"Enhancement Mode" MOSFETs:

ENHANCE the source to drain carrier flow by inverting the surface to form a conducting channel

"N-channel enhancement mode MOSFET"  "P-channel enhancement mode MOSFET"

Carriers (not current) travel from source to drain

Thus, important point #1:

N-channel: \( V_D > V_S \)  \[ \text{P-channel: } V_D < V_S \]
Important point #2:

When \( V_G \) is "above" \( V_T \), a surface inversion channel forms

\[
V_G > V_T \text{ for N-channel device (P-substrate)}
\]

"Above" =

\[
V_G < V_T \text{ for P-channel device (N-substrate)}
\]

Just think of the physics involved:

N-channel = Electron inversion layer: More positive \( V_G \) is, greater is attraction for negative electrons

P-channel = Hole inversion layer: More negative \( V_G \) is, greater is attraction for positive holes

Qualitative I-V Characteristics (plotting on identical axes for N and P channel devices):

a) For \( V_G \) "below" \( V_T \) inversion channel has not yet formed => no conduction
At right, being "below" threshold means $V_G$ is larger (more positive) than $V_{Tp}$

b) For $V_G$ "above" $V_T$ inversion channel forms, get conduction:

**N-channel (P-substrate):**

- $V_G > V_{Tn}$
- $V_G > V_{Tn}$

**P-channel (N-substrate):**

- $V_G < V_{Tp}$
- $V_G << V_{Tp}$

**c) $V_G$ well "above" $V_T$ => stronger inversion channel, stronger conduction:**

**N-channel (P-substrate):**

- $V_G >> V_{Tn}$

**P-channel (N-substrate):**

- $V_G < V_{Tp}$
- $V_G << V_{Tp}$
d) Increase $V_{DS}$ => Resistive drop in channel => Narrowing at drain end => Increased resistance:

N-channel (P-substrate):

- $V_G >> V_{Tn}$
- $V_G > V_{Tn}$

P-channel (N-substrate):

- $V_G < V_{Tp}$
- $V_G << V_{Tp}$

---

e) Finally, for $V_{DS} = V_{DS(sat)}$ go into saturation:

Channel "pinches off" at drain end

Further increase in $V_{DS}$ forms "waterfall" sudden energy drop at enter drain

Flow is $\sim$ constant (just changing height of waterfall at end of narrowing valley)

N-channel (P-substrate):

- $V_G >> V_{Tn}$
- $V_G > V_{Tn}$

P-channel (N-substrate):

- $V_G < V_{Tp}$
- $V_G << V_{Tp}$
Now we must QUANTIFY these trends!

As usual, focus on case of N-channel enhancement mode MOSFET (P-substrate) = Left column above

Inspection of diagrams how signs change for P-channel case

What is the value of $V_T$ (or for this case, $V_{Tn}$) ? Divide it into steps as we did in earlier lectures:

i) Must invert the semiconductor surface:

\[ q \cdot \phi_{FP} \]

=>

\[ E_i \]

\[ E_F \]

ii) Add voltage across the MOS oxide (calculate this from capacitor's charge on the semiconductor side):

\[ 2 \cdot \phi_{FP} \]
iii) Splice together by taking $\phi_{ms} = \phi_m - \phi_s$ (mismatch in material work functions) into account:

$$q\cdot V_{ox} \cdot q\cdot \phi_{Fp} \cdot q\cdot \Delta V_{semi} = 2q\cdot \phi_{Fp}$$

Giving final result of:

$$V_{Tn} = \phi_{ms} + 2\cdot \phi_{Fp} - \frac{(Q'_{ss} + Q'_{depletion\_layer})}{C'_{ox}}$$

(1)
Now, increase the voltage on the gate, \( V_g \), above this threshold of channel formation:

- Minuscule increase in depletion layer width
- Deepening of surface inversion layer

As we increase the gate voltage above \( V_{Tn} \), how much more voltage will be dropped across the oxide?

- \( Q'_{ss} = \text{oxide fixed charge} \)
- \( Q'_{\text{depletion}} = \text{Depletion layer charge, changes only very little} \)

But we will get a big change in the inversion layer charge, \( Q'_{\text{inversion}} \) (which is, after all, what we want!)

Using MOS capacitance equation, as a 1st approximation:

\[
\Delta V_{ox} = \frac{-Q'_{\text{inversion}}}{C'_{ox}}
\]

So for gate voltages above \( V_{Tn} \) can say:
\[ V_G = V_{Tn} - \frac{Q_{\text{inversion}}}{C'_{ox}} + \text{slight increase in semiconductor band bending} \]

Increase in semiconductor band bending = (bending now) - (bending at threshold) = \( \phi_s - 2\phi_{Fp} \)

\[ V_G = V_{Tn} - \frac{Q_{\text{inversion}}}{C'_{ox}} + \phi_s - 2\cdot\phi_{Fp} \tag{3} \]

Solve this for the inversion layer charge:

\[ Q_{\text{inversion}}(V_G) = -C'_{ox} \left[ V_G - V_{Tn} - (\phi_s - 2\cdot\phi_{Fp}) \right] \tag{4} \]

In MOSFET with \( V_{DS} > 0 \), have carriers flowing source to drain => variation in band bending (\( \phi_s \))
Cross-section #1: Down through source to P-substrate:

This is just a N-P junction, "band bending" \( \sim 2 \phi_{Fp} \)

Cross-section at #2: Down through inversion channel into P-substrate:

\[ 2 \phi_{Fp} \text{ at the source end (minimum to make inversion)} \]

by definition = \[ \phi_{S}(x) = 2 \phi_{FP} + V_{DS} \] at drain end

(i.e. band pulled down as in 3D drawings)

So, although may not know exactly how \( \phi_{S} \) changes, do know its end values at source and drain:
Have assumed here that channel is NOT yet "pinched-off" so that voltage merges smoothly with drain (no waterfall)

Now, if net electric field at surface of the semiconductor is perpendicular to the surface, can still apply equation (4) for the density of $\text{Q'}_{\text{inversion}}$ in the channel

If not perpendicular get in trouble w/ capacitor vs. charge equation

$\zeta$ is not perpendicular when:
1) Channel is "pinched off" at drain - waterfall = $\Delta V$ along channel
2) In a very small MOSFET w/ similar vertical & horizontal sizes

Avoiding those situations by operating below "pinch-off" in a larger MOSFET:

Will use the $\text{Q'}_{\text{inversion}}$ equation to solve for the channel resistance vs. position
Analyze channel as:

So each strip resistor has:

Length = $dx$

Cross-sectional area = $W \delta(x)$
So the resistance of each of these resistors is:

\[
\frac{dR}{dx} = \frac{dx}{A \cdot \rho(x)} = \frac{dx}{A \cdot \frac{1}{\sigma(x)}} = \frac{dx}{W \cdot \delta(x) \cdot \frac{1}{\sigma(x)}}
\]

But conductivity at \(x\) is:

\[\sigma(x) = (\text{mobility}) \cdot (\text{charge\_density}) = \mu_n \cdot Q_{\text{inversion}}(x)\]

Important: last term = charge / volume, \(Q\)

Not charge / area, \(Q'\)

Substitute in:

\[
\frac{dR}{dx} = \frac{dx}{W \cdot \delta(x) \cdot \frac{1}{\mu_n \cdot Q_{\text{inversion}}(x)}} = \frac{dx}{\mu_n \cdot W \cdot (\delta(x) \cdot Q_{\text{inversion}}(x))}
\]

but \(\delta(x) \cdot Q = Q'\) (thickness \(x\) charge per volume = charge / area)

\[
\frac{dR}{dx} = \frac{dx}{\mu_n \cdot W \cdot Q'_{\text{inversion}}(x)}
\]

Voltage drop across each of these mini resistors is just \(V = I \cdot R\):

\[
dV = I \cdot dR = I_{DS} \cdot \frac{dx}{\mu_n \cdot W \cdot Q'_{\text{inversion}}(x)}
\]

rearranging this:

\[
I_{DS} \cdot dx = \mu_n \cdot W \cdot Q'_{\text{inversion}}(x) \cdot dV
\]

Integrate from source to drain:
\[
\int_0^L I_{DS} \, dx = \int_0^L \mu_n W \cdot Q_{\text{inversion}}(x(x)) \, dV
\]

(9)

Left side is easy because \( I_{DS} \) is constant along the length of the channel

\[
\int_0^L I_{DS} \, dx = I_{DS} \cdot L
\]

Right side takes a bit more work:

\[
\int_0^L \mu_n W \cdot Q_{\text{inversion}}(x(x)) \, dV = \mu_n W \cdot \int_0^L Q_{\text{inversion}}(x(x)) \, dV
\]

Substitute in our equation (4) from way above on what the inversion charge/area will be vs. \( x \)

\[
= \mu_n W \int_0^L -C'_{ox} \left[ (V_G - V_{Tn}) - (\phi_S(x) - 2 \cdot \phi_{Fp}) \right] \, dV(x)
\]

\( V(x) \) is the surface voltage \hspace{1cm} \( \phi_S(x) \) is the band bending to the surface

Thus must have \( V(x) = \phi_S(x) \pm \text{Constant} \) \hspace{1cm} \( \Rightarrow \) \( dV(x) = d\phi_S(x) \)
\[
I_{DS} \cdot L = \mu_n \cdot W \cdot \int_{0}^{L} -C'_{ox} \left[ (V_G - V_{Th}) - (\phi_S(x) - 2 \cdot \phi_{Fp}) \right] d\phi_S(x)
\]

But from far above: \( \phi_S(x=0) = 2 \phi_{Fp} \)
\( \phi_S(x=L) = 2 \phi_{Fp} + V_{DS} \)

\[
= \mu_n \cdot W \cdot \int_{2 \cdot \phi_{Fp}}^{2 \cdot \phi_{Fp} + V_{DS}} -C'_{ox} \left[ (V_G - V_{Th}) - (\phi_S(x) - 2 \cdot \phi_{Fp}) \right] d\phi_S(x)
\]

Subtract 2 \( \phi_{Fp} \) from limits, compensate by adding it to \( \phi_S \) in integral

\[
I_{DS} = \mu_n \cdot W \cdot \int_{0}^{V_{DS}} -C'_{ox} \left[ (V_G - V_{Th}) - \phi_S \right] d\phi_S
\]

\[
I_{DS} = \frac{-\mu_n \cdot W \cdot C'_{ox}}{L} \left[ (V_G - V_{Th}) \cdot \phi_S - \frac{1}{2} \cdot \phi_S^2 \right] \text{ evaluated at } \phi_S = V_{DS} \text{ minus value at } \phi_S = 0
\]

\[
I_{DS} = \frac{-\mu_n \cdot W \cdot C'_{ox}}{L} \left[ (V_G - V_{Th}) \cdot V_{DS} - \frac{1}{2} \cdot V_{DS}^2 \right]
\]
Finally, generalize to possibility source is not at ground: $V_G => V_G - V_S = V_{GS}$

and follow convention that $I_{Drain} = I$ into drain wire = $- I_{DS}$

$$I_D = \frac{\mu_n \cdot W \cdot C'_{ox}}{L} \left[ (V_{GS} - V_{Tn}) \cdot V_{DS} - \frac{1}{2} \cdot V_{DS}^2 \right]$$

For N-channel MOSFET below "pinch-off"

= Parabolas

$\frac{d}{dV_{DS}} I_D = \frac{\mu_n \cdot W \cdot C'_{ox}}{L} \cdot (V_{GS} - V_{Tn} - V_{DS}) = 0$

So at maximum, $V_{DS}$ is:

$$V_{DS(\text{at_max})} = V_{GS} - V_{Tn}$$

Hold it!

Understand initial increase with $V_{DS}$, but why would it then turn around and decrease?

To figure out, solve for maximum:
I allege that this is exactly the $V_{DS}$ that causes channel charge $\Rightarrow 0$ at drain end (i.e. to "pinch off the channel")

To prove, go back to our equation earlier in lecture for charge in channel:

$$Q'_{\text{inversion}}(V_{GS}) = -C'_{ox}\left[ V_{GS} - V_{Tn} - \left( \phi_s - 2\cdot\phi_Fp \right) \right]$$

(4)

Plug in the fact (also from above) that at drain end, band bending is: $\phi_s = 2\cdot\phi_Fp + V_{DS}$ get:

$$Q'_{\text{inversion\_at\_drain}}(V_G) = -C'_{ox}\left( V_G - V_{Tn} - V_{DS} \right)$$

Plug in our maximum $V_{DS}$ from above:

$$Q'_{\text{inversion\_at\_drain}}(V_G) = -C'_{ox}\left( V_G - V_{Tn} - V_{DS\text{\_at\_max}} \right)$$

$$= -C'_{ox}\left( V_{GS} - V_{Tn} - V_{GS} + V_{Tn} \right) = 0$$

So at the top of the parabola, our channel has pinched off, and we go into "saturation"

$$V_{DS\text{\_at\_max}} = V_{DS\text{\_sat}}$$

Plug this value into current to get $I$ at peak (which then becomes our "saturation" current through the device):
\[ I_D(sat) = \frac{\mu_n \cdot W \cdot C'_{ox}}{2 \cdot L} \cdot V_{DS(\text{at_max})}^2 = \frac{\mu_n \cdot W \cdot C'_{ox}}{2 \cdot L} \cdot V_{DS(sat)}^2 = \frac{\mu_n \cdot W \cdot C'_{ox}}{2 \cdot L} \cdot (V_{GS} - V_{Tn})^2 \]

Restating I and V values at this "saturated peak"

\[ V_{DS(sat)} = V_{GS} - V_{Tn} \]
\[ I_D(sat) = \frac{\mu_n \cdot W \cdot C'_{ox}}{2 \cdot L} \cdot (V_{GS} - V_{Tn})^2 \]

Our derivation assumed no "pinch-off" so we can only use derivation UP to this point = left half of parabola

Above this point, further increase in \( V_{DS} \) => building height of "waterfall" making ~ no change in flow

So just draw flat lines above the saturation / maximum point:

Different \( I_D \) vs. \( V_{DS} \) curve for each choice of \( V_{GS} \)

Larger \( V_{GS} \), stronger inversion channel => more current
Below saturation (rising segment of parabolas):

$V_G > V_{Tn}$

Above Saturation (flat line extensions):

$V_G > V_{Tn}$

"Pinched-off" channel / "Waterfall"
REMARKABLE COMPARISON:

Enhancement Mode MOSFET (as above):

PHYSICS: Voltage induced inversion layer => Voltage controlled resistor

Bipolar Junction Transistor (the "other" kind of transistor covered earlier in course):

PHYSICS: Carriers topping a potential barrier => diffusion and recombination as pass through

Static Input Power:

MOSFET: $I_{Gate} =>$ once has been turned on $= \text{no power from input circuit}$

BJT: $I_{Base}$ always finite $= \text{always putting some power in}$

BUT THE OVERALL $I$ vs. $V$ CURVES ARE ALMOST IDENTICAL !!!!!

Choose which one is better for application (e.g. based on power consumption)

THEN can use virtually the same circuits because their $I$-$V$'s are so similar !!!!

"plug and play compatible!"