

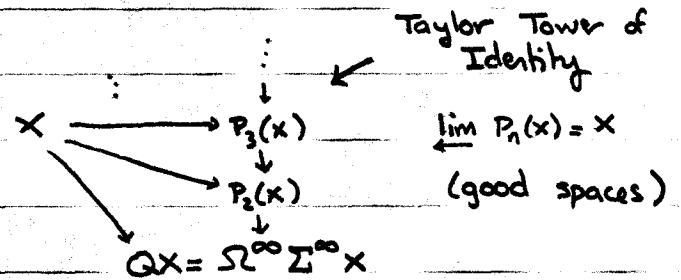
Delooping the Connecting Maps in the Taylor Tower

Greg Arone
1/18/07

(Joint w/ W. Dwyer & K. Lesh)

I. Given X , get a tower of fibrations

Gives a way to pass from stable to unstable information



Know a lot about this:

$$D_n(X) = \text{Fiber}(P_n(X) \rightarrow P_{n-1}(X))$$

$$\Omega^\infty((W_n \wedge X^n)_{hZ_n}) \quad , \text{ so just like } \frac{1}{n!}$$

derivative X^n

the Taylor series. Since these are ∞ loop spaces, have maps

$$P_n(X) \leftarrow D_n(X)$$

$$\downarrow$$

$$BD_n(X) \leftarrow P_{n-1}(X)$$

\Rightarrow connecting (k -invariant style) maps $D_{n-1}(X) \rightarrow BD_n(X)$

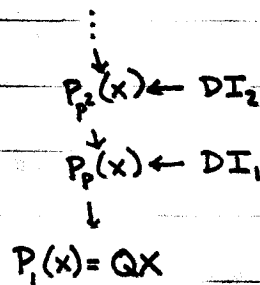
This gives the d_i differential in the SS for $\pi_*(X)$, using the tower.

Fix a prime $p \neq 2$ let $X = S^m$, m odd, $p > 2$, m anything, $p = 2$.

Thm In this case, $D_n(X) \simeq *$ unless $n = p^k$.

We can then regrade the tower

$$DI_k = D_{p^k}(X)$$



Want a connecting map, and since

$$P_{p^k}(X) \xrightarrow{\simeq} P_{p^{k-1}}(X)$$

$$\downarrow \quad \uparrow$$

$$BDI_k \leftarrow \alpha_{k-1} \leftarrow DI_{k-1}$$

this is our connecting map

Have another functor

$$V \longmapsto BU(V)$$

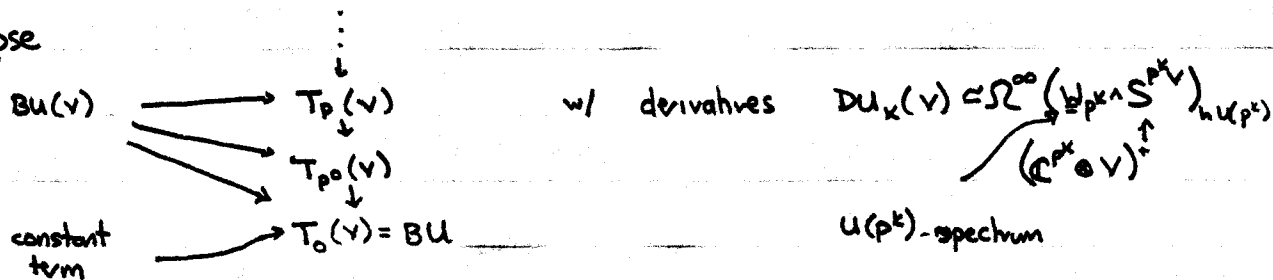
\mathbb{C} -vect \longrightarrow Spaces
100% product

Have a Weiss calculus starting with

$$BU = \varinjlim BU(V)$$

after p -completing, again have faster

collapse



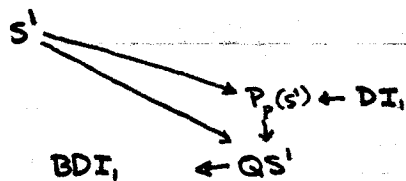
Again get connecting maps $DU_k \xrightarrow{\beta_k} BDU_{k+1}$

These are not infinite loop maps but

Thm $\alpha_k \neq \beta_k$ are k -fold loop maps.

i.e. $\exists \tilde{\alpha}_k : B^k DI_k \rightarrow B^{k+1} DI_{k+1}$ w/ $\alpha_k \simeq \Omega^k \tilde{\alpha}_k$

Example S^1 (even for this, α_k not ∞ -loop map)



Have a sequence of spaces using the deloopings:

$$S^1 \xrightarrow{\alpha_1} QS^1 \xrightarrow{\alpha_2} BDI_1 \xrightarrow{\alpha_3} B^2 DI_2 \rightarrow \dots \quad \xrightarrow{\alpha_k} = \text{not } \infty \text{ loop maps}$$

f_k : $\leftarrow \dots = \infty$ loop map coming from $Sp^\infty(X) \simeq K(H_k X) \rightsquigarrow$

$$S^0 \hookrightarrow Sp^2(S^0) \hookrightarrow \dots \hookrightarrow Sp^\infty(S^0) = H\mathbb{Z}$$

gives a filtration of $H\mathbb{Z}$. Also have (p -locally) $Sp^n(S^0)/Sp^{n+1}(S^0) \simeq *$

unless $n = p^k$. Also get $H\mathbb{Z} \leftarrow S^0 \leftarrow \Sigma^{-1} Sp^p(S^0)/Sp^1(S^0) \leftarrow \dots$

Looping other sequence gives

$$\mathbb{Z} \leftarrow QS^0 \leftarrow \Omega BDI_1 \leftarrow \dots = \Omega^\infty (\text{above tower})!$$

$$\Omega B\mathbb{Z}_p$$

Have an analogous filtration in the orthogonal case:

$$bu \rightarrow A_1 \rightarrow \dots \rightarrow H\mathbb{Z}$$

The Weiss calculus gives

$$\begin{array}{ccccccc} BU(1) & \rightarrow & BU & \rightarrow & BDU_0 & \rightarrow & B^2DU_1 \rightarrow \dots \\ \parallel & & & & \parallel & & \\ \mathbb{C}P^\infty & & & & \mathbb{Q}CP^\infty & & \end{array}$$

taking Ω^2 , get

$$\mathbb{Z} \rightarrow \mathbb{Z} \times BU \rightarrow \mathbb{Q}CP^\infty \rightarrow \Omega^2 B^2DU_1 \rightarrow \dots \quad \begin{array}{c} \xrightarrow{f_1} \\ \xleftarrow{g_1} \end{array}$$

same as the filtration from above

Conjecture: There exist deloopings α_k, β_k that act as a contracting homotopy

$$f_{k+1} \circ \alpha_k + \alpha_{k+1} \circ f_{k+1} \simeq \text{Id}$$

\Rightarrow The "chain complexes" are "exact"

\Rightarrow homotopy SS collapses at E_2 .

Idea of proof: How can we show a map is a k -fold loop map?

$$\Omega^k X \rightarrow \Omega^k Y \quad \text{! find the image of } \Omega^k: \text{Map}(X, Y) \rightarrow \text{Map}(\Omega^k X, \Omega^k Y)$$

These are too big. Instead look at $\text{Nat}(B^k(-), B^{k+1}(-)) \xrightarrow{\Omega^k} \text{Nat}(DI_k(-), BDI_{k+1}(-))$

(really consider $\forall r \rightarrow S^1 \wedge S^r$ in Weiss story!)

$$\text{Care about } \Omega^d \Omega^{\infty} \Omega^{\infty} X \xrightarrow{h_{\Omega^k} \Omega^k} \Omega^d \mathbb{Q} X \xrightarrow{h_{\Omega^{k+1}} \Omega^{k+1}}$$