If $\mathcal{H}$ is a Hilbert space of analytic functions on the unit disk and $\varphi$ is an analytic function mapping the disk into itself, then for $f$ in $\mathcal{H}$, the equation

$$C_{\varphi}f = f \circ \varphi$$

defines a composition operator on $\mathcal{H}$. On many common functional Hilbert spaces, conditions for boundedness and compactness of composition operators and results about their spectra and cyclicity are known. For example, the Littlewood Subordination Principle shows that $C_{\varphi}$ is bounded on the Hardy Hilbert space for any analytic function $\varphi$ that maps the unit disk into itself. In spite of this progress, many interesting and seemingly basic problems remain open. This talk will discuss the problem that for $\varphi$ an arbitrary analytic function mapping the unit disk into itself, no satisfactory formula for the adjoint $C_{\varphi}^*$ has been known. Recent work of the speaker with Eva Gallardo-Gutierrez describes the adjoint when the symbol is a rational function. This description, in turn, inspires the definition of a new type of operator on functional Hilbert or Banach spaces that generalizes standard composition or weighted composition operators.