The Psychology of Perspective and Renaissance Art

Michael Kubovy
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Chapter 2

The elements of perspective

Here the total artifice reveals itself
As the total reality.

Wallace Stevens, from “Someone Puts a Pineapple Together,” 1947 (Stevens, 1972, p. 299)

Look at Masaccio’s Trinity (Figure Trinity), the oldest surviving painting that uses perspective rigorously. Why it looks compellingly three-dimensional will be explained in Chapter 3. In this chapter, we will discuss the geometry that underlies perspective; in Chapter we will look at the origins of perspective.

We, in the late twentieth century, take photography for granted as the prototypical physical embodiment of picture taking, and perspective as its mathematical model. But for the artists-scientists of the Renaissance the introduction of perspective required a complex mesh of innovations: They had to define the very concept of taking a picture, to understand the optics implied by this definition, to abstract the geometry underlying the optics, and finally to discover ways of translating these abstractions into practical rules of thumb that anyone could apply in order to draw scenes in perspective. In the Introduction, we have already encountered Alberti’s description of his key concept of taking a picture. Let me quote it again:

First of all, on the surface on which I am going to paint, I draw a rectangle of whatever size I want, which I regard as an open window through which the subject to be painted is to be seen. (See Figure 2.2)

Before we analyze the relation between the Alberti window and perspective, a distinction must be made between the study of perspective as the theory of picture taking and the practice of drawing in perspective. In the remainder of this chapter, we will deal mostly with the theory of perspective, that is, with the nature of the geometric transformation that allows us to represent a three-dimensional scene on a two-dimensional surface, and with certain features that all perspective representations have in common. After presenting these notions in some detail, we will dwell briefly on the procedure that Alberti invented, but because this is a very complicated topic to which numerous textbooks have been devoted, the reader should not expect to learn from this book how to draw a perspectively correct representation.

Perspective is easiest to understand once we are familiar with the camera obscura (“dark chamber” in Latin) illustrated in Figures 2.3 and 2.4. Although the issue is shrouded in uncertainty, there is some evidence that the device was invented by Alberti (Pastore and Rosen, 1984). It is no more than a box, or a room, with a relatively small hole in it, called pinhole. a the box is to serve its purpose as a camera obscura, light should not enter it except through the pinhole. The side of the box opposite the pinhole is called the picture plane. If the picture plane is painted white and all the other sides are lined with light-absorbing black velvet, we can be sure that all the light that

1I have found Gill (1974, 1975) very useful in this regard.
falls on the picture plane has traveled in a straight line from an object outside the box through the pinhole and that none of it has been reflected from the walls. So, moving into geometry, a camera obscura creates an image $x$ of an object point $v$ by ensuring that one and only one ray of light, called a projecting ray, coming from $v$ hits the picture plane at $x$ after passing through the pinhole. Unless the camera obscura is a room in which a spectator can stand and look at the picture plane (in which case the picture will be both large and very faint), we must devise a way of showing the picture it takes. There are two ways to do that: Either replace the wall of the picture plane with a piece of ground glass and view the image from outside, or replace it with a photosensitive plate that can be developed into a photograph. In the latter case, we will have a pinhole camera, that is, a photographic camera with a pinhole for a lens.

A camera obscura does not correspond exactly to Alberti’s window, for it inverts right for left and top for bottom (as Figure 2.3 shows). To understand the basis of perspective as discovered by Alberti, consider Figure 2.5. We project a point $M$ onto a point $m$ on a front picture plane ($P$), replacing the pinhole with a center of projection ($O$), and replacing the optical process underlying the camera obscura with a geometric transformation called central projection. It is

\[\text{Figure 2.1: Masaccio, Trinity (ca. 1425). Fresco. Church of Santa Maria Novella, Florence.}\]

\[\text{Figure 2.2: Representation of Alberti’s window (perspective drawn using a front picture plane). Engraving (modified) from G. B. Vignola, La due regole della prospettiva practica, 1611.}\]
essential to keep in mind that the scheme of central projection is a convenient geometric fiction: No optical system can be devised to perform such a transformation in this way.\footnote{Such a projection onto a front picture plane would be physically realizable only by a two-stage optical process: The first stage would essentially be a camera obscura projecting a scene onto a rear picture plane, which produces an upside-down picture, and then projecting the upside down picture again to produce an upright picture.} Because central projection is a geometric abstraction, it can be misleading to link it too closely with vision — which is based on an optical process — as textbooks almost invariably do. It is better, I believe, to first comprehend perspective as a purely geometric procedure for the representation of a three-dimensional world on a two-dimensional surface before one tries to understand its Perceptual effects on viewers. One should avoid thinking of it as a method that mimics, in some sense, what we see.

We will see later (Chapter 4) that there is a sense in which a perspective representation mimics what could be seen from a certain point of view, but I have found it useful to postpone that interpretation of perspective and to consider it in the course of an exploration of the reasons why a perspective picture looks compellingly three-dimensional (see Chapter 4). The traditional representation of Figure 2.1, which places an eye at the center of projection and calls the center of projection the “vantage point” or the “station point,” is a convenient pedagogical device except that it takes the step of relating vision to central projection before the purely geometric nature of the transformation is made clear.

Up to this point, we have been talking as if central projection and perspective were the same. The truth is that central projection is a somewhat more gen-

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**Figure 2.3:** Earliest known illustration of a camera obscura. Engraving from R. Gemma Frisius, *De radio astronomico et geometrico liber*, 1545. The legend translates: “Observing solar eclipse of 24 January 1544.”

**Figure 2.4:** Geometry of the camera obscura

**Figure 2.5:** Main features of central projection

**Box 2.1:** Drawback of the pinhole camera

The main drawback of the pinhole camera is the dimness of the image it creates. If one enlarges the aperture, the image becomes blurred. In modern cameras, lenses are used to focus the light coming in through a relatively large aperture, thus assuring a sharply defined image.
eral geometric transformation than perspective. In central projection, one defines (see Figure 2.5) a picture plane $P$ and a center of projection $O$ (which is not contained in the plane $P$). The central projection of any point (distinct from $O$) is the point $m$, which is the intersection of the projecting ray $OM$ with the plane $P$. It is important to keep in mind that this definition does not restrict the location of $M$ with respect to $P$: It could be anywhere in space relative to the picture plane — in front of it or behind it. Now because perspective is a model of the picture-taking process, it restricts itself to a $180^\circ$ field of view from the vantage point of $O$, pointing toward the picture plane. That is why perspective applies only to the projection of points $M$ contained in the half-space (an infinite region of space bounded by a surface) that contains the picture plane $P$ and that is bounded by a plane $P'$ that contains the center of projection $O$ and is parallel to the picture plane $P$. This much constraint upon the geometry is imposed by the optical nature of the phenomenon we are trying to model: The maximum field of a pinhole camera is $180^\circ$. This formulation of perspective is still somewhat more general than Alberti’s window, which implies that only objects behind the window can be projected onto the picture plane. Indeed, in the large majority of the cases, perspective is restricted to the region beyond the window.

There are two ways of formulating this point, geometric and perceptual. Geometrically speaking, the points $M$ are limited to the closed half-space which does not contain $O$, and is bounded by the picture plane, $P$ (see Figure 2.5). The geometric evidence for this point can be found in the size of the depictions of known objects. The geometry of perspective implies that the painting of an object which is in front of the picture plane will be larger-than-life; since Renaissance painters very rarely painted larger-than-life figures, most figures must be behind the picture plane.\footnote{You must make the foremost figure in the picture less than the size of nature in proportion to the number of braccia at which you place it from the front line…” (Leonardo da Vinci, 1970, 538, p. 324).}

Psychologically speaking, most pictures look as if they were seen through a window — none of the objects seems to pop out into the space of the room; the scenes they depict appear to be entirely behind the surface of the painting. The evidence for this psychological observation comes from the exception to this rule. In these rare exceptions, we see the lengths to which an artist must go to coax the spectator into relinquishing the assumption that the entire scene is behind a window. Take for instance an anonymous illumination of the twelfth century (Figure 2.6). Because the fish occludes part of the frame, we assume that it is flying in front of the page. But to attribute this perceptual effect to the occlusion of a section of the frame implies that, in the absence of this device, the scene would be perceived to be entirely beyond the page. In other words, even in pre-perspectival pictorial representations, we tend to perceive a picture frame as the frame of a window through which we can look into the virtual space depicted by the picture. One of the most brilliant applications of this method is in Jan van Eyck’s Annunciation (Figure 2.7). In this diptych\footnote{A diptych is a pair of painted panels hinged together.} we see the angel and Mary represented in a gray simulation of figures sculpted in the round, standing on octagonal pedestals. Behind them, just touching the far surface of the pedestals, we see a black mirror-like surface in which the statues appear to be reflected, framed by dark moldings, part of which are occluded by the angel’s left wing and by Mary’s cloak.

It is possible to achieve the same effect by propagation, that is, to have object $A$ occlude the frame, and to suggest (whether by occlusion or other means) that object $B$ is in front of object $A$: The result is that object $B$ seems to be in front of the picture plane. Such is one of the interesting perspective effects used by Mantegna in his frescoes for the Ovetari Chapel (Figure 2.8) Martyrdom of Saint James, the railing appears to be attached to the front of the picture frame; that is why the torso of the soldier leaning over it appears to emerge into the space of the chapel, above the floor onto which Saint James’s head will roll when it is severed.

Returning now to our exposition of the elements of perspective, there are three geometric properties of central projection that we must understand in or-
der to proceed with our analysis of the psychology of perspective.

Proposition 1. The perspectival image of a straight line that does not pass through the center of projection is always a straight line (the image of a straight line that does pass through the center of projection, i.e., of a projecting ray, is a point).

The second property of perspective concerns the representation of sets of parallel lines (such as you would have to contend with in drawing a box). First, consider a set of vertical lines. If the picture plane is not vertical, the images of all vertical lines converge onto the one and only vertical vanishing point. Figure 2.9 illustrates this property as it occurs in a picture of a box projected onto a picture plane tilted sharply downward. Next, consider a set of horizontal lines. They too converge onto a vanishing point. To specify the location of this vanishing point, we must first define the horizon line of the picture (see Figure 2.10), which is the line defined by the intersection the picture plane (which need not be vertical) and a horizontal plane that contains the center of any two horizontal lines that are parallel to each other intersect the horizon line at the same point, which is their horizontal vanishing point.

Proposition 2. The perspectival images parallel lines that are also parallel to the picture plane are parallel to each other; the perspectival images parallel lines that are not parallel to the picture plane converge onto a vanishing point (which is not necessarily within the confines the picture).

It can be confusing at first to realize that when one looks at a picture like Figure 2.9 one cannot tell whether the box was tilted and the picture plane was vertical, or the box was upright and the picture plane tilted (as we described it to be). Had we de-
CHAPTER 2. THE ELEMENTS OF PERSPECTIVE

Figure 2.8: Mantegna, *Martyrdom of Saint James* (1451–5). Fresco. Ovetari Chapel, Eremitani Church, Padua.

scribed it as the picture a tilted box projected onto a vertical picture plane, then we would have had to re-label the vanishing points, for there would be neither horizontal nor vertical vanishing points and it would therefore be incorrect to label a line connecting two the vanishing points “horizon.”

This point can be further clarified by noting that in Figure 2.2 one projecting ray is singled out and given a name its own: It is the principal ray, that is perpendicular to the picture plane. (It is along this line that one measures the distance between the center of projection and the picture plane.)

In Figure 2.4 the principal ray intersects the picture plane at the horizon. This is true only in the special case when the picture plane is vertical. Nevertheless, although it is inappropriate to call a horizontal line drawn through the foot of the principal ray the “horizon,” this line is meaningful and important. It is the locus of the vanishing points of lines orthogonal to the picture plane and parallel to each other.

We are now ready for the third property of perspective:

**Proposition 3.** The location of an object point cannot be determined uniquely by its image. However, it is possible, by making assumptions about properties of the scene, to solve the inverse problem of perspective, namely, given the central projection of a scene, to reconstruct its plan and elevation.

For instance, Sanpaolesi (1962) proposed the

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Alberti (1966, p. 48) called the principal ray the “prince of rays.”

This problem was analyzed in great depth by Jules de la Gournerie (1814–83) in his monumental *Traité de Perspective Linéaire* (1884). Methods such as his have been applied to a fair number of works of Renaissance art. For a recent bibliography, see Welliver (1973).

Although Janson (1967, p. 88, footnote 25) argues convincingly that Sanpaolesi’s reconstruction contains errors, I have chosen to reproduce his rather than Janson’s because its elevation shows the location of the figures, whereas Janson’s shows
reconstruction of Masaccio’s *Trinity* (Figure 2.1), shown in Figure 2.11.

To gain some insight into what Alberti taught his contemporaries, we should examine what Renaissance artists came to call the *construzione legittima* (legitimate construction) of the perspective drawing of a pavement consisting of square tiles. Figure 2.12 summarizes the geometry underlying the construction. To carry it out, one needs a plan (view from above) and an elevation (side view) of the pavement, on which are indicated the picture plane and the center of projection. In Figure 2.12 have numbered the steps involved in constructing the perspective: (1) Use the elevation to draw the horizon. (2) Use the only the architecture. (See also Battisti, 1971.)
Figure 2.11: Plan and elevation of Masaccio’s *Trinity* (according to Sanpaolesi, 1962, figure C, opp. p. 52)

plan to determine the vanishing point on the horizon. (3) Use the plan to mark off the front of the pavement on the bottom of the picture. (4) Connect these points to the vanishing point. (5) Transfer the locations of the tiles from the plan to the elevation. (6) Connect these points to the center of projection in the elevation. (7) Transfer the intersections of these lines with the picture plane in the elevation. Figure 2.13 shows how Leonardo represented the procedure. Because the tiles in the pavement are square, Alberti (and Leonardo after him) was able to combine steps (3) and (5).

To verify the correctness of the construction, Alberti recommended that the artist draw the two sets of diagonals of the square tiles. Because each set consists only of parallel lines, each should converge to a vanishing point on the horizon; these are the *distance points* \( D_1 \) and \( D_2 \) in Figure 2.12. These distance points are important for a reason suggested by their name: In the *construzione legittima*, the distance between the vanishing point and each distance point is equal to the distance \( d \) between the center of projection and the picture plane (see Box 2.2).

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*The distance points are known as a *conjugate pair* of vanishing points. For future reference, I wish to define this term: The perspective images of any two lines pass through their respective vanishing points. If the lines to be represented intersect, and if the angle of their intersection is a right angle,
Figure 2.12: Construction of perspective representation of a pavement consisting of square tiles
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Box 2.2: The distance between the vanishing point and a distance point equals the distance between the center of projection and the picture plane.

To demonstrate this fact, find the central projection of a horizontal line, passing through the center of projection O and forming a 45° angle with the picture plane (line 9 in the plan of Figure 2.12). Because, as we have seen, all lines that pass through the center of projection are represented as a point, the representation of this line is the intersection of line 9 with the picture plane. This intersection is D₁ for line 9 is parallel to the diagonals that converge at that point. Now consider the triangle OVD₁. Because it is a right triangle with one 45° angle, it is isosceles; and because the length of OV is d, the length of D₁V is also d. QED.

their respective vanishing points are said to form a conjugate pair.