NAME: ______________________ ☐ TTH ☐ MWF 10AM ☐ MWF 11AM

PLEDGE: ____

____

SIGNATURE: ______________________________

To get credit for a problem, you must show all of your reasoning and calculations. No calculators may be used.

Problem	Score
1	
2	
3	
4	
5	
Total	

1. (a) (12 Points) Find the adjoint, determinant and inverse of

$$A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

(b) (8 Points) Compute the determinant of B using elimination method

$$B = \begin{bmatrix} 2 & 3 & 0 & 2 \\ 2 & 6 & -1 & 5 \\ 2 & 3 & -1 & -4 \\ 4 & 12 & -3 & 7 \end{bmatrix}.$$

2. (a) (10 Points) Check whether the vectors

$$w_1 = (-1, -16, -7) \text{ and } w_2 = (1, 2, 3)$$

lie in the space generated by the vectors

$$v_1 = (1, -2, 1),\ \ v_2 = (-2, 1, -3) \text{ and } v_3 = (-1, -4, -3).$$

If w_1 or w_2 lie in the span$\{v_1, v_2, v_3\}$ write it as a linear combination of v_1, v_2, v_3 (Hint: Solve the two resulting systems simultaneously).

(b) (10 points) Check if the following set of matrices linearly independent:

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ -2 & 6 \end{bmatrix}.$$

3. (a) (20 Points) Determine a basis for the kernel and range of the transformation defined by the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -2 \\ 2 & 20 & 22 \end{bmatrix},$$

and verify the Rank-Nullity theorem.

4. (20 Points)Find eigenvalues and and the set of all eigenvectors for each eigenvalue of the following matrix

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$$

.

5. (20 Points) Use Gram-Schmidt process to obtain an orthogonal basis for $\mathbb{R}^3$ from the basis $\{v_1 = (1,1,1),\ v_2 = (-1,0,1),\ v_3 = (-1,2,3)\}$.