Today: Calculate current density, $J$, across the diode as a function of the applied voltage

Might expect: Start with $n_0$ and $p_0$ => calculate chance of crossing, add velocities => current

No, doesn't work out that way: Carriers can also come BACK across junction

So instead of firing carriers across junction, we need to work out the balance of flows both ways

"Balances" => use of equilibrium relationships including Fermi functions

1) Assume "Steady-State"

$V_{\text{applied}}$ constant in time (or has been at present value a "long" time)

2) Assume "Low-Level Injection"

Despite movement of carriers, assume MAJORITY carrier concentrations are not significantly affected
Can apply Fermi distributions to find how many have enough energy to make across step (ones w/ arrows)

Consider electrons above at right

\[
\text{Number}_c(E_c \text{ Step}) = \frac{f_F(E_c + \text{Step})}{f_F(E_c)} \times \text{Boltzman}(E_c + \text{Step}) \]

Boltzman Function = tail of \( f \), \( e^{-\Delta E/kT} \)

\[
\text{Number}_c(E_c \text{ Step}) = e^{-\frac{\text{Step}}{kT}}
\]

But:
1. Step = \( V_{bi} - V_{applied} \)
2. Number at \( E_c \) ~ all ~ \( n_0 \)

\[
\text{Number}_c(E_c \Rightarrow V_{bi} - V_{applied}) = n_0 e^{-\frac{q(V_{bi} - V_{applied})}{kT}}
\]

This gives us a handle on how many carriers will have enough thermal energy to make it across the barrier

Final assumption:

Assumption 3) All carriers that start across the junction make it to the other side

Makes sense for MINORITY carriers:

Electric field pushes minority carriers across (that is what the fall in the bands indicates)

Will move so fast have no time to recombine!

MAJORITY Carriers: Because minority carriers (above) whip across junction, there are \( \sim 0 \) there at any time

~ Nothing for majority carriers to recombine WITH! Majority also likely to make

Similarly for the energetic hole above (one with arrow)

\[
\text{Number}_h(E_c \Rightarrow V_{bi} - V_{applied}) = \frac{q(V_{bi} - V_{applied})}{kT}
\]

\[
\text{Number}_h(E_c \Rightarrow V_{bi} - V_{applied}) = p_0 e^{-\frac{q(V_{bi} - V_{applied})}{kT}}
\]

This gives us a handle on how many carriers will have enough thermal energy to make it across the barrier

Final assumption:

NOTATION BREAK - New notation to better keep track of things: \([\text{carrier}][\text{where it is}]\)

<table>
<thead>
<tr>
<th>carrier</th>
<th>where it is</th>
</tr>
</thead>
<tbody>
<tr>
<td>Majority carriers:</td>
<td>( p_n ) = Holes on P-side</td>
</tr>
<tr>
<td></td>
<td>( n_n ) = Electrons on N-side</td>
</tr>
</tbody>
</table>

If ~ all carriers make it across => same density of electrons on both sides / same density of holes on both sides

\[
\text{num}_p(p_n - x_p) = \text{num}_n(p_n - x_n)
\]

\[
\text{num}_p = \frac{q(V_{bi} - V_{applied})}{kT}
\]

\[
\text{num}_n = \frac{q(V_{bi} - V_{applied})}{kT}
\]
Reiterate arguments (for electrons)

\[ n_p(-x_p) = \text{Value of MINORITY electron concentration at edge of undepleted P-region} \]

= Boundary value that we need to solve for the entire electron distribution on left \( \Rightarrow J_n \)

From above (or its P-type analogy):

\[
\begin{align*}
\text{P-side:} & \quad n_p(-x_p) = \frac{-q(V_{bi} - V_{\text{applied}})}{kT} \\
\text{N-side:} & \quad n_n(x_n) = \frac{-q(V_{bi} - V_{\text{applied}})}{kT}
\end{align*}
\]

(form 1) (equation 2)

That is, if \( V_{\text{applied}} = 0 \), concentration of carrier steps down by \( e^{-qV_{bi}/kT} \) as cross junction

This is also embedded in our definition of \( V_{bi} \):

\[
V_{bi} = \frac{kT}{q} \ln \left( \frac{N_a N_d}{N^2} \right)
\]

\( N_a \) is acceptor concentration on P-side \( \Rightarrow p_{po} \)

\( N_d \) is donor concentration on N-side \( \Rightarrow n_{po} \)

\[
V_{bi} = \frac{kT}{q} \ln \left( \frac{p_{po} n_{po}}{n^2} \right) = \frac{kT}{q} \ln \left( \frac{n_{po}}{n^2} \right) = \frac{kT}{q} \ln \left( \frac{n_{po}}{n^2} \right)
\]

but: \( n_{po} = n^2 \) so get:

\[
V_{bi} = \frac{kT}{q} \ln \left( \frac{n_{po}}{n^2} \right) = -\frac{qV_{bi}}{kT}
\]

reproducing equation 3 above

Plug equations 3 and 4 into equations 1 and 2, respectively
Remember the "Minority Carrier Continuity Equations". Time to put them to truly useful work:

\[
\frac{d\delta n_p}{dt} = \frac{d}{dx} \left( \mu_n n_p \delta n_p \right) - \frac{\delta n_p}{\tau_n} + G_{\text{non-thermal}}
\]

Outside of the junction (to left and right where there are still carriers) make three simplifications:

1. Steady-State: \( \frac{d}{dt} \approx 0 \)
2. Small Electric Field: \( \xi \approx 0 \)
3. No non-thermal generation sources: \( G \approx 0 \)

So when solve outside depletion region, can use:

**P-side:**
\[
D_n \frac{d^2\delta n_p}{dx^2} - \frac{\delta n_p}{\tau_n} = 0
\]

**N-side:**
\[
D_p \frac{d^2\delta n_n}{dx^2} - \frac{\delta n_n}{\tau_p} = 0
\]

So we have equations describing minority carriers AND from law of junction, we know values at edges:

**SOLUTIONS:**

\[
\delta n_p(x) = A \cdot e^{L_n \cdot \frac{x_p}{x}} - B \cdot e^{-L_n \cdot \frac{x_p}{x}}
\]

\[
\delta n_n(x) = C \cdot e^{L_p \cdot \frac{x_n}{x}} - D \cdot e^{-L_p \cdot \frac{x_n}{x}}
\]

Where:

\( L_n = \sqrt{\frac{D_n}{\tau_n}} \) "electron diffusion length"

\( L_p = \sqrt{\frac{D_p}{\tau_p}} \) "hole diffusion length"

"Thick Diode" or more precisely, "Thick Layer" Case

If layers are thick compared to diffusion lengths:

- One term falls to zero
  - Makes sense: carriers diffuse deeper and recombine
- One term goes to infinity
  - Makes no sense: minority carriers increasing away from junction

Solution is to set coefficient of growing terms equal to zero (B=0 and D=0 above)

**P-side** (\( x<0 \)): \( \delta n_p(x) = A \cdot e^{\frac{x_p}{L_n}} \)

**N-side** (\( x>0 \)): \( \delta n_n(x) = C \cdot e^{\frac{x_n}{L_p}} \) Thick Layers
And we KNOW the values at the edges of the regions from the "Law of the Junction"

P-side (x = -x_p):
\[ \delta n_p(x = -x_p) = A_p = n_p(-x_p) - n_{po} = n_p e^{\frac{q V_{applied}}{kT}} - n_{po} \]

N-side (x = x_n):
\[ \delta n_n(x = x_n) = A_n = n_n(x_n) - p_{no} = n_n e^{\frac{q V_{applied}}{kT}} - 1/e \]  

Yielding for thick layers:

P-side (x<0):
\[ \delta n_p(x) = n_{po} \left( \frac{q V_{applied}}{kT} - 1 \right) / e \]  

N-side (x>0):
\[ \delta n_n(x) = p_{no} \left( \frac{q V_{applied}}{kT} - 1 \right) / e \]  

Similarly for holes on right side of junction:
\[ J_p(x > x_n) = q D_p \frac{p_{no}}{L_p} e^{\frac{q V_{applied}}{kT} - 1} / e \]  

Solve both of the above right at the edges of their regions:  
\[ J_n(x = -x_p) \quad J_p(x = x_n) \]

Finally, invoke assumption that ~ all carriers entering junction make it across:
\[ J_n(x = -x_p) + J_p(x = x_n) = \text{J total} \]

Further, form original assumption of steady-state, reasoned that J_total must be same throughout!
\[ J = J_n(x = -x_p) + J_p(x = x_n) \]  

Can now quickly get the currents: Already assumed \( \xi = 0 \) outside junction, so only diffusion current is large

\[ J_{diffusion_n}(x < -x_p) = q D_n \frac{d}{dx} n_{po} = q D_n \frac{d}{dx} n_{po} \cdot \delta n_p = q D_n \frac{d}{dx} n_{po} \]  

plugging in equation 5:

\[ J_{diffusion_n}(x < -x_p) = q D_n \frac{d}{dx} n_{po} \left( \frac{q V_{applied}}{kT} - 1 \right) / e \]  

\[ J_n(x < -x_p) = q D_n \frac{d}{dx} n_{po} \left( \frac{q V_{applied}}{kT} - 1 \right) / e \]  

\[ J_p(x > x_n) = q D_p \frac{p_{no}}{L_p} \left( \frac{q V_{applied}}{kT} - 1 \right) / e \]  

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