

Extra Credit!

Due December 4th, 5 PM

The following computational problems use the inner product space $C[-\pi, \pi]$ of all continuous, real-valued functions on $[-\pi, \pi]$, together with the inner product

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x)dx.$$

1. Show that the set $\{\frac{1}{\sqrt{2}}, \sin x, \sin 2x, \dots, \cos x, \cos 2x, \dots\}$ is an orthogonal set. Show also that all vectors have length 1.
2. Find the Fourier expansion of $f(x) = x$. In other words, find the coefficients in

$$\hat{f} := \langle f(x), \frac{1}{\sqrt{2}} \rangle \frac{1}{\sqrt{2}} + \sum_{n=1}^{\infty} \langle f(x), \sin nx \rangle \sin nx + \sum_{n=1}^{\infty} \langle f(x), \cos nx \rangle \cos nx.$$

3. Use Parseval's Identity: $\langle f, g \rangle = \langle \hat{f}, \hat{g} \rangle$ to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$