Last time: Equations governing population of minority carriers

**P-type:** \( N_a >> N_d \quad p >> n \)  
\( p = \text{majority carrier} \quad n = \text{minority carrier} \)

**N-type:** \( N_d >> N_a \quad n >> p \)  
\( n = \text{majority carrier} \quad p = \text{minority carrier} \)

Then for minority electrons in **P-type** material:

\[
\frac{d}{dt} \delta n = \mu_n \cdot \xi \cdot \frac{d}{dx} \delta n + D_n \cdot \frac{d^2}{dx^2} \delta n - \frac{\delta n}{\tau_n} + G_{\text{non\_thermal}}
\]

And for minority holes in **N-type** material:

\[
\frac{d}{dt} \delta p = -\mu_p \cdot \xi \cdot \frac{d}{dx} \delta p + D_p \cdot \frac{d^2}{dx^2} \delta p - \frac{\delta p}{\tau_p} + G_{\text{non\_thermal}}
\]

Source: | Drift | Diffusion | Recombination | Non-thermal Generation

Where use:

\[
\delta n(x, t) = n(x, t) - n_O
\]

\[
\delta p(x, t) = p(x, t) - p_O
\]

Deviations from equilibrium or "excess" concentrations
ASSUMPTIONS / LIMITATIONS:

1) Only apply to a material's MINORITY CARRIERS

2) Only apply with LOW-LEVEL INJECTION

Assumption that whatever we did to change the minority carrier populations
the alteration was not so great that majority carrier populations changed

That is: if have P-type material, require that \( \delta p \ll p_0 \)

if have N-type material, require that \( \delta n \ll n_0 \)

What about the term dealing with the net thermal Generation - Recombination rate?

For minority holes: \( \delta p / \tau_p \)

For minority electrons: \( \delta n / \tau_n \)
1) If dealing with a pure DIRECT BANDGAP SEMICONDUCTOR (e.g. GaAs) physical process is:

Electron just loses energy falling directly into hole

Very efficient process, energy often given up as light

\[ \tau_p = \frac{1}{\alpha_r n_0} \]

\[ \tau_n = \frac{1}{\alpha_r p_0} \]

2) If dealing with an INDIRECT BANDGAP SEMICONDUCTOR (such as silicon) physical process is:

Electron cannot combine directly with hole

Also has to shed some "crystal momentum" (\(h_\text{bar} k\))

Our treatment of electrons as almost free breaks down:

- Not just adequate to change \(m\) to \(m_{\text{effective}}\)
- Details require QM treatment of crystal and electron together
Electron at bottom of conduction band (at top right) is not really moving

But detailed QM still says it must transfer some momentum to atoms of crystal before it can fill hole

So two things must occur: i) Transfer of momentum  ii) Loss of energy

Works best if occurs in two steps facilitated by a "TRAP"

TRAP = Impurity atom that produces a quantum energy level near the middle of the bandgap

So don't want atoms from adjacent columns of periodic table (donors & acceptors)

Instead want atoms very different from semiconductor atoms, i.e. from elsewhere in periodic table

Process of recombination in an indirect bandgap semiconductor (using simpler E vs. x diagrams):

STEP 1: =>

Electron first captured at trap level shedding momentum and some energy
Works better the farther down the trap energy level is - less chance of it jumping back up to conduction band:

"Shallow Trap" jumps back out

"Deep Trap" gotcha!

Now need to combine with the hole

STEP 2: Hole jumping up = electron falling down

Process complete
However, what if had used really "deep" trap that had level too close to valence band:

Would be easy for other electron in valence band (are lots even if don't show them) to jump up and undo the last step:

Best compromise giving good electron capture AND then hole capture is trap level ~ middle of bandgap

The "ideal" trap

For silicon crystal this is the case for metal atoms such as gold and silver

Net recombination rate for an indirect bandgap semiconductor then becomes:
Net recombination rate = \( \frac{\delta_{\text{carrier}}}{\tau_{\text{due to traps}}} \)  

More traps => faster recombination => shorter \( \tau \)

Minority carrier lifetime in indirect bandgap silicon is not a fundamental property!  

Won't find in appendix B!

Varies piece of silicon to piece of silicon:  

Greater purity (greater $) => lower recombination  

Lower purity (lower $) => greater recombination

What purity gives what lifetime?

![Graph showing the relationship between silicon minority carrier lifetime (\( \tau \)) and concentration of gold. The x-axis represents metal concentration (1/cc) ranging from \( 10^{14} \) to \( 10^{18} \), and the y-axis represents \( \tau \) in seconds ranging from \( 10^{-10} \) to \( 10^{-6} \). The graph shows an inverse relationship, with lower metal concentration resulting in a longer \( \tau \).]
In general (but not always) it is good to have minimal recombination: Minority carriers live longer, go further.

Would then want highest purity Si that the product could afford:

**Integrated Circuits:** Want $\tau = 10 - 100 \ \mu\text{sec} \Rightarrow$ metals $10^{13} - 10^{14} / \text{cc} \sim 1 \text{ part per billion}$

**Solar Cells:** Want $\tau = 10 - 100 \ \text{msec} \Rightarrow$ metals $10^{10} - 10^{11} / \text{cc} \sim 1 \text{ metal per } 10^{12} \text{ Si atoms !!}$

*Why workers in Si technology are PARANOID about gold contamination!*

*Also why Si technology waited until 1960's: Took that long to figure out how to purify Si!*

Will use this input to get a handle on the tau's appearing in the minority carrier continuity equations.

How to simplify and apply the "Minority Carrier Continuity Equations"
Simplification #1: "Steady-State"  Everything has settled in and is now ~ constant in time

Consider P-silicon where the minority carrier = n

\[
\frac{d}{dt}\delta n = 0 = \mu \cdot \xi \cdot \frac{d}{dx}\delta n + D \cdot \frac{d^2}{dx^2}\delta n - \frac{\delta n}{\tau} + G_{\text{non-thermal}}
\]

Assume I now shine high energy (UV) light onto one end of a P-silicon bar:

Chose UV light because its energy is much greater than Si's bandgap => is absorbed very quickly

Assume a delta function of added carriers ~ at left end of Si bar
$G_{\text{non\_thermal}}$ delta function blip of extra electrons $\sim$ at left end surface

Solve for region to *deeper* in the bar: $\sim$ left end to deep into bar

Simplification #2: $G_{\text{non\_thermal}} = 0$ AWAY from surface, light already absorbed $\Rightarrow$ no non-thermal generation

$0 = \mu_n \cdot \xi \frac{d}{dx} \delta n + D_n \cdot \frac{d^2}{dx^2} \delta n - \frac{\delta n}{\tau_n}$

Simplification #3: $\xi$ electric field $\sim 0$ There is no applied field! Only small field from imbalance of carriers

$0 = D_n \cdot \frac{d^2}{dx^2} \delta n - \frac{\delta n}{\tau_n}$ Reduced original 5 terms down to 2 terms: Equation now solvable!

**ELIMINATION OF TERMS = GENERAL SOLUTION STRATEGY**

$0 = \frac{d^2}{dx^2} \delta n - \left( \frac{1}{D_n \cdot \tau_n} \right) \cdot \delta n$ Define "Diffusion Length" $= \sqrt{D_n \cdot \tau_n}$
\[ 0 = \frac{d^2\delta n}{dx^2} - \frac{1}{L_n^2} \cdot \delta n \]

Look past strange \( \delta n \) - You *know* the solution to this PDE!

\[
\delta n = A \cdot e^{-\frac{x}{L_n}} + B \cdot e^{\frac{x}{L_n}}
\]

Where \( A \) and \( B \) are as yet arbitrary constants

If the bar is VERY long (on scale of diffusion length), second term would blow up at large \( x \)

So for long bar must have \( B = 0 \)

\[
\delta n(x) = A \cdot e^{-\frac{x}{L_n}} \quad \text{then} \quad \delta n(x=0) \Rightarrow A \quad \text{so:} \quad \delta n(x) = \delta n(0) \cdot e^{-\frac{x}{L_n}}
\]

From intensity of UV light I cold figure out what the concentration of excess electrons was in that surface blip

Putting it all together, expect:
Let's work this case with some real numbers to see values and test assumptions

Assume P-type silicon doped with

\[ N_a := 1 \times 10^{18} \frac{1}{cc} \]

(and negligible \( N_d \)) at \( T := 300K \)

Say that absorption of UV light a surface adds:

\[ \delta n_{at-0} := 10^9 \frac{1}{cc} \]

Because light promotes electrons from valence band to conduction band it creates new electrons and holes in pairs

\[ \delta p_{at-0} := 10^9 \frac{1}{cc} \]

First question: Do we indeed have the "low level" injection we assumed?

Doping is modest so we can assume all of the acceptors ionize creating holes

\[ p_o := N_a \quad p_o = 1 \times 10^{18} \frac{1}{cc} \]

then using Si value of:

\[ n_i := 1.5 \times 10^{10} \frac{1}{cc} \]

\[ n_o := \frac{n_i^2}{p_o} \quad n_o = 225 \frac{1}{cc} \]
Low level injection requires that change in MAJORITY carrier be small:

\[ p_0 = 1 \times 10^{18} \frac{1}{\text{cc}} \quad \delta p_{\text{at}_0} = 1 \times 10^9 \frac{1}{\text{cc}} \]

\( \delta p \) is as large as it gets at the surface, \( x = 0 \)

So indeed \( \delta p \ll p_0 \) and have low-level injection

Moving on, need to know what the value of the "Diffusion Length" is

"Diffusion Length" = \( \sqrt{D \cdot \tau} \)

Say we have a ~ typical piece of IC grade Si that has minority carrier lifetime: \( \tau_n := 10^{-6} \cdot \text{sec} \)

To get \( D \) we have to recall Einstein

\[ \frac{D_n}{\mu_n} = \frac{k_B \cdot T}{q} \]

\( k_B := 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}} \)
\( q := 1.6 \cdot 10^{-19} \cdot \text{coul} \)
\( \text{micron} := 10^{-6} \cdot \text{m} \)

TEST ALERT: Misunderstanding of following = One of John's Most Popular Test Errors

Need the mobility of our electrons in this P-type bar

Which curve to use and at what value?
Want ELECTRON mobility so use red curve

Total Ion Concentration?

\( n_o \)? NO! Electrons are not ions!

\( p_o \)? NO! Holes are not ions!

Only ions are the acceptors \( N_a^- \) that created the holes

Total Ionized Ion Concentration = \( N_a = 10^{18}/cc \)
Only because $N_a$ concentration was moderate, could we assume ALL $N_a$'s ionized creating holes

That (and only that) gave us the result that $p$ had the same value of $10^{18}$/cc

Would NOT have been the case if $N_a$ were high (e.g. $10^{19}$/cc) and did not all ionize

Would NOT have been the case if $N_a$'s were partially compensated by $N_d$'s

Horizontal axis is NOT carrier concentration - Make sure you understand why !!!

\[ \mu_n (\text{at } 10^{18} \text{ ions/cc}) = 270 \text{ cm}^2 / \text{V-s} \quad \text{NOT } 1350 \text{ cm}^2 / \text{V-s}! \quad \text{NOT } 400 \text{ cm}^2 / \text{V-s}! \quad \text{NOT } 130 \text{ cm}^2 / \text{V-s}! \]

Going back to Einstein using:

\[ \mu_n := \frac{270 \text{ cm}^2}{\text{V-s}} \]
\[ D_n := \mu_n \left( \frac{k_B T}{q} \right) \]
\[ D_n = 6.986 \frac{\text{cm}^2}{\text{sec}} \]

\[ L_n := \sqrt{D_n \cdot \tau_n} \]
\[ L_n = 2.643 \times 10^{-5} \text{ m} \quad \text{or with more common unit: } L_n = 26.432 \text{ micron} \]

Justifies throwing away growing exponential term (setting $B=0$) because bar could easily be long compared to $L_n$

Bar would be "very" long even if only 1mm in length ( = 39 times $L_n$)
So for our $10^{18}$ / cc P-type Si bar, with light-produced "excess" carrier concentration of $10^9$ / cc at left surface:

$$\delta n(\text{per cc})$$

$$\delta n(x) = \left(10^9 \cdot \frac{1}{\text{cc}}\right) \cdot e^{-\frac{x}{L_n}}$$

$L_n = 26 \ \mu m$

What about our assumption that could throw away term involving electric field $\xi$?

$$0 = \mu_n \cdot \xi \cdot \frac{d}{dx} \delta n + D_n \cdot \frac{d^2}{dx^2} \delta n - \frac{\delta n}{\tau_n}$$

Threw away

$$\frac{\text{Term}_1}{\text{Term}_2} = \frac{\mu_n \cdot \xi \cdot \frac{d}{dx} \delta n}{D_n \cdot \frac{d^2}{dx^2} \delta n} = \left(\frac{\mu_n}{D_n}\right) \cdot \xi \cdot \left(\frac{q}{k_B \cdot T}\right) \cdot \xi \cdot \frac{d}{dx} \delta n$$

So approximation was actually that Term 1 $\ll$ Term 2 or 3
Because $\delta n(x) = A \cdot e^{-x/L_n}$, the effect of a derivative is to just kick down factor of $-1/L_n$.

\[
\frac{\text{Term}_1}{\text{Term}_2} = \left( \frac{q}{k_B \cdot T} \right) \cdot \xi \cdot \left( \frac{-1}{L_n} \right) \cdot \delta n \cdot \left( \frac{1}{L_n^2} \right) = \left( \frac{q}{k_B \cdot T} \right) \cdot \xi \cdot (-L_n)
\]

Have already calculated everything in final expression EXCEPT for the electric field $\xi$.

If we knew the charge profile we could calculate $\xi$ from Gauss's law.

**What DO we know of charge profile?**

\[\rho = \delta p - \delta n\]

We KNOW:  1) Surface values of each (they are the same)  2) The full electron profile (red)
Taking that difference by eye:

\[ \rho = \delta p - \delta n \]

Need to know hole (blue) profile

Assume worst case (most charge): Holes = delta blip at surface

\[ \rho = \delta p - \delta n \]

\[ \approx \text{Negative of known excess electron profile} \]
Hauling out Gauss’s Equation

\[ \nabla \cdot \xi(x) = \frac{\rho}{\varepsilon} \quad \text{or now:} \quad \frac{d}{dx} \xi(x) = -\frac{q \cdot \delta n(x)}{\varepsilon} \]

Integrate both sides to \(+ \infty\)

\[ \int_{0}^{\infty} \frac{d}{dx'} \xi(x') \, dx' = \int_{0}^{\infty} \frac{-q \cdot \delta n(x')}{\varepsilon} \, dx' \]

Using fact that \(\xi(\infty)\) must be 0

\[ \xi(0) = -\frac{q}{\varepsilon} \int_{0}^{\infty} \delta n(0) \cdot e^{-\frac{x'}{L_n}} \, dx' = \frac{q \cdot L_n}{\varepsilon} \cdot \delta n(0) \]

This is maximum of \(\xi\) so is again worst case

\[ \frac{\text{Term}_1}{\text{Term}_2} = \left( \frac{-q}{k_B \cdot T} \right) \cdot \xi(x) \cdot L_n < \left( \frac{-q}{k_B \cdot T} \right) \left( \frac{q \cdot L_n}{\varepsilon} \cdot \delta n(0) \right) \cdot L_n \]

To evaluate Mathcad still needs:

Permittivity of free space \(\varepsilon_o := 8.85 \cdot 10^{-14} \text{ farad/cm}\)

Relative dielectric constant of Si \(k_{Si} := 11.7\)
Then \( \varepsilon := k_{Si} \cdot \varepsilon_0 \) and ratio of 1st (neglected) term to others becomes:

\[
\left( \frac{q}{k_B \cdot T} \right) \left( \frac{q \cdot L_n}{\varepsilon \cdot \delta n_{at_0}} \right) \cdot L_n = 0.042 \quad \text{YES is small - especially as I made worst case assumptions}
\]

WAS justified in throwing away \( \xi \) containing term = Validation of final simplification made