12-8: Breaking the piano wire.

A steel piano wire is 1.60 m long with a diameter of 0.20 cm. Approximately what tension force would break it?
A horizontal beam will be under both tensile and compressive stress due to its own weight. Therefore, it must be made of a material that is strong under both compression and tension.
Conceptual Example 12-9: A tragic substitution.

Two walkways, one above the other, are suspended from vertical rods attached to the ceiling of a high hotel lobby. The original design called for single rods 14 m long, but when such long rods proved to be unwieldy to install, it was decided to replace each long rod with two shorter ones as shown. Determine the net force exerted by the rods on the supporting pin A (assumed to be the same size) for each design. Assume each vertical rod supports a mass \( m \) of each bridge.
If \( m = 1.8 \text{ kg}, \ L = 1 \text{ meter} \) and \( M = 2.7 \text{ kg} \) at \( L/4 \) from the left end. What do the scales read?

Normal Force = \( F_l \) and \( F_r \)

\[ y \text{-dir.} \quad F_l + F_r - Mg - mg = 0 \]

Set net torque = 0 about right end

\[
(0)(F_r) + \left(\frac{3L}{4}\right)Mg + \left(\frac{L}{2}\right)(mg) - (L)(F_l) = 0
\]

\[
F_l = 0.75(26.46) + 0.5(17.64) = 28.67 \text{ N}
\]

\[
F_r = Mg + mg - F_l
\]

\[
F_r = 26.46 + 17.64 - 28.67
\]

\[
F_r = 15.43 \text{ N}
\]
Frictionless

Free body diagram

Given
\( L = 12\, m \)
\( M = 72\, kg \)
\( m = 45\, kg \)
\( h = 9.3\, m \)

Find
\( F_w \)
\( F_{py} \)
\( F_{px} \)
Frictionless

System

Take moments about which axis since net torque is 0
Choose O eliminate two variables

\[-(h)(F_w) + (a/2)(Mg) + (a/3)mg + (0)(F_{px}) + (0)(F_{py}) = 0\]

\[a = \sqrt{L^2 - h^2} = \sqrt{12^2 - 9.3^2} = 7.58\, m\]

\[F_w = ga(M/2 + m/3) / h\]

\[F_w = 9.8(7.58)(72/2 + 45/3) / 9.3 = 407\, N\]

Now use \( F_{\text{net,x}} = 0 \) and \( F_{\text{net,y}} = 0 \)

\[F_w - F_{px} = 0\]

\[F_{px} = 407\, N\]

\[F_{py} - Mg - mg = 0\]

\[F_{py} = Mg + mg = (72 + 45)(9.8) = 1146.6\, N\]
Problem

Suppose we have a steel ball of mass 10 kg hanging from a steel wire 3 m long and 3mm in diameter. Young’s modulus is $2 \times 10^{11} \text{ N/m}^2$. How much does the wire stretch?

$$\frac{\Delta L}{L} = \frac{F_T}{A} \frac{1}{Y}$$

$F_T = 9.8(10) = 98 \text{ N}$

$A = 3.14(0.003)(0.003) = 2.8 \times 10^{-5} \text{ m}^2$

$$\Delta L = \frac{L}{Y} \frac{F_T}{A} = \frac{3}{2 \times 10^{11}} \left(\frac{98}{2.8 \times 10^{-5}}\right) = 52.5 \times 10^{-6} \text{ m}$$