Quiz 3 for Math 231

Tuesday - February 7, 2006

Student's Name: KEY

Instructions: Show all your work for full credit. Indicate your answers clearly.

Problem 1. If a curve has the property that the position vector $\vec{r}(t)$ is always perpendicular to the tangent vector $\vec{r}'(t)$, show that the curve lies on a sphere with center at the origin.

$\vec{r}(t) \perp \vec{r}'(t) \Rightarrow \vec{r}(t) \cdot \vec{r}'(t) = 0.$

Let $d: \mathbb{R} \to \mathbb{R}$ $d(t) = |\vec{r}(t)|^2 = \vec{r}(t) \cdot \vec{r}(t).$

Then

$$d'(t) = 2\vec{r}(t) \cdot \vec{r}'(t) = 0 \text{ by hypothesis.}$$

This shows $d(t) \equiv c$ for some $c \geq 0$ and ?

Therefore $|\vec{r}(t)|^2 = c \geq 0.$

Hence $\vec{r}(t)$ describes a curve on the sphere with center $(0,0,0)$ and radius $\sqrt{c}$.