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# Perceived complexity and the grouping effect in band patterns

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## Abstract

Symmetry properties have been shown to determine the perceived complexity of certain patterns. We used a paired-comparison method to obtain judgments of relative complexity for a family of two-dimensional regular patterns called *band patterns*. Although the complexity of these patterns is well predicted by their symmetry properties we were unable to explain an interaction observed between two of these properties for our experimental patterns. We discuss the implications of our results for two predominant approaches to the perceived complexity of patterns. Neither approach takes into account that the presence of grouping sometimes makes it difficult to perceive a relationship within the pattern that would—were it not for the grouping—simplify it. We conclude that grouping can mask simplicity and that this phenomenon is crucial to the understanding of perceived complexity.

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## 1. Introduction

Garner (1970, 1974) proposed a transformational approach to account for the perceived complexity of patterns like those in Fig. 1. Palmer (1982, 1983, 1991) offered a subtly different, and more specific, version of this approach. In contrast, Leeuwenberg (1968) and his colleagues (van der Helm & Leeuwenberg, 1991, 1996) have taken a “non-transformational” coding approach to the salience of regularities that determine the perceived complexity of patterns. Both of these

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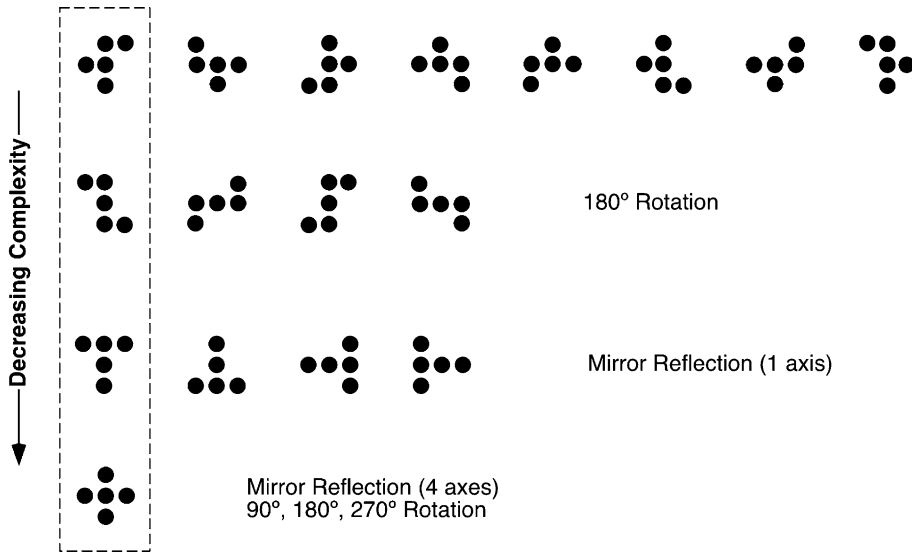


Fig. 1. Four finite patterns in order of decreasing complexity (adapted from Palmer, 1991).

approaches concern pattern symmetries. In the present article, we argue that predicting the judged complexity of a pattern requires an understanding of its perceptual organization that is not entirely predicted by pattern symmetries. We present data on the judged complexity of a class of regular patterns called *band patterns* and show that our data cannot be understood without recognizing the crucial role played by the masking of simple structural relations by grouping.

### 1.1. Symmetry and perceived complexity

A key notion in Garner's (1974) approach to judged complexity is the *inferred set of stimuli*, a general and powerful idea (extended by Kahneman & Miller, 1986). According to Garner, every pattern evokes in the perceiver an implied set of alternative patterns. This implied set is determined by certain properties of the pattern but makes no explicit reference to the perception of these properties. The perceived complexity of a pattern increases with the size of the implied set.

Palmer (1982, 1983, 1991), focusing on the stimuli Garner developed for his research on perceived complexity (Fig. 1), notes that in these stimuli, the size of the implied set is perfectly correlated with the internal structure of the pattern. To describe this structure, one needs the notion of *plane isometry*—a transformation in the plane that does not change the metric properties of the pattern (Fig. 2, see also Martin, 1982). An isometry that leaves a pattern unchanged is a symmetry of the pattern. The results summarized in Fig. 1 suggest that the number of symmetries of a pattern determines perceived complexity for some patterns.

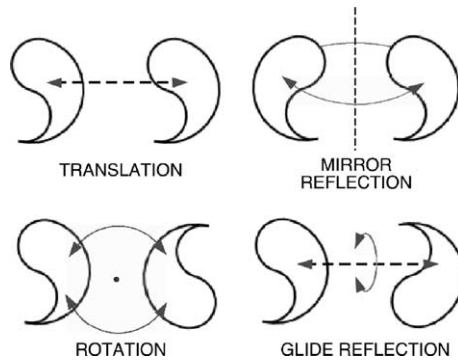


Fig. 2. The four plane isometries. This classification is exhaustive (Martin, 1982, Theorem 8.6).

Palmer (1982, 1983) makes predictions similar to Garner (1970, 1974) but Palmer (1991) goes further by differentiating symmetries and their relative effects on judged complexity. For instance, Palmer found that although the two patterns in the center of the rectangular region in Fig. 1 have the same number of symmetries, the pattern with mirror symmetry was judged as less complex than the one with 180° rotational symmetry. Palmer also examined the perceived complexity of patterns with oblique axes of mirror reflection and found that these symmetries produced effects comparable to 180° rotational symmetry. In summary, the work of Garner and Palmer shows that, although the number of symmetries is the strongest predictor of the perceived complexity of a pattern, different symmetries predict different degrees of complexity.

### 1.2. Beyond finite patterns

The approach taken by Garner and Palmer (henceforth *GP*) deals with patterns that display only two types of symmetry: mirror reflection and rotation. Such patterns are *finite* (Grünbaum & Shephard, 1987). The results obtained by *GP* do not offer an obvious means of predicting the perceived complexity of patterns that exhibit repetition such as those in Fig. 3. This restriction does not apply to the approach taken by Leeuwenberg (1968) and van der Helm and Leeuwenberg, 1991, 1996 (henceforth *LH*) which uses the idea of repetition extensively in coding stimuli.

The *LH* approach evolved from *structural information theory* (Leeuwenberg, 1969) which, like other coding theories, provides a means of generating multiple descriptions of objects or patterns. The simplest of these descriptions or codes is presumed to be the one that is perceived. In other words, a coding theory provides a mechanism by which the Gestalt idea of *Prägnanz* can be realized. *Ipsa facto* it also provides a theory of perceptual organization. Further, the complexity of the code is believed to reflect the complexity of what is described by the code. Leeuwenberg (1968) showed that judged complexity can be predicted by structural information.

A comparison of *GP* and *LH* is hampered by differences in scope and differences in the stimuli they consider. *GP* is both broader and narrower than *LH*. Garner's idea of



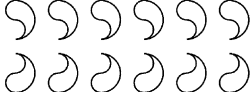



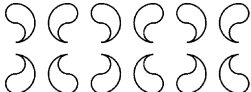
Group-Theoretic Notation		Symmetries
p111		translation
pm11		translation, <i>tm</i> reflection
p1m1		translation, <i>lm</i> reflection
p112		translation, 180° rotation
p1a1		translation, glide reflection
pma2		translation, glide reflection, rotation, <i>tm</i> reflection
pmm2		translation, rotation, <i>tm</i> & <i>lm</i> reflection

Fig. 3. The seven band patterns, their notation and symmetries (transverse and longitudinal mirror reflection abbreviated *tm* and *lm*). This classification is exhaustive (Martin, 1982, Theorem 10.2).

inferred sets is quite general; however, it offers little guidance regarding its application outside the stimuli he studied. Palmer’s group-theoretic version of GP is specific in its formulation: the larger the number of symmetries a form has, the simpler it is. We will see that this specificity restricts its applicability to stimuli to which group-theoretic considerations readily apply. LH, as embodied in coding theory, has been applied to classic problems in form perception, such as predicting how an occluded figure will be amodally completed. It is broader in scope and applicability than Palmer’s version of GP, and it is not restricted to a particular class of stimuli, although it has for the most part been applied to line drawings.

In an attempt to bring the two theories to bear on a common set of stimuli, we chose band patterns because they have some features of the typical stimuli used by both GP and LH. Like the stimuli used by LH, band patterns are repetitive. And like those used by GP, our band patterns are made up of discrete elements. We therefore extend the study of complexity to so-called *infinite* patterns.<sup>1</sup> In

<sup>1</sup> We assume our patterns to be infinite in order to apply the mathematical nomenclature to them. We make no commitment to the perceptual reality of this assumption. However, just as we can perceive the amodal completion of one object occluded by another, we may be able to perceive amodally that a pattern extends indefinitely. We will not hesitate to abandon the mathematical classification of patterns when it is inconsistent with our evidence.

particular, we will study patterns that repeat along one axis (Fig. 3), called *band patterns* (which we will describe in detail in the next section).

The stimuli studied by GP are discrete and can easily be seen as a collection of motifs, whereas LH stimuli most often have continuous contours (except for a finite number of corners), and tend to be seen as single objects. In this respect our stimuli are closer to GP's stimuli than to those used by LH. It is possible that the importance of the perception of repetition in the judgment of complexity applies only to discrete stimuli, and may not generalize to continuous ones. Why is discreteness important? Because translation is perceived as a *between-object* relation, whereas mirror symmetry is perceived as a *within-object* relation (Baylis & Driver, 1995; Bertamini, Friedenberg, & Kubovy, 1997). In this respect glide reflection is similar to translation (glide reflection suggests an object that has been translated and reflected) and rotation is similar to reflection (as Fig. 1 shows, rotation is a property of finite patterns or grouped objects).

According to a naive interpretation of GP, complexity should vary inversely with the number of symmetries of a pattern. Of the patterns in Fig. 3 the one at the top should be the most complex (it has only one symmetry), the next four patterns should be simpler (because each has two symmetries), and the last two should be simplest (because each has four symmetries). Patently, this is not the case.

Since, as Palmer (1991) pointed out, the group-theoretic approach to symmetry does not conform to the way we perceive band patterns, could a modified version of GP allow us to count symmetries to predict complexity? We believe not. Group theory prescribes *global* transformations, whereas we generally perceive *local* transformations (Kubovy & Strother, in press; Palmer, 1982). Group theory requires that the isometries leave the *whole* band pattern unchanged. Consider translation. From a mathematical point of view, a band pattern remains invariant when it is shifted by  $n \cdot |t|$ , where  $|t|$  is the shortest distance between repeated motifs and  $n$  is any integer (which is why band patterns are by definition infinite). But we do not perceive global translation. Instead we perceive a band pattern as a repeating pattern—a string of repeated motifs (such as, *...pppp...*) or a string of repeated strings of motifs (such as, *...pqbdpqbdpqbd...*). This is why van der Helm and Leeuwenberg (1996) prefer the term *repetition* over *translation*.

As a second example, consider band patterns with transverse mirror reflection. According to group theory, a band pattern has transverse mirror reflection symmetry (i.e., it is *pm11*, *pmm2*, or *pma2*), if and only if the band pattern remains invariant when it is rotated 180° in depth *in its entirety* about an axis in the same plane as the axis of translation, but perpendicular to it (for instance, *pma2*: *...pqbdpqbdpqbdpq...*). Here again perception is different: we perceive such a band pattern as pairs or collections of motifs for which mirror reflection is a symmetry (for instance, *...pq bd pq bd pq bd pq...*, or alternatively, *...p qbdp qbdp qbdp q...*, where more closely spaced letters indicate perceptually grouped motifs).

Because of the local nature of symmetry perception we do not foresee a simple relation between the number of symmetries of a band pattern and the perceived complexity. On the other hand, symmetry clearly influences the perceived complexity of

these patterns so we will not abandon the possibility that a modified GP approach will predict the complexity of band patterns.

## 2. An experiment

To assess the perceived complexity of band patterns we designed a paired-comparison study.

### 2.1. Our band patterns and their symmetries

To describe our stimuli, we need to say more about the seven band patterns (Fig. 3). Every band pattern has a *motif*, which is its smallest unit. In our case it is the “half-yin-yang shape” (to which we will return).

As mentioned earlier, band patterns can have up to four symmetries (Fig. 2). The first, *translation*, is unique; it is a symmetry of all band patterns. If we imagine our pattern to be infinite (see the discussion of infinite patterns in the Introduction), we can slide it horizontally until all of the motifs coincide again, leaving the pattern as it was. A standard notation of band patterns consists of four alphanumeric characters, the first of which is always *p*, which stands for *pattern* (but could be taken to mean translation).

If the pattern has a mirror perpendicular to the direction of translation, a *transverse mirror*, the second character of the notation is *m*. A transverse mirror can be thought of as a rotation in depth around an axis in the plane (vertical in our case), perpendicular to the axis of translation. If the band pattern does not have a transverse mirror, the second character is *l*.

If the pattern has a mirror parallel to the direction of translation, a *longitudinal mirror*, the third character is *m*. A longitudinal mirror can be thought of as a rotation in depth around an axis in the plane (horizontal in our case), parallel to the axis of translation. An alternative to a longitudinal mirror is *glide reflection*. When present, the third character is *a*. It is a concurrent translation (along half the distance of the translation in the band pattern) and a reflection in a longitudinal mirror. A glide reflection can be seen in the tracks of a walking person. If neither longitudinal mirror reflection nor glide reflection is a symmetry of the pattern, the third character is *l*.

If the pattern has a 180° rotation (henceforth *rotation*), the fourth character is *2*. The axis of rotation is orthogonal to the plane. It is sometimes called *two-fold rotational symmetry* or a *half-turn*. If rotation is not a symmetry of the pattern, the fourth character is *l*.

Two band patterns in Fig. 3 have two rows. We would have liked to use these in our experiment, except that they would have had twice as many motifs, and might be judged more complex for that reason. This is what Strother (2001) observed in an experiment similar to the one we report here. To avoid this pitfall, we departed from the set of band patterns in Fig. 3 and equated the number of motifs in each pattern by using two sets of stimuli: one-row and two-row patterns. We used the five single-row band patterns (the first five patterns in first column in Fig. 4). From them, we

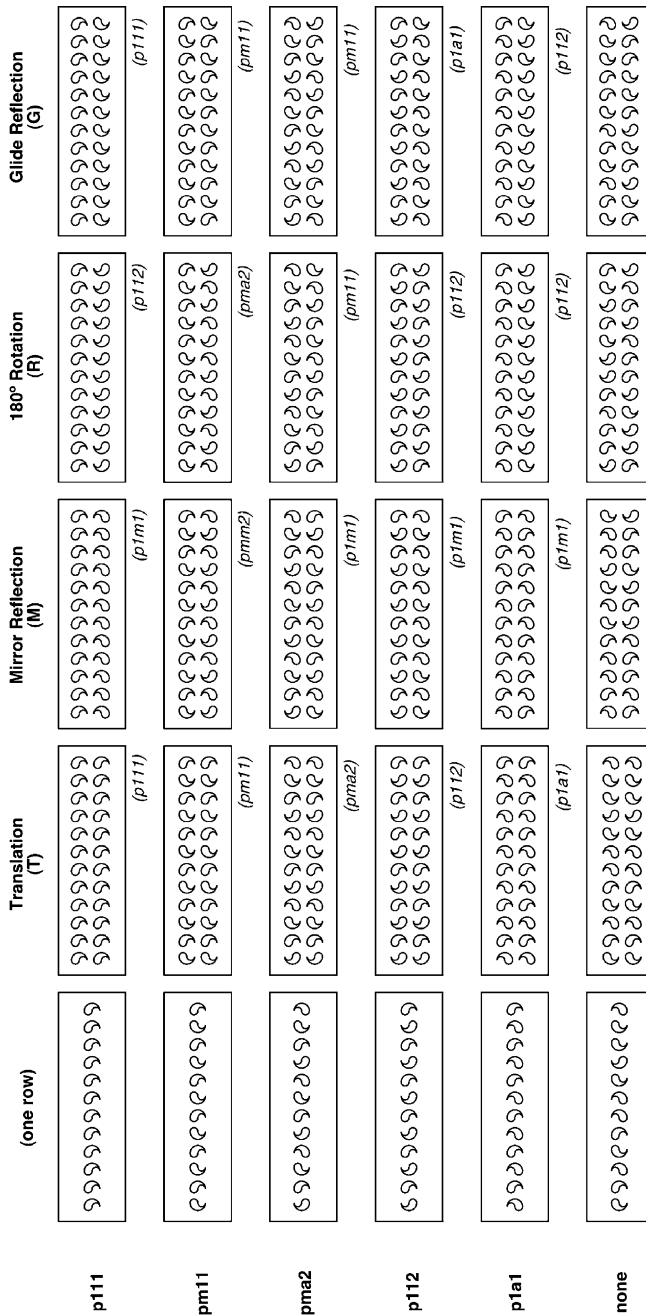


Fig. 4. Rows: five base designs and control patterns (*none*). Columns: patterns with one row of motifs and those with two rows with vertical symmetry relations  $\tau$  and  $M$ . Group-theoretic notation for two-row patterns is shown in parentheses.

generated 20 types of two-row patterns (the first five patterns in the remaining columns in Fig. 4).

To notate these patterns we use a two-part notation. The first part is the standard four-character group-theoretic notation for each row, which we call its *base design*. To this we add a suffix, T, M, R or G, to indicate whether the vertical transformation that relates the two rows was a translation, reflection, rotation, or glide reflection. We could have treated these patterns as standard band patterns (their conventional notation is shown below each pattern in Fig. 4). We do not, however, because in this set of stimuli patterns with the same group-theoretic notation often look different, and patterns with different group-theoretic notations sometimes look alike (Kubovy & Strother, in press). This is because *local* transformations, or “local symmetries” (Palmer, 1982), between pairs of motifs in band patterns are perceptually relevant even if they are not symmetries of the pattern (Kubovy & Strother, in press). We do not claim that the two-part notation conveys the perceptual subtleties of these patterns. It does, however, capture all local transformations within rows because they are always symmetries. In sum, it is more refined than its conventional group-theoretic alternative and will prove to be more useful to our study.

## 2.2. Experimental band patterns

We chose our motif, the half-yin-yang, because of its *asymmetry*. We conjecture that this is the *most asymmetric*<sup>2</sup> *simple shape*.<sup>3</sup> The figure is easily described (just three 180° circular arcs), but it is not a “good” figure in the GP sense because its rotation and reflection set is of size 8 (see Fig. 1).

To our set of stimuli we added patterns of maximal complexity. They consisted of one or two rows which have no discernible horizontal structure (bottom row of Fig. 4); we labeled them *none*. In the two-row *none* patterns, the motifs in the two rows were identical (*noneT*), reflections of each other (*noneM*), 180° rotations of each other (*noneR*), glide reflections of each other (*noneG*), or unrelated *none2* (not shown).

## 2.3. Method

Fig. 5 shows a typical display. To generate all possible patterns, we began with a base design that faded into the background at both ends. We then generated four versions of the base design by rotating and reflecting the motif. We next shifted the position of the base design horizontally by a random amount. Finally, as mentioned earlier, we generated the second row of the two-row patterns by applying one of the four transformations.

We asked observers to assess which of the two patterns was simpler or whether the two were similar in complexity by choosing one of five response levels. We offered

<sup>2</sup> This is admittedly a vague term, which we would be happy to see defined precisely.

<sup>3</sup> This is another vague term, here defined as a closed curve that has only one point of inflection and one discontinuity.

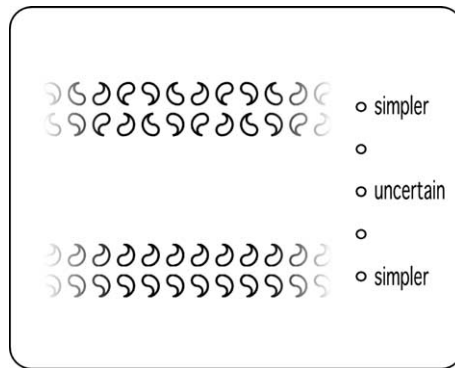


Fig. 5. Example of stimulus pair and response options. On the screen all contours were white on black background. Patterns subtended 10° of visual angle (dva) horizontally, and the two-row ones subtended 2.5 dva vertically. Pairs of band designs were always separated by at least 5 dva. Motifs subtended 0.7×1.0 dva.

them a heuristic, “A pattern is complex if it would take many words to describe to a friend, and simple if it would take few words”.

Patterns were paired at random during the experiment, which consisted of two one-hour sessions. During the first part of each session observers compared 36 pairs of single-row patterns. They then made judgments of 465 pairs of two-row patterns. Thus observers always compared pairs of patterns with the same number of rows.

Ninety-four undergraduate students from the University of Virginia participated for course credit.

## 2.4. Results

### 2.4.1. Treatment of data

We treated the responses as ratings of relative complexity. Call one of the two patterns presented on a trial the *focal pattern* and the other the *comparison pattern*. The code for each of the five responses is summarized in Table 1. We scaled the complexity of each pattern by considering it focal and averaging the coded responses for all pairs in which that pattern appeared (Thurstone, 1927). From the pattern’s *scaled*

Table 1  
Coding of the judged complexity (C) of patterns *a* and *b*

C(a)?C(b)	Code when		C(b)?C(a)
	<i>a</i> is focal	<i>b</i> is focal	
≪	0.00	1.00	≫
<	0.25	0.75	>
=	0.50	0.50	=
>	0.75	0.25	<
≫	1.00	0.00	≪

complexity we obtained its mean scaled complexity (MSC) by averaging over observers. We excluded comparisons of patterns with themselves from computations of MSC, because they never differed from 0.50.

#### 2.4.2. Analysis of one-row and $\tau$ two-row patterns

Fig. 6 suggests that the MSC of two-row patterns with identical rows (those in column 2 of Fig. 4) is no greater than the complexity of single-row patterns (those in column 1, Fig. 4). This is an encouraging finding, considering that we obtained the two sets of data in different blocks of the experiment. The single-row MSC values summarize all the data from single-row patterns (two blocks of 36 trials), whereas the two-row MSC values summarize about 7% of the two-row comparisons in which both patterns fell in the  $\tau$  category (out of two blocks of 465 trials). In our single-row data, we included the unstructured *none* patterns. In the two-row data, we included comparisons with the least-structured *none2* patterns (and excluded comparisons

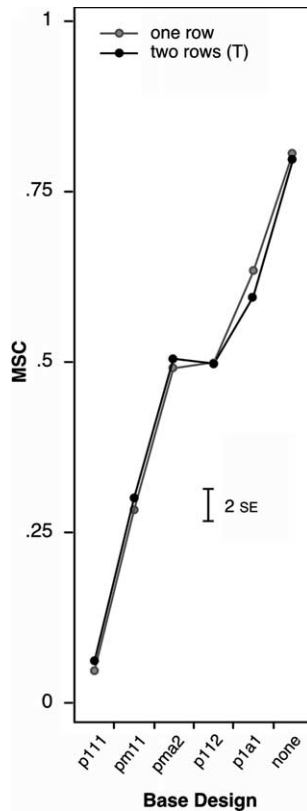


Fig. 6. Mean scaled complexity (MSC) for patterns with one row and those with two identical rows ( $p \cdots \tau$ ). Error bars represent 2 standard errors, averaged over conditions (they did not differ materially from condition to condition).

with *noneT* patterns, which have some structure). After a repeated measures ANOVA with *base design* and *number of rows* as nominal factors, a post hoc analysis—Tukey's HSD—confirmed our impression that the number of rows had no effect for any of the five base designs.

What is the effect of base design on judgments of complexity? Our post hoc analysis showed that all differences in MSC between base designs were significant, except for *pma2* and *p112*. If we conjecture that the more easily one perceives the translation in a band pattern, the simpler it is, we can explain the ordering of the MSC values of the patterns. For ease of exposition, we will concentrate on single-row patterns.

*MSC(p111)* is lowest. *p111* has only one form of the motif, whereas all other patterns have two or four. Thus translation is most easily perceived in *p111*.

*MSC(pm11) < MSC(p112)*. A 180° rotation is less visible than reflection. Therefore, all other things being equal, it is more likely that an observer will group two objects related by mirror symmetry, than by 180° rotation (Mach, 1914/1996). Hence groupings of pairs of motifs are more readily formed in *pm11* than in *p112*. When such groupings ( $\Psi$ -motifs) emerge, the fact that they can be matched by translation becomes perceptible.

*MSC(pma2)  $\approx$  MSC(p112)*. The complexity of these two patterns is about equal, because they are hard to distinguish. When observers are trained to identify symmetries in band designs (Strother, 2001), they successfully detect the translation and the rotation in *pma2* and *p112* patterns (80% correct), but they often fail to detect the mirror symmetry or glide reflection in *pma2*. Thus, *pma2* patterns often look like *p112* patterns.

*MSC(pma2)  $\approx$  MSC(p112) < MSC(pla1)*. We conjecture two causes for this difference. (1) Glide reflection is less visible than rotation, because, (a) it is unfamiliar, and (b) it is seen as a compound transformation: reflection + translation. (2) Because *pla1* offers no opportunities for grouping, it is more complex. Grouping creates  $\Psi$ -motifs, which simplify a pattern. *pma2* and *p112* offer opportunities for grouping (both tend to group by closure around their centers of rotation, and *pma2* can also group around its transverse mirrors).

#### 2.4.3. Two-row patterns: comparison of *M*, *R*, and *G* to *T*

Having shown that the values of MSC for single-row patterns and two-row patterns with identical rows are similar, we used the latter as a control to ask whether the MSC profiles within vertical transformations are different. We based our inferences on a repeated measures ANOVA with *base design* and *vertical transformation* as nominal factors. The fit accounted for  $R^2_{\text{adjusted}} = 26\%$  of the variance. The main effects and the interaction were highly significant with large effect sizes (*base design*:  $F[5, 465] = 470$ , *vertical transformation*:  $F[3, 279] = 225$ , *interaction*:  $F[15, 1395] = 35.5$ , all  $p < 0.0001$ ).

Fig. 7 will help us examine the interaction between base design and vertical transformation. We first concentrate on the lower left panel. It represents the MSC for *M* (solid dark line), compared to the MSC for *T* (solid gray line). To better see the interaction, we take the difference between the MSC for *p111T* and the MSC for *p111M* as a

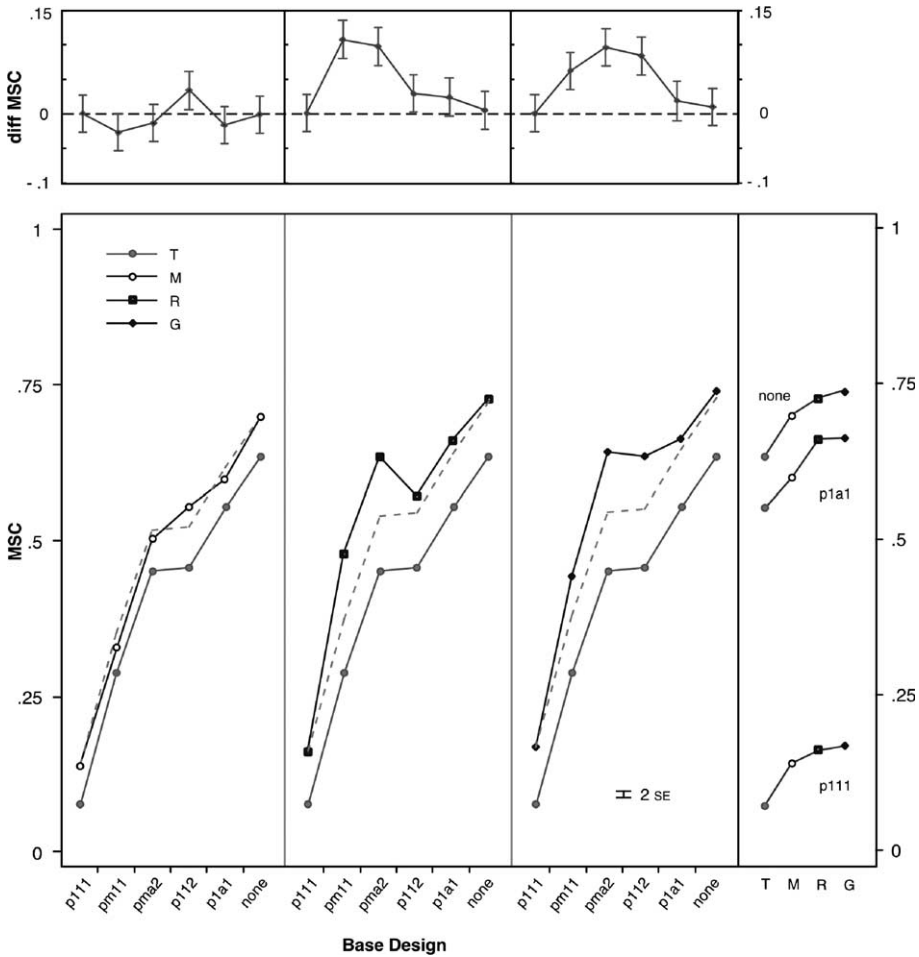


Fig. 7. Mean scaled complexity (MSC) as a function of band pattern and vertical transformation. In the three lower panels, the graph for  $\tau$  (solid gray line) is repeated, and used as a reference. The solid dark lines represent the MSC for  $M$  (left panel),  $R$  (middle panel), and  $G$  (right panel). In each panel we shift the reference graph for  $\tau$ , so that the MSC for  $p111\tau$  coincides with the MSC for  $p111M$  (left panel),  $p111R$  (middle panel), or  $p111G$ . The dashed gray line is the shifted function for  $\tau$ . Error bars indicate 2 mean standard errors. In the upper panels, we present the difference between the shifted reference function and the function for the other vertical transformation (*diff MSC*). The vertical lines are conservative estimates of 95% confidence intervals for the differences between the two curves; if they do not straddle 0.0, the difference is significant.

measure of the difference in complexity between the  $\tau$  and  $M$  vertical transformations, which is not of concern here. We therefore removed this difference by shifting the function for  $\tau$  so that the MSC for  $p111\tau$  was equal to the MSC for  $p111M$ . The dashed gray line is this shifted function. The upper left panel shows the difference between the shifted  $\tau$  function and the  $M$  function. The vertical lines are conservative

estimates of 95% confidence intervals for the differences between the two curves; if they do not straddle zero (in Fig. 7, top) the difference is significant.

The three upper panels of Fig. 7 show that six band patterns are more complex than we would predict from the shifted graph for  $\tau$  (dashed line):  $p112M$ ,  $pm11R$ ,  $pma2R$ ,  $pm11G$ ,  $pma2G$ , and  $p112G$ . None of the patterns was simpler than predicted by the reference function.

The interaction we have just described is generated by the second, third and fourth band patterns of Fig. 4. The right-hand panel of Fig. 7 shows the interaction plot for the other three band patterns. (The principal independent variable here is vertical transformation.) The data for  $p111$ ,  $p1a1$ , and *none* are essentially additive:  $\tau$ ,  $M$ ,  $R$ , and  $G$  produce equal increments of complexity for these three band patterns.

## 2.5. Discussion

We first observe that of the 26% of the total variance accounted for by our linear model, 93.9% was accounted for by the additive effects of our factors, base design and vertical transformation (which accounted for 78.4% and 15.5% of the variance respectively). We first discuss the main effects, and then turn to the interaction.

### 2.5.1. Effect of base design (single-row patterns) on judged complexity

We cannot explain the effect of base design in the spirit of a literal GP approach. Neither the inference of a set of stimuli from which these band patterns were drawn, nor a counting of symmetries suggest a parsimonious account of these data. Instead, they can be explained by four hypotheses, the first of which is consistent with Palmer's (1991) observation that different symmetries have different effects on judged complexity and is probably related to van der Helm and Leeuwenberg's (1991, 1996) characterization of translation (repetition), mirror reflection, and rotation:

1. The visibility of local symmetries is ordered. The most visible local symmetry is translation, followed by reflection, 180° rotation, and glide reflection.
2. The more visible a local symmetry (other than translation), the more likely it is to group two (or more) motifs into a  $\Psi$ -motif.
3. If all motifs are *either* identical ( $p111$ ) *or* grouped into  $\Psi$ -motifs, then the band pattern is likely to be exhaustively parsed into identical motifs or  $\Psi$ -motifs.
4. The more likely it is that a band pattern will be seen to be exhaustively parsed into identical motifs or  $\Psi$ -motifs, the simpler it is judged to be.

### 2.5.2. Effect of vertical transformation on judged complexity

The ordering of the effects of the four transformations is  $\tau$ ,  $M$ ,  $R$ , and  $G$ . We reached this conclusion by relying on the  $p111$ -,  $p1a1$ - and *none*- two-row patterns, for which there is no interaction between the effect of GP and vertical transformation. This order is just the one which we had proposed for the visibility of the local symmetries (hypothesis 1 in the preceding paragraph). In other words, the more visible the relationship between the two rows in a pattern, the simpler the pattern.

2.5.3. *The interaction*

We find new clues to perceived complexity in the interaction between our independent variables. We summarize our account of the interaction in Table 2. The column to the left of the “<” includes patterns whose complexity was predicted by the additive combination of the main effects. It lists factors that *may* be responsible for preserving the level of complexity. The right-hand column includes patterns responsible for the interaction. It lists factors that *may* be responsible for increasing the level of complexity.

Although this account of the interaction is not unique, it leads to an important insight: *grouping can mask simple relations*. An example of this phenomenon is described in Table 2: in both *pm11G* and *p112R*, the rows are identical but offset by  $0.5 \cdot |t|$ . So how is it that *pm11G* is one of the patterns that deviates from additivity, whereas *p112R* does not? Because *pm11G* has  $2 \times 2$   $\Psi$ -motifs, whereas *p112R* does not. In other words, the presence of grouping makes it difficult to perceive a relationship within the pattern; observers would see the relationship only if they could disregard the grouping.

This phenomenon is not confined to our stimuli. It is related to a venerable Gestalt insight that Kanizsa (1979) described so well in his chapter on “Prägnanz as an obstacle to problem solving.” There Kanizsa shows how some problems of geometry are difficult not because of a conceptual difficulty, but because a good perceptual organization must be broken before these problems can be solved.

Table 2  
An account of the interaction (see Fig. 4)

$\Psi$ -motifs are formed from pairs of motifs around transverse mirrors within rows; the mirrors are aligned between rows, and form $2 \times 2$ $\Psi$ -motifs		
MSC(T,M)	<	MSC(R,G)
<p><i>pm11</i> and <i>pma2</i></p>	<p>The relation between rows in the <math>\tau</math> and <math>\mathbf{M}</math> <math>\Psi</math>-motifs is easy to describe. For example, “same” for <math>\tau</math> and “upside-down” for <math>\mathbf{M}</math>.</p>	<p>The relation between rows in the <math>\mathbf{R}</math> and <math>\mathbf{G}</math> <math>\Psi</math>-motifs is hard to describe. For example, “inside out” for <math>\mathbf{G}</math> and “inside-out and upside-down” for <math>\mathbf{R}</math>. (The perception of <math>2 \times 2</math> <math>\Psi</math>-motifs reduces the salience that the two rows in <i>pm11G</i> are identical but offset by <math>0.5 \cdot  t </math>, where <math> t </math> is the translation distance.)</p>
$\Psi$ -motifs are formed from pairs of motifs around centers of rotation within rows; no $\Psi$ -motifs are formed between rows.		
MSC(T,R)	<	MSC(M,G)
<p><i>p112</i></p>	<p>For <math>\tau</math> the rows are identical. For <math>\mathbf{R}</math> the rows are identical but offset by <math>0.5 \cdot  t </math>. These relations are easy to see because the linkage between rows is perceptually weak (which is not the case for <i>pm11G</i>).</p>	<p>For <math>\mathbf{M}</math>, and <math>\mathbf{G}</math>, the relation between the rows is hard to describe.</p>

### 3. Conclusion

We have seen that the number of symmetries of a band pattern does not predict its perceived complexity. More importantly, although base design predicts some of the observed trends in the judged complexity of our band patterns, we have seen that perceived complexity is not always easily predicted by the *kinds* of symmetries of each band pattern. This is because the perceptual “potency” of symmetries does not predict the emergent properties, whether these are true symmetry properties or the psychological counterparts of symmetry.

An objection to our analyses might be that our system of classifying band patterns, in terms of base design and vertical transformation, does not adequately discriminate local and global symmetries of our stimuli—a purely psychological distinction since symmetries are global by definition. If grouping masks some symmetry relations in favor of others then we must know which symmetries are perceived before we can use these symmetries to predict perceived complexity. LH could conceivably reproduce our additive model *a priori* by coding our stimuli. However, LH could not *in principle* account for the remaining variance. LH is designed to *account* for perceptual organization, and hence for perceived complexity. In order to do so it would have to account for the fact that grouping can mask simple relations. We have shown that finding a code for the complexity of a stimulus requires a prior understanding of the laws of perceptual organization. And, since a theory that is based on a set of laws cannot possibly explain them, LH will not, in the end, work for all of our data. We are in favor of improving our current code for band patterns. The new code must be informed by the workings of the visual system as it encounters these patterns. In doing so we will approach a more complete theory of perceptual organization.

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