THE IMAGES OF NON-COMMUTATIVE POLYNOMIALS EVALUATED ON MATRICES.

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We survey results about the evaluations of a polynomial $p$ in several non-commuting variables taken in a central simple algebra $A$ over an infinite center $F$. It is well known that when $p$ is multilinear, the space generated by the image $p(A)$ of $p$ is either:

- zero,
- $F$,
- the set $sl(A)$ of matrices of reduced trace 0,
- all of $A$.

Conjecture (attributed to Kaplansky, for $A = M_n(F)$): For any $n$, the same answer holds for $p(A)$ itself. This question turns out to be considerably more difficult, and decisive results are known only for low values of $n$ (especially $n = 2$ and $n = 3$). More recently, interest has turned to Lie polynomials on $sl_n$, but here such basic questions as the degree of a minimal Lie identity remain open. We shall discuss the relation between regular associative polynomials and Lie polynomials.