Problem Set #3

Due November 15th

1. Find the Jordan Form and Jordan Basis (expressible via the Change of Basis matrix) of the following matrices:

   (a) $\begin{bmatrix} -3 & 3 & -2 \\ -7 & 6 & -3 \\ 1 & -1 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 1 & -1 \\ -4 & 4 & -2 \\ -2 & 1 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & -3 & 1 & 2 \\ -2 & 1 & -1 & 2 \\ -2 & 1 & -1 & 2 \\ -2 & -3 & 1 & 4 \end{bmatrix}$

2. Find the Jordan form and Jordan basis for the following linear operators.

   (a) T is the operator on $M_2(\mathbb{R})$ defined by

   $$T(A) = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} \cdot A - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot A^t.$$

   (b) V is the vector space of all polynomials in variables x and y over $\mathbb{R}$ of degree exactly 2. S is defined by

   $$S(p(x,y)) = y\frac{\partial f}{\partial x}(x,y) + x\frac{\partial f}{\partial y}(x,y).$$

3. Show that if

   $$A = \begin{bmatrix} 0 & 0 & \dots & 0 & -a_0 \\ 1 & 0 & \dots & 0 & -a_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -a_{n-1} \end{bmatrix},$$

   then

   $$p_A(\lambda) = (-1)^n(a_0 + a_1\lambda + \cdots + a_{n-1}\lambda^{n-1} + \lambda^n).$$

4. Chapter 8: Problem 23