Last Time: Movement of Carriers = "Carrier Transport"

1) "DRIFT" = Movement induced by electric field, $\xi$

Scattering Limited:

\[
\text{"Carrier Mobility"} = \frac{v_{ave}}{\xi} = \frac{q \cdot \tau}{m_{eff}}
\]

Two Scattering Sources:

a) Crystalline Nuclei ("Lattice Scattering")

b) Ionized Impurities

Dominated by Ionized Impurity Scattering
(from ionized donor & acceptor impurities)

Dominated by Lattice Scattering
Valid at low to moderate $\xi$ field intensities

At very high $\xi$ fields, scattering increases and velocity "saturates" $\Rightarrow \mu$(high field) is smaller

(more about this later)

How does one measure mobility? With the "Hall Effect"

$$\text{mobility} = \frac{v_{\text{ave}}}{\xi}$$

So want to measure the average carrier drift velocity vs. $\xi$ field intensity

Use this set up:

Applied voltage forces charges along bar

Immersed in perpendicular magnetic field

"Lorentz Force" equation kicks in:

$$\mathbf{F} = q \cdot (\mathbf{v} \times \mathbf{B})$$

That is:
\( \xi_{\text{applied}} \) (induced along the bar by \( V_{\text{applied}} \)) drives charge along bar

- Velocity along bar interacts with \( B_{\text{perpendicular}} \)
- Force pushes charge towards front surface of bar (out of page)
- Builds up charge dipole side to side (= \( V_{\text{induced}} \))

\[ V_{\text{induced}} = \text{"Hall Voltage" } \alpha \text{ velocity along bar} \]

So use it to calculate that velocity:

\[
\frac{v_{\text{along}}}{\left( \frac{V_{\text{applied}}}{\text{Length}} \right)} = \frac{v_{\text{along}}}{\xi_{\text{applied}}} = \mu
\]

From known values of \( V_{\text{applied}}, B, \) and length along with measurement of \( V_{\text{induced}} \) can infer velocity, and hence mobility, of the carrier

(full derivation Neaman section 5.4)

Derivations and measurements above give full expression for \( \xi \) field induced DRIFT of holes and electrons:

\[
J_{\text{drift}} = q \cdot (\mu_p \cdot \mathbf{p} + \mu_n \cdot \mathbf{n}) \cdot \xi
\]

SECOND POSSIBLE SOURCE OF CURRENT:  Spontaneous redistribution of carriers:
2) DIFFUSION of carriers

At $t=0$, start with a blip of electrons

What happens at $t > 0$?

Obviously, it is going to spread out!

To quantify, look at a small section of the above distribution:

$n_1$ and $n_2$ approximate by steps.
What is the net "flux" (flow) of electrons across the boundary at \( x_0 \)?

To simplify, choose \( L = \) average distance an electron travels between collisions:

\[
L = v_{\text{thermal}} \cdot \tau_{\text{scattering}}
\]

With this assumption, during interval \( t = 0 \) to \( t = \tau_{\text{scattering}} \) only carriers within \( L \) of \( x_0 \) can reach the boundary.

(\text{so don't have to worry about carriers from volumes farther to the left or right})

**Net Flow in positive x direction / Interfacial Area** = Flux at \( x = \phi(x) \)

\[
\phi(x_0) = \text{Flux from volume 1} - \text{Flux from volume 2}
\]

\[
= \frac{1}{2} \cdot \frac{(\text{Number in 1 per area})}{\tau} - \frac{1}{2} \cdot \frac{(\text{Number in 2 per area})}{\tau}
\]

\[\text{Factor of } \frac{1}{2} \text{ because 50% move away from boundary!}\]
\[
\frac{1}{2 \cdot \tau} \cdot (n_1 \cdot L) - \frac{1}{2 \cdot \tau} \cdot (n_2 \cdot L)
\]

\[
\frac{L}{2 \cdot \tau} \cdot (n_1 - n_2)
\]

But \( L \) is small so can express \( n_2 \) from linear extrapolation of \( n_1 \) and slope:

\[
n_2 = n_1 + L \cdot \frac{d}{dx} n
\]

Put this into \( n_1 - n_2 \) above:

\[
\phi(x) = \left( \frac{L^2}{2 \cdot \tau} \right) \cdot \frac{d}{dx} n(x)
\]

\[
\phi(x) = -D_n \cdot \frac{d}{dx} n(x)
\]

\( D_n \) is the "Diffusion Constant"

\[
D_n = \frac{L^2}{2 \cdot \tau} = \frac{\text{cm}^2}{\text{second}}
\]

\( \tau \) = average time between scattering

\( L \) = average distance between scattering
Things diffuse (spontaneously rearrange) from regions of high concentration to low concentration.

**NOTE:**

*Diffusion moves things downhill*  \(\Rightarrow\) Flow is proportional to NEGATIVE of concentration slope!!

In general, scattering times and lengths are different for holes and electrons (no surprise) so:

\[
D_n = \frac{L_n^2}{2 \cdot \tau_n} \quad \text{does not equal comparable hole quantity:} \quad D_p = \frac{L_p^2}{2 \cdot \tau_p}
\]

Finally, remembering that current density is just flux multiplied by charge, get:

\[
J_{\text{diffusion}} = q \cdot \text{Flux}_{\text{holes}} + (-q) \cdot \text{Flux}_{\text{electrons}}
\]

\[
= q \left( -D_p \frac{d}{dx} p \right) + (-q) \left( -D_n \frac{d}{dx} n \right)
\]

\[
= -q \cdot D_p \frac{d}{dx} p + q \cdot D_n \frac{d}{dx} n
\]

**NOTE SIGNS!**

P term is negative!
Signs are opposite and strange because:

- FLUX is always \(- D \frac{d}{dx} \) (concentration)
- electron charge = -
- hole charge = +

\[ J_{\text{diffusion}_n} = (-)(-) = + \]
\[ J_{\text{diffusion}_p} = (-)(+) = - \]

Combine these new DIFFUSION currents with last lecture’s DRIFT currents to get TOTAL currents:

\[
J_{\text{total}_n} = q \cdot \mu_n \cdot n \cdot \xi + q \cdot D_n \cdot \frac{d}{dx} n \\
J_{\text{total}_p} = q \cdot \mu_p \cdot p \cdot \xi - q \cdot D_p \cdot \frac{d}{dx} p
\]

Again, note the single negative sign for the hole diffusion current!

We have now admitted the possibility that \( \xi \), the electric field, may not be zero

But: \( \xi = -\nabla V \) where \( V \), the voltage = Potential Energy / Charge

Thus a non-zero \( \xi \) implies a gradient in the potential energy
But don't our bands denote potential energy? Yes (for negative electrons)

a) If $\xi = 0$, then $V = \text{constant}$ then bands should be constant

b) If $\xi > 0$ then $V$ has a gradient then bands should be sloped

$E_i = \text{intrinsic Fermi level}$

Proved in lecture 8 that is $\sim \text{midgap}$

For here on, will assume is \textit{exactly} midgap

$E_i = \frac{E_c + E_v}{2}$

No drift

Electric field must induce tilt in the bands!

Indeed, slopes at left would produce

movement in direction expected for $\xi$ field
Voltage = Potential Energy / Charge

(Charge) Voltage = Potential Energy

But bands = Potential Energy (give or take a constant)

For electrons:

\[ V(x) = \frac{-1}{q} \cdot (E_i + \text{constant}'') \]

\[ \xi(x) = -\nabla V(x) = -\frac{d}{dx} \left[ \frac{-1}{q} \cdot (E_i + \text{constant}') \right] = \frac{1}{q} \cdot \frac{d}{dx} E_i \]  

(Equation 1)

\[ \xi(x) = \frac{1}{q} \cdot \frac{d}{dx} E_i \]

Relationship between electric field and bands: \( \xi = (1/q) \) Band Slope
Go back to total current expressions:

\[ J_{\text{total}_p} = q \cdot \mu_p \cdot p \cdot \xi - q \cdot D_p \cdot \frac{d}{dx} p \]
\[ J_{\text{total}_n} = q \cdot \mu_n \cdot n \cdot \xi + q \cdot D_n \cdot \frac{d}{dx} n \]

Where mobilities \( \mu = q \cdot x \text{ (scattering time)} / \text{(effective mass)} \)
And diffusivities \( D = \text{(scattering length)}^2 / \text{(scattering time)} \)

Sort of suggests might be a relationship!

But must be very careful, scattering lengths and times are all volume averaged quantities

Rigorous way of deriving relationship between \( \mu \) and \( D \): Consider a bar on non-uniformly doped semiconductor

Donor concentration increasing steadily to right of bar

(exact profile doesn't matter)
Further, for simplicity, assume:

i) No acceptors

ii) $N_d$ low enough that all donors ionize ("non-degenerate")

Then $n = N_d^+(x) \sim N_d(x)$ and must also have:

But from equation for $E_i$ (lecture 8) and:

$$n(x) = N_C \cdot e$$

Can show that will also have:

$$n(x) = n_i \cdot e$$

So if $n$ increases to the right, $E_F - E_i$ must ALSO increase to the right:

In band diagram have invoked the fact that the bar is in equilibrium (after all, it is just sitting there alone)

In equilibrium, $E_F$ (electron filling level) must be constant
If not, would be net rearrangement of electrons => net current => not equilibrium

THEN band diagram given is the only way to get the necessary $E_F - E_i$ increase to right

Makes sense, but then have gradient in $E_i$

$\nabla E_i(x)$ does not equal zero, then from equation 1 above:

$$\xi(x) = \frac{1}{q} \cdot \nabla E_i(x) = \text{non-zero}$$

Doping gradient implies non-zero electric field in bar!

But if there is an electric field, we will have a drift current density

Current? How can this be at equilibrium?

Equilibrium requires that TOTAL current (drift plus diffusion) be equal to zero:

$$J_{\text{total,n}} = q \cdot \mu_n \cdot n \cdot \xi + q \cdot D_n \cdot \frac{d}{dx} n$$

First and second terms (drift and diffusion currents) must cancel!

OK, but let's first diddle a bit with the second (diffusion) term. From above:

$$n(x) = n_i \cdot e^{\frac{(E_F-E_i)}{k_B \cdot T}}$$

Differentiate as required in second term:
\[ \frac{d}{dx} n(x) = \frac{n_i}{k_B T} \cdot e^{\frac{(E_F - E_i)}{k_B T}} \cdot \frac{d}{dx}(E_F - E_i) \]

But \( E_F \) is constant (the equilibrium argument)

Term in square brackets is just \( n(x) \) and

last term \( \frac{d}{dx} E_i = q \cdot \xi \) (equation 1 again)

\[ \frac{d}{dx} n(x) = \frac{-1}{k_B T} \cdot \left[ \frac{(E_F - E_i)}{k_B T} \right] \cdot \frac{d}{dx} E_i \]

Put this into second term of total electron current equation (which had to equal zero):

\[ 0 = J_{\text{total}_n} = q \cdot \mu_n \cdot n \cdot \xi + q \cdot D_n \cdot \left( \frac{-q \cdot n}{k_B T} \cdot \xi \right) \]

collect terms a bit

\[ 0 = q \cdot n \cdot \xi \cdot \left( \mu_n - \frac{q \cdot D_n}{k_B T} \right) \]

neither \( q \) nor \( n \) nor \( \xi \) out front are zero, so term in ( ) must be zero:

\[ \mu_n = \frac{q \cdot D_n}{k_B T} \quad \text{also works for holes:} \quad \mu_p = \frac{q \cdot D_p}{k_B T} \]

Or generalizing for either carrier type:
Way of remembering - Think of it as an engineer's lame attempt at poetry: "D over \( \mu \) equals \( k_B \cdot T \) over \( q \)"

Don't Laugh!

*Got e-mail in 2002 from student thanking me for rhyme which allowed him to remember the "Einstein Relationship" during his Ph.D. qualifying exam!*

What is really going on in bar? Imagine we introduced donors atoms instantaneously at \( t=0 \)

Donor and electron populations would be precisely equal
But there is a gradient in the electron population => Diffusion current to left

Reaches equilibrium:

- $n(x)$ some of which have diffused to left
- $N_d^+(x)$ which can't move

Is now an excess of negative charge at left
And excess of positive charge at right

CREATES electric field to left!

DIFFUSION of electrons to left (down gradient) balanced by electric field driven DRIFT to right

1) Einstein Relationship between D and $\mu$ is ultimately based on common underlying scattering mechanisms

2) If know $\mu$ for a carrier (n or p) can use Einstein Relationship to get D for that carrier
   - Can use relatively easy "Hall Effect" measurement (beginning of lecture) to get $\mu$'s
   - Then calculate the D's - which are MUCH harder to measure directly!
Could almost have derived from definitions and straight algebra:

\[ \mu = \frac{q \text{ (scattering time)}}{\text{(effective mass)}} \quad D = \frac{(\text{scattering length})^2}{\text{(scattering time)}} \]

But \( L = v \tau \) where \( v \) is the thermal velocity of the carriers

And kinetic energy of carrier is \( \sim \frac{3}{2} kT \)

\[ \frac{1}{2} m \cdot v^2 = \frac{3}{2} k_B \cdot T \quad \text{so:} \quad v^2 = \frac{3 k_B \cdot T}{m} \quad \text{thus get:} \quad L = \sqrt{\frac{3 k_B \cdot T}{m}} \cdot \tau \]

Incorrect factor of 3 appears because did not do proper geometric averaging of \( l's \) and \( v's \)

OK, but if don't trust theoretical derivation, is there also a way of measuring \( D's \) so can confirm?

Yes (of course): There is the "Shockley - Haynes" Experiment (or recent PC: "Haynes - Shockley")
V_{DC} Drift inducing voltage

Semiconductor Bar

Narrow pulse of electrons

Current Pulser (input)

Current sensing oscilloscope (output)

Pick up lead of oscilloscope can be shifted to left or right on bar (closer or farther from pulse injection point)

Oscilloscope Output:

Probe to left

Probe center

Probe to right

Spread vs. position => D value!

Very tricky: I've done once vs. made 100's of Hall Effect $\mu$ measurements!