

Example 5.1 A load impedance of $73 - j80$ ohm is required to be matched to a 50 ohm coaxial line having a lossless dielectric of dielectric constant 4, using a short circuited shunt stub at 500 MHz. Determine the position and length of the stub.

Solution The guide wavelength in *TEM* mode coaxial line is

$$\lambda = \frac{30}{0.5\sqrt{4}} = 30 \text{ cm}$$

Normalised load impedance $z_L = (73 - j80)/50 = 1.46 - j1.6$. The position and the length of the stub are found from the Smith chart of Fig. 5.13 as follows.

1. z_L is located on the Smith chart.
2. A circle is drawn through z_L , with centre on unity of real axis.
3. The normalised load admittance y_L is located at *A* diametrically opposite to z_L .
4. A distance d/λ is moved from *A* towards generator to reach a point *B* on the circle where $y = 1 + jb$. The values $d/\lambda = 0.12$ and $jb = j 1.38$ are read from the Smith chart.
5. To cancel the susceptance $j 1.38$ by the stub at *B* of length l , the stub should provide a susceptance of $-j 1.38$ corresponding to *C*. To find the stub length l a distance of $l/\lambda = 0.1$ is moved towards generator from the short circuit susceptance point $y = \infty$ to reach the stub susceptance point *C*, where $y = -j 1.38$.

Therefore, the required distance and length are

$$d = 0.12 \lambda = 3.6 \text{ cm.}$$

$$l = 0.1 \lambda = 3.0 \text{ cm.}$$

The coaxial line configuration of the stub is shown in Fig. 5.13.

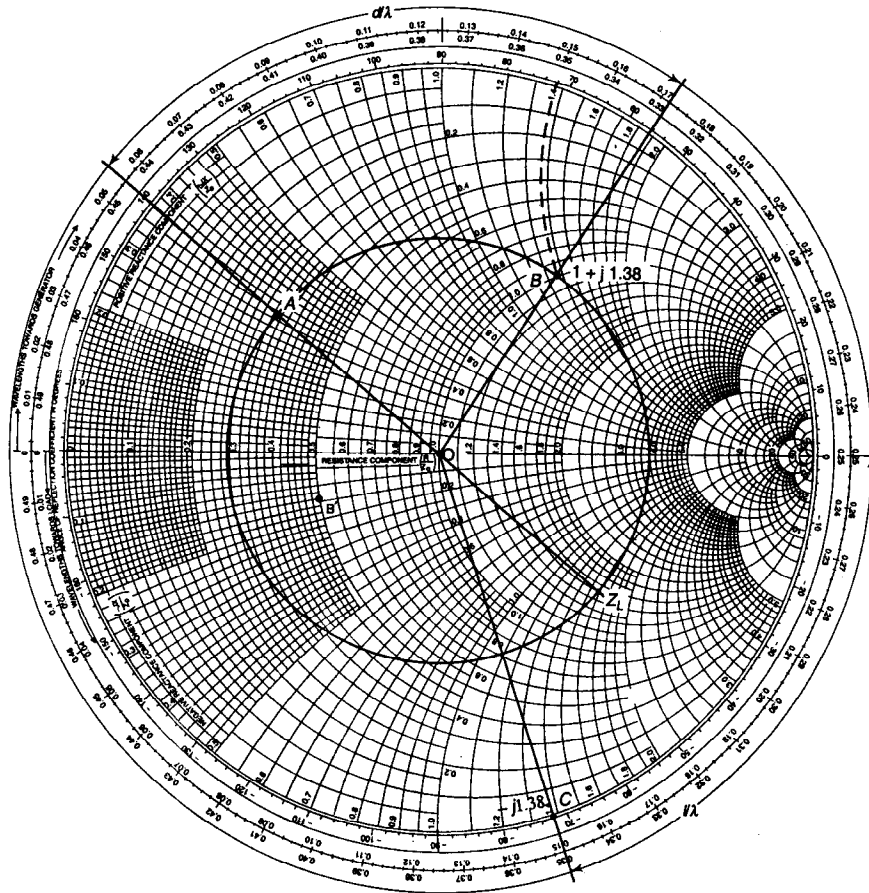
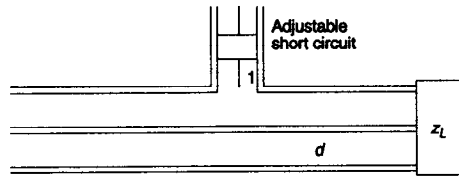


Fig. 5.13 Single coaxial stub design

Example 5.2 A 100 ohm line with air as dielectric is terminated by a load impedance of $75 + j40$ ohm and is excited at 1 GHz by a matched generator. Find the position of a single matching stub of 100 ohm impedance on the line, and determine the length of the stub. Explain how such a stub is realised in a rectangular waveguide circuit, excited in the dominant mode.

Solution

$$75 + j40$$

The wavelength $\lambda = c/f = 30$ cm.

The stub design is made using the Smith chart Fig. 5.14 as follows.

1. z_L is located on the Smith chart and the corresponding VSWR circle is drawn through z_L .
2. The normalised load admittance y_L is located at a diametrically opposite point A on the VSWR circle to obtain the value $y_L = 1.04 - j0.55$.
3. A distance d/λ is moved towards the generator from y_L to reach a normalised admittance point of intersection B between the VSWR circle and the unit conductance circle where the normalised admittance is $y_B = 1 + j0.55$. This gives $d/\lambda = 0.3$.
4. For matching, the normalised stub susceptance is $-j0.55$ which is located at C on the outer circle.
5. To find the length of the short circuited stub a distance l/λ is moved towards the generator from the short circuit infinite admittance point to reach C so that $l/\lambda = 0.17$.

Therefore,

$$d = 0.3 \times 30 = 9 \text{ cm}$$

$$l = 0.17 \times 30 = 5.1 \text{ cm}$$

A shunt stub can be realised in rectangular waveguide by joining it to the waveguide's narrow dimension so that magnetic loops can penetrate from the main waveguide into the stub as shown in Fig. 5.14. It is assumed that the tee junction formed between the waveguide and the stub do not create any field discontinuities d and l should be determined in terms of guide wavelength.

Example 5.3 A lossless 50 ohm air line has $V_{\max} = 2.5V$ and $V_{\min} = 1V$ when terminated with an unknown load. The distance between the successive voltage minima is 5 cm and the first voltage minimum from the load end is 1.25 cm. Design a short circuited single stub for impedance matching.

Solution

Distance between two successive minima is

$$\lambda/2 = 5 \text{ cm, or } \lambda = 10 \text{ cm}$$

$$\text{VSWR} = V_{\max} / V_{\min} = 2.5/1 = 2.5$$

Normalised distance of first minimum from load end is

$$d_{\min} / \lambda = 1.25/10 = 0.125$$

1. Using Smith chart of Fig. 5.15, a VSWR circle $S = 2.5$ is drawn.
2. Locate first minimum point A on the real axis where normalised impedance is $1/S = 1/2.5 = 0.4$.
3. From A, a distance d_{\min} / λ is moved towards load to reach the load point B where the normalised load impedance $z_L = 0.7 - j0.75$. An opposite point C is marked for load admittance $y_L = 0.7 + j0.73$.
4. From C, a distance d/λ is moved towards generator to reach the point D

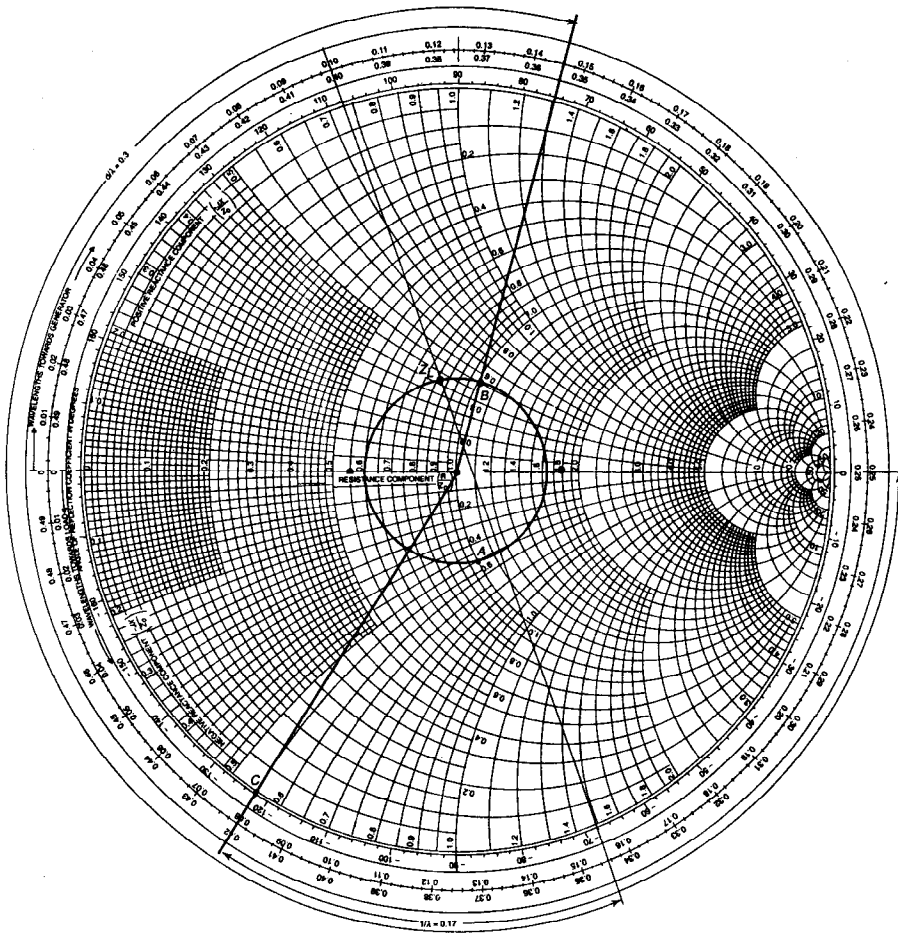
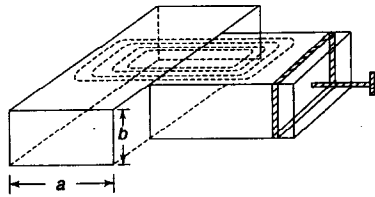


Fig. 5.14 Single waveguide stub design

tance = $1 + jb = 1 + j 0.95$. The distance of the stub from the load is $d/\lambda = 0.035$.

- The stub must provide a susceptance of $-j0.95$ for matching ($y_{in} = 1$). From the short circuit admittance ($g = \infty$) point *E*, a distance l/λ is moved towards generator to reach a point *F* on the outer circle where the normalised susceptance is $-j 0.95$. $l/\lambda = 0.13$, the required stub length.

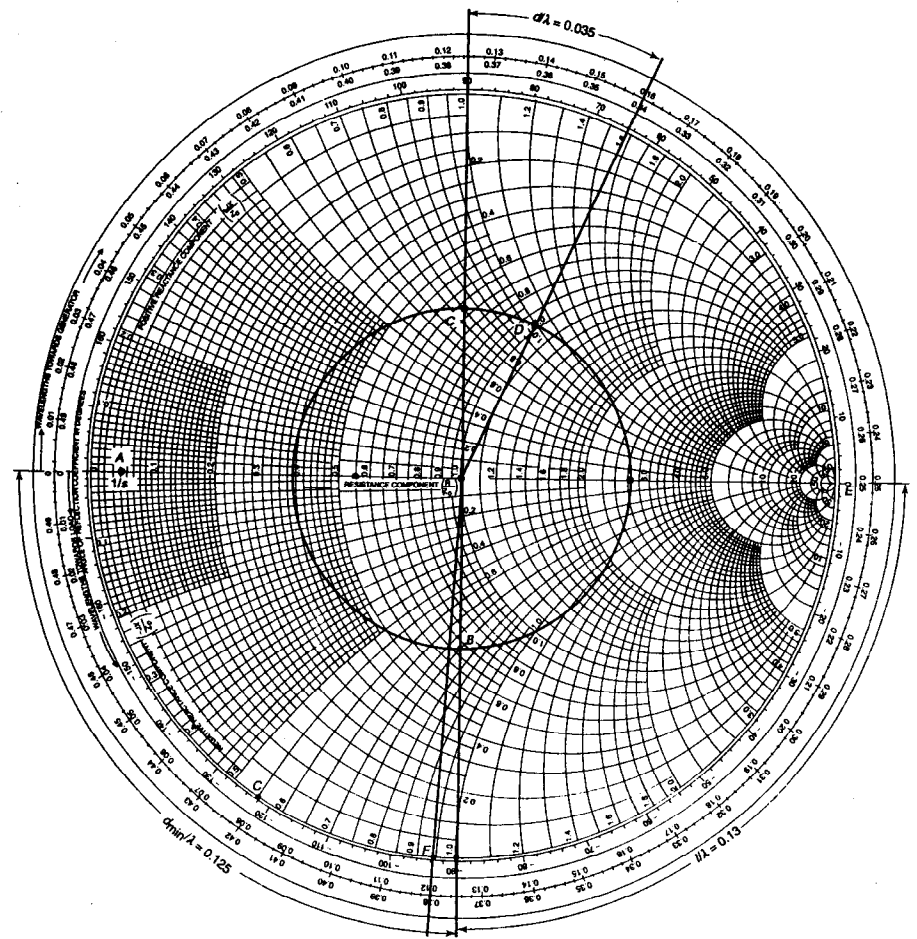


Fig. 5.15 Single stub design from load VSWR and voltage minimum position

Example 5.4 A 75 ohm air-filled coaxial line is terminated with a complex load of $109.5 - j120$ ohm. Design a double stub matching system using short circuited coaxial lines of characteristic impedance 75 ohm.

Solution

The normalised load impedance

$$z_L = (109.5 - j120)/75 = 1.46 - j1.6;$$

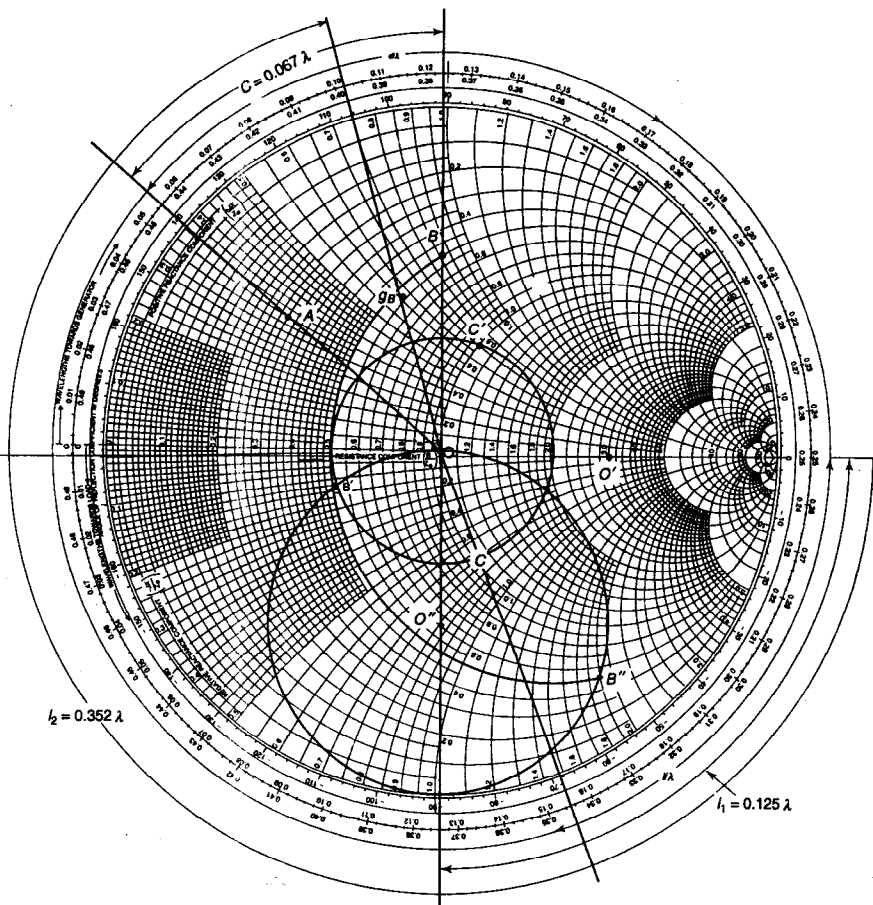
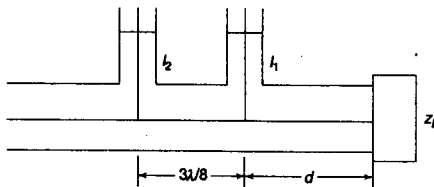
The load reflection coefficient

$$|\Gamma| = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| = \left| \frac{109.5 - j120 - 75}{109.5 - j120 + 75} \right|$$

$$= \left| \frac{34.5 - j120}{184.5 - j120} \right|$$

$$\begin{aligned} \text{VSWR} &= \frac{1 + |\Gamma|}{1 - |\Gamma|} \\ &= \frac{1 + 0.567}{1 - 0.567} = 3.6 \end{aligned}$$

The double stub system is designed using the Smith chart of Fig. 5.16 and the following steps.



1. Load impedance z_L is located on the Smith chart. A circle is drawn through z_L with center at $y = 1$ and normalised load admittance y_L is located on the circle at A diametrically opposite z_L , where $y_L = 0.31 + j 0.35$.
2. At an arbitrary distance $d/\lambda = 0.067$ from y_L towards the generator along the circle of radius $S = OA$ gives the position B of the first stub where the normalised admittance is

$$y_B = g_B + jb_B = 0.5 + j 0.85$$

3. The unit conductance circle ($g = 1$) with centre O' is rotated towards load by $3\lambda/8$ to reach the new centre O'' of the rotated circle on the line passing through the points $+j$ and $-j$.
4. The point of intersection B' between g_B circle and the rotated unit conductance circle of centre O'' gives the total admittance

$$\begin{aligned} y_{BS} &= g_B + jb_B + jb_{S1} \\ &= 0.5 + j 0.85 + jb_{S1} \\ &= 0.5 - j 0.14 \end{aligned}$$

or, $j 0.85 + jb_{S1} = -j 0.14$

or the susceptance of the first stub is

$$jb_{S1} = -j 0.14 - j 0.85 = -j 0.99$$

5. The length l_1 of the first stub is found by rotating towards the generator along the periphery of outer circle from the right-hand intersection of the unit circle and the real axis, which corresponds to the short circuit to obtain admittance of the stub, to its input susceptance value $-j 0.99$ on the outer circle

$$l_1/\lambda = 0.125$$

6. The point B' is rotated towards generator by $3\lambda/8$ to C on the original $g = 1$ circle (centre O') to get the total normalised admittance at C due to the line and the first stub:

$$y_C = 1 - jb_C = 1 - j 0.75$$

7. Matching by second stub is obtained by its susceptance $jb_{S2} = -jb_C = +j 0.75$ corresponding to the opposite point C' on the outer circle.
8. The length of the second stub l_2 is found by rotation along the outer circle towards generator from the right hand intersection of the unit circle and the real axis to the point $+j 0.75$:

$$l_2/\lambda = 0.352$$

Hence the required stub lengths are

$$l_1/\lambda = 0.125 \quad l_2/\lambda = 0.352$$

for distance of the first stub from the load $d/\lambda = 0.067$

Example 5.5 Design a double stub tuner with a stub separation of $3\lambda/8$ and first stub position at the load $z_L = 80 + j 60$ ohm, to match the load Z_L to a 50 ohm line. The generator end is matched.

Solution The normalised load impedance $z_L = (80 + j 60)/50 = 1.6 + j1.2$. The normalised load admittance y_L is at the diametrically opposite point A on the load VSWR circle, where $y_L = 0.4 - j 0.3$. The first stub should be located at the load end and the stub separation = $3\lambda/8$.

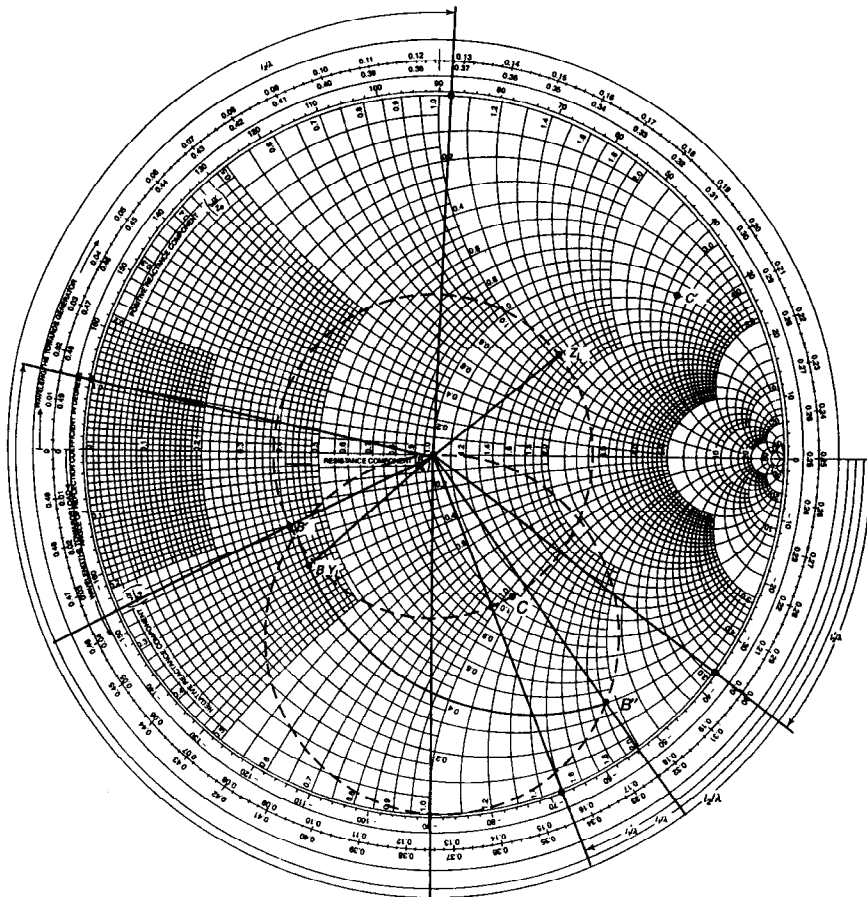
The following steps are performed on the Smith chart.

1. The point of intersection B' or B'' between $g = 0.4$ circle and the rotated unit conductance circle of centre O'' gives the total susceptance

$$y_{BS} = 0.4 - j 0.2 \quad \text{or} \quad 0.4 - j 1.8 = y_L + j b_{S1}$$

respectively. Therefore, the first stub must add a susceptance of

$$j b_{S1} = -j 0.2 + j 0.3 = +j 0.1 \quad \text{or} \quad -j 1.8 + j 0.3 = -j 1.5$$



- to the load admittance to result y_{BS} value as above at the first stub point.
2. The length of the first stub is found by rotating towards the generator along the outer circle starting from the $y = \infty$ point to the $j b_{S1}$. For the two solutions of $j b_{S1}$, the lengths are obtained as

$$l_1/\lambda = 0.268$$

$$l_2/\lambda = 0.094$$

3. The points B' and B'' are then rotated towards generator by $3\lambda/8$ to reach the point C or C' on the $g = 1$ circle where total admittance

$$y_C = 1 + j b = 1 - j 1.05 \quad \text{or} \quad 1.0 + j 3.0$$

4. Matching will be achieved when total admittance at C or C' is $y_{CS} = 1.0$. Therefore, the second stub must offer a susceptance of $+j 1.05$ or $-j 3.0$.
5. The length of the second stub is obtained in similar way as in (2):

$$l_2/\lambda = 0.378$$

or, $l_2'/\lambda = 0.052$