

Practice Exam 1 for Math 325

Wednesday - September 22, 2004

Student's Name: _____

Instructions: Show all your work for full credit. Indicate your answers clearly.

Problem # 1. Show that any separable equation $M(x) + N(y)y' = 0$ is also exact.

Problem # 2. Find a solution of the initial value problem

$$y' + \frac{2}{t}y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, \quad t > 0.$$

Problem # 3. Solve the initial value problem $y' = 2y^2 + xy^2$, $y(0) = 1$ and determine where the solution attains its minimal value.

Problem # 4. Solve the equation $y + (2x - ye^y)y' = 0$.

Problem # 5. Determine whether the following equation is exact:

$$ye^{xy} \cos(2x) - 2e^{xy} \sin(2x) + 2x + (xe^{xy} \cos(2x) - 3)y' = 0.$$

Problem # 6. Determine the values of r for which the equation $t^2y'' - 4ty' + 4y = 0$ has a solution of the form $y(t) = t^r$ for $t > 0$.

Problem # 7. Consider a population p of field mice that grows at a rate proportional to the current population, that is $\frac{dp}{dt} = rp$. Find the constant rate r if the population doubles in 30 days.

Problem # 8. True or False

- (a) $\frac{dy}{dt} + ty^2 = 0$ is second order differential equation.
- (b) $\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + 1 = 0$ is linear differential equation.
- (c) $u(x, y) = \ln(x^2 + y^2)$ is a solution of the partial differential equation $u_{xx} + u_{yy} = 0$.
- (d) A solution to the initial value problem $y' + \frac{1}{\ln t}y = \frac{\cot t}{\ln t}$, $y(2) = 3$, exists on the interval $(0, \pi)$.
- (e) $\mu(x) = xe^x$ is an integrating factor for $(x + 2) \sin y + (x \cos y)y' = 0$.