

## HW 11

### Problem 18.14

#### a. To Find:

- (a) Determine  $\rho_0$  and  $a$  in Equation 18.10 for pure copper, using the data in Fig.18.8
- (b) Determine 'A' in Equation 18.11 for nickel as an impurity in copper, using the data in Figure 18.8
- (c) Estimate the electrical resistivity of copper containing 1.75 at% Ni at 100°C using the results obtained in parts (a) and (b)

#### b. Given:

Fig.18.8.

#### c. Assumptions:

1. Fig.18.8 is accurate.
2. The alloys are undeformed (hence,  $\rho_d = 0$ )

#### d. Solution:

- (a) Two simultaneous equations are set up using two resistivity values (labeled  $\rho_{t1}$  and  $\rho_{t2}$ ) at two corresponding temperatures ( $T_1$  and  $T_2$ ). Thus, from equation 18.10:

$$\rho_{t1} = \rho_0 + aT_1$$

$$\rho_{t2} = \rho_0 + aT_2$$

$$a = \frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2}$$

$$\rho_0 = \rho_{t1} - T_1 \left[ \frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2} \right] \text{ or } \rho_0 = \rho_{t2} - T_2 \left[ \frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2} \right]$$

From equation 18.9,  $\rho_{\text{total}} = \rho_t + \rho_i + \rho_d$

There are no impurities in Cu  $\Rightarrow \rho_i = 0$

Copper is undeformed  $\Rightarrow \rho_d = 0$

Hence, the plot of  $\rho_{\text{total}}$  versus temperature in Fig.18.8 is equivalent to the variation of  $\rho_t$  versus temperature (only) for pure copper.

From Figure 18.8,  $T_1 = -150^\circ\text{C}$ ,  $T_2 = -50^\circ\text{C}$  gives  $\rho_{t1} = 0.6 \times 10^{-8} \text{ } (\Omega\text{-m})$ , and  $\rho_{t2} = 1.25 \times 10^{-8} \text{ } (\Omega\text{-m})$ :

$$a = \frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2} = \frac{[(0.6 \times 10^{-8}) - (1.25 \times 10^{-8})](\Omega\text{-m})}{-150^\circ\text{C} - (-50^\circ\text{C})} = 6.5 \times 10^{-11} \text{ } (\Omega\text{-m})/^\circ\text{C}$$

$$\begin{aligned} \rho_0 &= \rho_{t1} - T_1 \left[ \frac{\rho_{t1} - \rho_{t2}}{T_1 - T_2} \right] = (0.6 \times 10^{-8}) - (-150) \frac{[(0.6 \times 10^{-8}) - (1.25 \times 10^{-8})](\Omega\text{-m})}{-150^\circ\text{C} - (-50^\circ\text{C})} \\ &= 1.58 \times 10^{-8} \text{ } (\Omega\text{-m}) \end{aligned}$$

(b) The variation of  $\rho_{\text{total}}$  with temperature is plotted for three values of  $c_j$  viz. 0.0112, 0.0216, and 0.0332 in Figure 18.8. The curves in the figure allow us to tabulate values of  $\rho_{\text{total}}$  at some temperature for all values of  $c_j$ .

The following table has values of  $\rho_{\text{total}}$  at  $0^\circ\text{C}$  for all three values of  $c_j$ .

From equation 18.9,  $\rho_{\text{total}} = \rho_t + \rho_i + \rho_d$

The alloys are undeformed, hence,  $\rho_d = 0$ .

$$\Rightarrow \rho_i = \rho_{\text{total}} - \rho_t \quad \dots(1)$$

From equation 18.10,  $\rho_t = \rho_0 + aT$

$$T = 0 \Rightarrow \rho_t = \rho_0 = 1.58 \times 10^{-8} \text{ } (\Omega\text{-m}) \quad \dots(2) \text{ (from part a)}$$

$$\Rightarrow \rho_i = \rho_{\text{total}} - 1.58 \times 10^{-8} \text{ } (\Omega\text{-m}) \quad \dots(3)$$

The values of  $\rho_i$  are noted in the table next to the corresponding values of  $\rho_{\text{total}}$ .

The value of A may be determined from equation 18.11:

$$\rho_i = Ac_i(1 - c_i) \Rightarrow A = \rho_i / [c_i(1 - c_i)]$$

$c_i$	$1 - c_i$	$\rho_{\text{total}} (\Omega\text{-m})$	$\rho_i (\Omega\text{-m})$	$A (\Omega\text{-m})$
0.0112	0.989	$3.0 \times 10^{-8}$	$1.42 \times 10^{-8}$	$1.28 \times 10^{-6}$
0.0216	0.978	$4.2 \times 10^{-8}$	$2.62 \times 10^{-8}$	$1.24 \times 10^{-6}$
0.0332	0.967	$5.5 \times 10^{-8}$	$3.92 \times 10^{-8}$	$1.22 \times 10^{-6}$

The average value of A is  $1.25 \times 10^{-6} (\Omega\text{-m})$ .

(c) From equation 18.11,  $\rho_i = Ac_i(1 - c_i)$

For  $c_i = 0.0175$ , using the value of A from part(b) :

$$\rho_i = 1.25 * 10^{-6} * 0.0175 * (1 - 0.0175) = 1.25 * 10^{-6} * 0.0175 * 0.982 = 2.15 * 10^{-8} (\Omega\text{-m})$$

From equation 18.10,  $\rho_t = \rho_0 + aT$

For  $T = 100^\circ\text{C}$ , using the values of a and  $\rho_0$  from part (a):

$$\rho_t = (1.58 * 10^{-8}) + (6.5 * 10^{-11} * 100) = 2.23 * 10^{-8} (\Omega\text{-m})$$

The alloys is undeformed  $\Rightarrow \rho_d = 0$

From equation 18.9,  $\rho_{\text{total}} = \rho_t + \rho_i + \rho_d$

$$\Rightarrow \rho_{\text{total}} = (2.15 * 10^{-8}) + (2.23 * 10^{-8}) + 0 = 4.38 * 10^{-8} (\Omega\text{-m})$$

(a)  $a = 6.5 * 10^{-11} (\Omega\text{-m})/^\circ\text{C}$        $\rho_0 = 1.58 * 10^{-8} (\Omega\text{-m})$

(b)  $A = 1.25 * 10^{-6} (\Omega\text{-m})$

(c)  $\rho_{\text{total}} = 4.38 * 10^{-8} (\Omega\text{-m})$

### **Problem 18.17**

#### **a. To Find:**

The metals/alloys suitable for the given application.

#### **b. Given:**

The wire is cylindrical with diameter = 2mm. It is required to carry a current of 10 A with a minimum of 0.03 V drop per 300 mm of the wire.

Table 18.1 lists the electrical conductivities of some metals/alloys.

#### **c. Assumptions:**

The data in 18.1 is accurate.

#### **d. Solution:**

From equation 18.4:  $\sigma = \frac{1}{\rho}$

Expressing  $\rho$  in terms of V, A, I and l according to equation 18.3 and substituting in equation 18.4:  $\sigma = \frac{1}{\rho} = \frac{Il}{VA} = \frac{Il}{V\pi\left(\frac{d}{2}\right)^2}$

$$\Rightarrow \sigma = \frac{(10 \text{ A})(300 \times 10^{-3} \text{ m})}{(0.03 \text{ V}) (\pi) \left(\frac{2 \times 10^{-3} \text{ m}}{2}\right)^2} = 3.2 \times 10^7 (\Omega \cdot \text{m})^{-1}$$

Comparing with Table 18.1, we find that brass, iron, platinum, plain carbon steel and stainless steel are candidate materials.

**Brass, iron, platinum, plain carbon steel and stainless steel**

### **Problem 18.30**

#### **a. To Find:**

- (a) Determine whether the material is a p-type or n-type semi-conductor.
- (b) Calculate the electrical conductivity of the material.

#### **b. Given:**

- 1. Dopant (Sb) concentration =  $5 \times 10^{22} \text{ m}^{-3}$
- 2. All Sb atoms are ionized
- 3. Electron mobility =  $0.1 \text{ m}^2/\text{V-s}$ ; Hole mobility =  $0.05 \text{ m}^2/\text{V-s}$

#### **c. Assumptions:**

All Sb atoms are ionized at room temperature.

#### **d. Solution:**

- (a) Sb is a group VA element while Ge is a group IVA element. Hence, Sb is a donor in Ge and the material is an n-type semi-conductor.
- (b) The material is an n-type semiconductor. Hence, equation 18.16 is applicable. Each Sb atom will donate one electron. Hence, the electron concentration equals the Sb concentration (i.e.,  $n = 5 \times 10^{22} \text{ m}^{-3}$ ). Also,  $\mu_e = 0.1 \text{ m}^2/\text{V-s}$ . Hence,

$$\sigma = n |e| \mu_e = (5 \times 10^{22} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})(0.1 \text{ m}^2/\text{V-s}) = 800 (\Omega\text{-m})^{-1}$$

**(a) n-type semi-conductor**

**(b)  $800 (\Omega\text{-m})^{-1}$**

### **Problem 18.40**

#### **a. To Find:**

Calculate the electrical conductivity of silicon doped with  $10^{20} \text{ m}^{-3}$  of phosphorus atoms at  $85^\circ\text{C}$ .

#### **b. Given:**

Si is doped with P atoms.  
Dopant concentration =  $10^{20} \text{ m}^{-3}$

#### **c. Assumptions:**

All P atoms are ionized at the given temperature.

#### **d. Solution:**

P is a group VA element while Si is a group IVA element. Hence, P is a donor in Si and the material is an n-type semi-conductor.

Equation 18.16 is applicable. Each P atom will donate one electron (since all P atoms are ionized at the given temperature). Hence, the electron concentration equals the P concentration (i.e.,  $n = 10^{20} \text{ m}^{-3}$ ).

Figure 18.19a is used to determine the electron mobility: From the curve corresponding to impurity concentration  $< 10^{20} \text{ m}^{-3}$  at  $85^\circ\text{C}$  (358 K),  $\mu_e = 0.1 \text{ m}^2/\text{V}\cdot\text{s}$ .

Hence,  $\sigma = n/e |\mu_e| = (10^{20} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})(0.1 \text{ m}^2/\text{V}\cdot\text{s}) = 1.6 (\Omega\text{-m})^{-1}$

$$1.6 (\Omega\text{-m})^{-1}$$

### **Problem 18.D4**

#### **a. To Find:**

The type and concentration (in weight percent) of an acceptor impurity that produces a p-type silicon material with an electrical conductivity of  $50 (\Omega\text{-m})^{-1}$  at room temperature.

#### **b. Given:**

The p-type semi-conductor has an electrical conductivity of  $50 (\Omega\text{-m})^{-1}$  at room temperature.

#### **c. Assumptions:**

All dopant atoms are ionized at room temperature.

#### **d. Solution:**

Step 1: Identifying the type of acceptor impurity atoms

The elements which produce a p-type semi-conductor material when added to silicon render are group IIIA elements like boron, aluminum, gallium, and indium.

Step 2: Calculating the concentration of dopant atoms (atoms/  $\text{m}^3$ )

The electrical conductivity is calculated according to equation 18.17 for p-type semi-conductors.

$$\sigma = p |e| \mu_h$$

Hence, the conductivity is a function of both the hole concentration ( $p$ ) and the hole mobility ( $\mu_h$ ).

All dopant atoms are ionized at room temperature. Hence, the number of holes equals the number of acceptor impurities. (Hence, the number of holes per  $\text{m}^3$  equals the number of acceptor impurities per  $\text{m}^3$ ,  $N_a$ .)

The room-temperature hole mobility varies with impurity concentration, as shown in Figure 18.18. Had we known the exact equation of the variation of hole mobility with impurity concentration, we could have combined this equation with equation 18.17 to calculate  $p$ .

However, such an equation is not given in the text-book. Hence, we may adopt an iterative approach to tackle this problem. Two equivalent iterative approaches is described below.

### Approach #1:

We assume some acceptor impurity concentration (which equals the value of  $p$ ). We then look up the corresponding value of  $\mu_h$  from Fig.18.18. Please note that both axes in this figure are logarithmically scaled. If the calculated value of  $\sigma$  is higher than  $50 (\Omega\text{-m})^{-1}$ , we need to repeat the process with a lower value of the impurity concentration. If the calculated value of  $\sigma$  is lower than  $50 (\Omega\text{-m})^{-1}$ , we need to repeat the process with a higher value of the impurity concentration. We need to repeat this procedure until the calculated value of  $\sigma$  equals  $50 (\Omega\text{-m})^{-1}$ .

Let us start by assuming that  $N_a = p = 10^{22} \text{ m}^{-3}$ .

From Figure 18.18, at this impurity concentration, the hole mobility  $\mu_h$  is  $10^{(-2+(1.2/2.1))} = 0.0373 \text{ m}^2/\text{V}\cdot\text{s}$ .

Using equation 18.17, the calculated  $\sigma = 10^{22} * 1.602 * 10^{-19} * 0.0373 = 59.7546 (\Omega\text{-m})^{-1}$ . This value is  $> 50 (\Omega\text{-m})^{-1}$ .

Next, let us assume that  $N_a = p = 5 \times 10^{21} \text{ m}^{-3}$ .

From Figure 18.18, at this impurity concentration, the hole mobility  $\mu_h$  is  $10^{(-2+(1.3/2.1))} = 0.0416 \text{ m}^2/\text{V}\cdot\text{s}$ .

Using equation 18.17, the calculated  $\sigma = 5 * 10^{21} * 1.602 * 10^{-19} * 0.0416 = 33.3216 (\Omega\text{-m})^{-1}$ . This value is  $< 50 (\Omega\text{-m})^{-1}$ .

Thus, the correct impurity concentration lies between  $5 \times 10^{21}$  and  $10^{22} \text{ m}^{-3}$  (and should be closer to the latter). At  $N_a = p = 7.92 * 10^{21} \text{ m}^{-3}$ , the hole mobility  $\mu_h$  is  $10^{(-2+(1.25/2.1))} = 0.0394 \text{ m}^2/\text{V}\cdot\text{s}$ . Using equation 18.17, the calculated  $\sigma = 7.92 * 10^{21} * 1.602 * 10^{-19} * 0.0394 = 49.99 (\Omega\text{-m})^{-1}$ . This value is  $\approx 50 (\Omega\text{-m})^{-1}$ .

### Approach #2:

Instead of comparing the actual and calculated values of  $\sigma$ , we may calculate the value of  $\mu_h$  using a re-arranged version of equation 18.17:  $\mu_h = \frac{\sigma}{p|e|}$  and compare it with the actual value of  $\mu_h$  obtained from Fig.18.18.

When  $N_a = p = 10^{22} \text{ m}^{-3}$ :

$$\text{the calculated value of } \mu_h = \frac{\sigma}{p|e|} = \frac{50 (\Omega\text{-m})^{-1}}{(10^{22} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})} = 0.0312 \text{ m}^2/\text{V}\cdot\text{s}$$

This value is lower than than 0.0373, the value obtained from Fig.18.18

When  $N_a = p = 5 \times 10^{21} \text{ m}^{-3}$

The calculated value of  $\mu_h = \frac{\sigma}{p |e|} = \frac{50 (\Omega - \text{m})^{-1}}{(5 \times 10^{21} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})} = 0.0624 \text{ m}^2 / \text{V} - \text{s}$

This value is higher than 0.0416 , the value obtained from Fig.18.18

At  $N_a = p = 7.92 \times 10^{21} \text{ m}^{-3}$

The calculated value of  $\mu_h = \frac{\sigma}{p |e|} = \frac{50(\Omega - \text{m})^{-1}}{(7.92 \times 10^{21} \text{ m}^{-3})(1.602 \times 10^{-19} \text{ C})} = 0.0394 \text{ m}^2 / \text{V} - \text{s}$

This is equal to the value obtained from the figure.

Step 3: Converting the concentration of dopant atoms (atoms/  $\text{m}^3$ ) to atom percent

To calculate the concentration of acceptor impurities in atom percent, we need to calculate the number of silicon atoms per cubic meter,  $N_{\text{Si}}$ . From equation 4.2:

$$N_{\text{Si}} = \frac{N_A \rho_{\text{Si}}'}{A_{\text{Si}}} = \frac{(6.022 \times 10^{23} \text{ atoms/mol})(2.33 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{28.09 \text{ g/mol}} = 5.0 \times 10^{28} \text{ m}^{-3}$$

(where  $\rho_{\text{Si}}'$  is the density of silicon)

The concentration of acceptor impurities in atom percent ( $C_a'$ ) equals the ratio of  $N_a$  and ( $N_a + N_{\text{Si}}$ ) multiplied by 100 as

$$C_a' = \frac{N_a}{N_a + N_{\text{Si}}} \times 100 = \frac{7.92 \times 10^{21} \text{ m}^{-3}}{(7.92 \times 10^{21} \text{ m}^{-3}) + (5.0 \times 10^{28} \text{ m}^{-3})} \times 100 = 1.58 \times 10^{-5} \text{ at\%}$$

Step 4: Converting the concentration of dopant atoms from atom percent ( $C_a'$ ) to weight percent ( $C_a$ )

$$\text{From Equation 4.7a : } C_a = \frac{C_a' A_a}{C_a' A_a + C_{\text{Si}}' A_{\text{Si}}} \times 100$$

where  $A_a$  and  $A_{Si}$  represent the atomic weights of the acceptor and silicon respectively.

Thus, the concentration of the acceptor in weight percent depends on the particular acceptor type.

For boron:

$$C_B = \frac{C'_B A_B}{C'_B A_B + C'_{Si} A_{Si}} \times 100 = \frac{(1.58 \times 10^{-5} \text{ at\%})(10.81 \text{ g/mol})}{(1.58 \times 10^{-5} \text{ at\%})(10.81 \text{ g/mol}) + (99.999984 \text{ at\%})(28.09 \text{ g/mol})} \times 100$$
$$= 6.08 \times 10^{-6} \text{ wt\%}$$

Similarly, for aluminum:  $C_{Al} = 1.52 \times 10^{-5} \text{ wt\%}$

For gallium:  $C_{Ga} = 3.92 \times 10^{-5} \text{ wt\%}$

For indium:  $C_{In} = 6.46 \times 10^{-5} \text{ wt\%}$

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**NOTE:** Instead of first converting atoms/  $m^3$  to atom percent and then atom percent to weight percent, we can directly convert the impurity concentration from atoms/  $m^3$  to weight percent as follows. We will consider the impurity to be B. The same procedure is applicable for Al, Ga and In.

$N_a = p = 7.92 \times 10^{21} \text{ m}^{-3}$  : Number of impurity (boron) atoms present/ $m^3$

Weight of 1 mol of B atoms = 10.81 g

⇒ Weight of  $6.023 \times 10^{23}$  B atoms = 10.81 g

⇒ Weight of 1 boron atom =  $10.81 \text{ g} / (6.023 \times 10^{23})$

⇒ Weight of  $7.92 \times 10^{21}$  atoms =  $7.92 \times 10^{21} \times 10.81 \text{ g} / (6.023 \times 10^{23}) = 0.1421 \text{ g}$

⇒ Weight of impurity (boron) atoms =  $0.1421 \text{ g/m}^3$

From equation 4.2:

$$N_{\text{Si}} = \frac{N_A \rho_{\text{Si}}}{A_{\text{Si}}} = \frac{(6.022 \times 10^{23} \text{ atoms/mol})(2.33 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{28.09 \text{ g/mol}} = 5.0 \times 10^{28} \text{ m}^{-3} : \text{Number of silicon atoms present/m}^3$$

Weight of 1 mol of Si atoms = 28.09 g

⇒ Weight of  $6.023 \times 10^{23}$  atoms = 28.09 g

⇒ Weight of 1 atom =  $28.09 \text{ g} / (6.023 \times 10^{23})$

⇒ Weight of  $5.0 \times 10^{28}$  atoms =  $5.0 \times 10^{28} * 28.09 \text{ g} / (6.023 \times 10^{23}) = 2.3319 \times 10^6$

⇒ Weight of Si atoms =  $2.3319 \times 10^6/\text{m}^3$

$$\text{Total weight/m}^3 = \text{Weight of impurity (boron) atoms/m}^3 + \text{Weight of Si atoms/m}^3 = 2.3319 \times 10^6 / \text{m}^3$$

$$\text{Weight \% of impurity} = \text{Weight of impurity atoms (per m}^3) * 100 / (\text{Total weight (per m}^3)) = 0.1421 * 100 / (2.3319 \times 10^6) = 6.08 \times 10^{-6}$$

$$C_{\text{B}} = 6.08 \times 10^{-6} \text{ wt}\%$$

$$C_{\text{Al}} = 1.52 \times 10^{-5} \text{ wt}\%$$

$$C_{\text{Ga}} = 3.92 \times 10^{-5} \text{ wt}\%$$

$$C_{\text{In}} = 6.46 \times 10^{-5} \text{ wt}\%$$